Valuation of ‘Razorback’ Executive Stock Options:

A Simulation Approach

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Abstract

Executive stock options with a time-varying strike price are a recent innovation in Australia and New Zealand. These options have a strike price that increases at a prespecified rate and also have a dividend protection feature that reduces the strike price by the amount of any dividend payment. With an upward trend and dividend-induced drops, the path of the strike price over time appears jagged. Hence the label ‘razorback’ options. Standard option pricing methodology is not easily applied to value razorback options, since the strike price is typically a path-dependent function of the stock price. For example, suppose the company pays a dividend yield as a constant percentage of the stock price. In this case, the cumulative dividend adjustment to the strike price depends on the particular path of the stock price. While analytic valuation appears intractable, these path-dependent options can be valued using a least squares Monte Carlo approach developed by Longstaff and Schwartz (2001). We examine the effects of changing volatility, maturity, vesting period and dividend yield on these executive stock option values. Our results indicate that valuations can differ quite significantly from those obtained from standard Black-Scholes valuations.

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1. Introduction

Performance and reward measures such as economic value added (EVA) that explicitly takes into account the cost of capital have gained worldwide acceptance.[1] Some well-known companies in Australia and New Zealand have extended this concept to the design of executive stock options. These executive stock options have two distinguishing features: 1) they are dividend-protected and, 2) they have a strike price that drifts upward by a prespecified amount usually related to a cost of equity estimate. Consequently, the strike price for these options follows a jigsaw pattern over time –steadily rising through option maturity but falling each time a dividend is paid. We refer to these executive stock options as 'razorback' options.[2] Companies that have issued razorback options include The Warehouse Limited, Sky City Entertainment Group, Pacific Dunlop Limited, Santos Limited and others.

While early exercise may not be optimal for standard dividend-protected call options, this may not be the case for razorback options. The continual increase in the strike price may trigger an optimal early exercise. The conditions for optimal early exercise are discussed in section 2, which also contains a brief discussion on the difficulties involved in using standard option pricing methodologies to value razorback options.

In section 3, we review the simulation methodology developed in Longstaff and Schwartz (2001) and apply the methodology to value razorback options. In actual practice as in, for example, financial statement reporting, these options are typically valued using the Black-Scholes (1973) formula. The Black-Scholes formula generally undervalues razorback options for two reasons: 1) it does not account for the path dependency of the dividend-adjusted strike price, and 2) it does not account for the optimal early exercise that is possible any time after vesting. We study the valuations effects of changing volatility, maturity, vesting period, and dividend yield on the size of the valuation bias in section 4. Our conclusions and a summary of our findings follow in section 5.

2. Optimal Early Exercise

2.1 Early exercise for a dividend-protected call option

It is well known that it may be optimal to exercise an American call option early when the underlying stock pays a dividend.[3] When a dividend is paid, the share price is

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[1] For example, see www.eva.com for a partial worldwide listing of companies that adopted the EVA measure.

[2] Razorback is an American nickname for a wild boar and refers to the pattern of bristles running down a boar's spine. The University of Arkansas fondly calls its football team the 'razorbacks.' Besides the executive options discussed in this paper, another example of a razorback option is endowment warrants traded on the Australian Stock Exchange (ASX).

[3] See Merton (1973).

expected to fall but the strike price of a standard American call option does not change. Therefore, the call option will lose some value after the dividend payment. In this case, the option holder may find it beneficial to exercise the option before the ex-dividend date so as to become a stockholder and capture the dividend.

In the case of razorback options, the incentive to exercise early is tempered since the strike price falls by the amount of the dividend. While the gain in option value from a lower strike price is generally less than the fall in option value due to a lower ex-dividend share price,[4] Geske et al (1983) shows why it is not optimal to exercise a dividend-protected American call option immediately before a dividend payment.[5] The argument is straightforward. Consider that the exercise value, i.e., the intrinsic value, of a dividend-protected option is the same both in the instant before and the instant after the ex-date for a known dividend payment. Thus an option holder is indifferent to exercise immediately before or immediately after an ex-date. Consequently, if early exercise is not optimal immediately after the ex-date, then it is also not optimal immediately before the ex-date.

2.2 Early exercise for a call option with an increasing strike price

The strike price of a razorback option grows at a constant rate, usually set to reflect a cost of equity estimate. When the strike price of an American call option grows at the constant rate g, there exists some value, say g^*, that may trigger an early exercise. Note that immediate exercise is triggered when the value of immediate exercise exceeds the value of a comparable European call option, i.e., $S - K > C(K^*\exp(g^*(T-t))$, where $C(K^*\exp(g^*(T-t))$ denotes the value of a European call with a strike price at maturity of $K^*\exp(g^*(T-t)$. So long as the option is in the money, i.e., $S - K > 0$, a sufficiently large g^* yields a strike price that will trigger immediate exercise.

However, based on a result in Margrabe (1978) the rate g^* must be larger than the riskless rate. Specifically, Margrabe shows that it is not optimal to exercise an American-style exchange option early.[6] A call option with a strike price that grows at the risk-free rate is simply a special case of an exchange option in which one of the two assets to be exchanged is non-stochastic. It follows that an American call option with a strike price that grows at the risk-free rate would not be exercised early. Hence, the riskless rate is a lower bound for the rate g^* that will trigger an early exercise.[7]

2.3 Early exercise for razorback options and valuation difficulties

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[4] For instance, in the case of the standard Black-Scholes formula, the share price is multiplied by $N(d_1)$ while the present value of the strike price is multiplied by $N(d_2)$. Therefore, an equal drop in the share price and the strike price does not generally have a neutral effect on the value of the call.

[5] Strictly speaking this is only true if the dividend is less than the exercise price. The contrary is highly unlikely for razorback options given that the exercise price increases at the cost of equity over time.

[6] An exchange option is the option to exchange one risky asset for another. The proof of this statement can be found in pp.180 of Margrabe (1978).

[7] Consider any g, say g', where g'<r. No early exercise when g=0 and g=r implies:

$$\max(0, S_t - K_t) < C(K_t e^{r(T-t)}) < C(K_t e^{g'(T-t)}) < C(K_t)$$

Clearly if $C(K_t e^{g'(T-t)})$ does not lie between $C(K_t e^{r(T-t)})$ and $C(K_t)$, there will be arbitrage opportunities. An arbitrageur can simply sell the higher price option that has the higher exercise price and use the proceeds to buy the lower price option with the lower exercise price to earn an arbitrage profit.

While the dividend-protection feature serves to preclude early exercise for a razorback option, the increasing strike price introduces the possibility of early exercise. Given that the cost of equity can be substantially higher, but not less than the riskless rate, razorback options could have a significant early exercise premium. This possibility of optimal early exercise in combination with a path-dependent strike price makes razorback options quite difficult to value analytically.

The strike price of a razorback option is path-dependent due to the dividend-protection feature. For instance, if dividends are paid as a constant yield on a stock price, the strike price at a given point in time will depend on the particular path the stock price has taken.[8] This path-dependency in the strike price makes it difficult, if at all possible, to derive a pricing formula for razorback options.[9] The problem becomes significantly more complicated when the possibility of early exercise is introduced.

Since an analytical solution for the value of razorback options presents intractable difficulties, we turn to numerical methods based on Monte Carlo simulations. Until recently, the Monte Carlo simulation approach has not been regarded as suitable for valuing American options.[10] However, the recently developed least squares Monte Carlo (LSM) approach suggested by Longstaff and Schwartz (2001) allows path-dependent American-style options to be valued via Monte Carlo simulations.

3. The Valuation Approach

3.1 The least squares Monte Carlo simulation (LSM) approach

The Monte Carlo simulation approach was originally used to estimate the conditional expectation of the payoff for a European option without the complications of early exercise.[11] In the case of an American option, it is necessary to compare the immediate exercise payoff on each decision date for each simulated price with the corresponding conditional expected payoff from continuation without exercise. It was not clear previously how the conditional expectation could be estimated for any given price path.

Longstaff and Schwartz (2001) propose a simple least squares Monte Carlo (LSM) approach to estimate these conditional expectations. The key insight is that the information required for estimating conditional expectations is already contained in the full set of simulated price paths. To process the information, the LSM approach uses a cross-sectional least squares regression on each decision date to estimate the functional relationship between the 'ex-post' realised payoffs and current prices. The

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[8] It is possible that the strike price will be adjusted downwards to zero for some extreme 'high' price paths. Since negative strike prices have no economic meaning, these possibilities also make it difficult to derive a pricing formula for razorback options.

[9] An alternative is to model dividends as non-stochastic. However, this approach is questionable, particularly given the medium- to long-term maturities of razorback options.

[10] For instance, see comments in Hull (2000) pp.408. that Monte Carlo simulation "… cannot easily handle situations where there are early exercise opportunities."

[11] The simulation and hence all conditional expectations are performed under the risk-neutral measure. See, for example, Boyle (1977).

fitted values from this cross-sectional regression provide estimates of the conditional expectations required to make the early exercise decision.

The LSM approach works as follows. On the last early exercise decision date, the set of discounted simulated payoffs at option maturity (the dependent variable) is regressed against the set of simulated prices (the independent variable). A least squares regression using the dependent variable and functions of the independent variable are used to generate fitted values.[12] These fitted values, or estimates of conditional expected payoffs, are then compared to the immediate exercise values at the corresponding prices. If a fitted value is lower than the immediate exercise value (i.e. when early exercise is optimal), the discounted payoff is replaced by the immediate exercise value. The procedures are repeated at each decision date recursively. The American option value is given by the average of discounted payoffs of the simulated price paths.

Table 1: Parameters for a representative razorback option	
Initial price	$1.00
Strike price	$1.00
Maturity	6 years
Volatility	25.00% p.a.
Interest rate	6.00% p.a.
Dividend yield	3% p.a.
Vesting period	2 years
Cost of capital	12% p.a.

3.2 Applying the least square Monte Carlo approach to value razorback options

We apply the LSM approach to value a hypothetical razorback option.[13] Representative parameters affecting option value are shown in Table 1. Option values are obtained from 50,000 thousand random price paths. Early exercise possibilities are checked recursively each month from maturity back to the end of the vesting period. Along with American-style option values, we also calculate corresponding European-style option values to assess the value of early exercise. As a standard benchmark, we also calculate option values using the Black-Scholes formula, in which expected dividends are based on a constant dividend yield. Thus the Black-Scholes formula used: is: $C = Se^{-yT}N(d_1) - Ke^{(g-y)T}e^{-rT}N(d_2)$.[14] Calculation results are presented in Table 2.

[12] A standard approach is to use a nth-degree polynomial fit using powers of the independent variable for some n. Also, while fitted values are calculated for the full set of data, the regression coefficients are estimated using only the set of values where the immediate exercise payoff is positive. See Longstaff and Schwartz (2001) for details.

[13] Since the strike price of a razorback option changes over time, discounted payoffs are regressed against intrinsic values of the option instead of just share prices. Also, third-degree Chebyshev polynomials are used in the least squares regression.

[14] Note that the terminal exercise price is simply $Ke^{(g-y)T}$ since a razorback option is assumed to be issued at-the-money and that dividends are assumed to be paid continuously. Also, in our simulations under the LSM approach, we assume dividends after the first 12 months to be a constant yield of the

Table 2: Razorback option values (simulation standard errors in brackets) obtained from 50,000 Monte Carlo experiments		
	LSM values	*Black-Scholes value*
European	\$0.1103 (0.0016)	\$0.1027
American	\$0.1320 (0.0012)	

As shown in Table 2, with an initial share price and strike price of \$1.00, the representative American-style razorback option has a value of 13.20 cents. The corresponding European-style option value is 11.03 cents, indicating an early exercise premium of 2.17 cents (with a 2-year vesting period), or 16.4 percent of the value of the option. Notice also that the Black-Scholes formula understates the value of the razorback option by 2.93 cents, or 22.2 percent. This example clearly indicates that both the effects of path dependency and the early exercise premium might lead to a significant valuation bias from the Black-Scholes formula. Consequently, the LSM valuation approach provides a notable improvement over the Black-Scholes approach in the valuation of razorback options. Immediately below, we investigate factors that lead to cross-sectional differences in the size of the valuation bias.

4. The Valuation Bias

It is a common practice to use the Black-Scholes formula to value executive stock options.[15] However, as suggested above, the Black-Scholes formula is a biased estimator of razorback option value. In this section, we investigate factors affecting variations in this bias. Specifically, we examine how variations in the following factors affect the valuation bias:

- Volatility
- Maturity
- Vesting period
- Cost of equity
- Dividend yield

The sensitivity of the valuation bias to changes in each variable is discussed below. All parameters other than the variable under consideration remain unchanged in accordance with values shown in Table 1. We define valuation bias to be the valuation error as a percentage of the value obtained from the LSM approach.

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average share price in the previous 12 months. The average share price in the first 12 months is calculated as the average share price in the previous available months.

[15] For instance, see the US requirements of Statement of Financial Accounting Standards No. 123 - "Accounting for Stock-Based Compensation".

Table 3: Effects of volatility on the Black-Scholes valuation bias			
Volatility	*LSM value*	*B-S value*	*Bias*
20%	$ 0.0913	$ 0.0648	29.0%
25%	$ 0.1320	$ 0.1027	22.2%
30%	$ 0.1715	$ 0.1427	16.8%
35%	$ 0.2154	$ 0.1834	14.9%
40%	$ 0.2609	$ 0.2241	14.1%
45%	$ 0.3013	$ 0.2643	12.3%

4.1 Volatility

As expected, Table 3 reveals that higher volatility leads to higher option value and makes early exercise relatively less attractive. While the absolute difference between the value of the razorback option and its Black-Scholes benchmark increases with higher volatility levels, the percentage difference becomes smaller. In Table 3, the valuation bias decreases from 29.0 percent to 12.3 percent as volatility increases from 20 percent to 45 percent.

Table 4: Effects of maturity on the Black-Scholes valuation bias			
Maturity (Years)	*LSM value*	*B-S value*	*Bias*
3	$ 0.1069	$ 0.0964	9.8%
4	$ 0.1169	$ 0.1004	14.1%
5	$ 0.1245	$ 0.1023	17.8%
6	$ 0.1320	$ 0.1027	22.2%
7	$ 0.1370	$ 0.1023	25.3%
8	$ 0.1395	$ 0.1011	27.5%
9	$ 0.1444	$ 0.0995	31.1%
10	$ 0.1472	$ 0.0976	33.7%

4.2 Maturity

Table 4 reveals that the valuation bias increases with the maturity of the option, from 9.8 percent to 33.7 percent as razorback option maturity rises from 3 years to 10 years. It is interesting to note that while razorback option values obtained via the LSM approach increase monotonically with maturity,[16] this is not the case for the Black-Scholes formula values. As shown in Table 4, Black-Scholes values increase with maturities up to 6 years and decline in value thereafter.

[16] The possibility of early exercise means that the razorback option value must be monotonic in terms of time to maturity. If the rising strike price actually lowers the European option value after some time, as it does in this example, the razorback option will be exercised early before the value starts to decline.

Table 5: Effects of vesting period on Black-Scholes valuation bias			
Vesting Period (Years)	*LSM value*	*B-S value*	*Bias*
0	$ 0.1377	$ 0.1027	25.4%
1	$ 0.1337	$ 0.1027	23.2%
2	$ 0.1320	$ 0.1027	22.2%
3	$ 0.1279	$ 0.1027	19.7%
4	$ 0.1238	$ 0.1027	17.0%
5	$ 0.1172	$ 0.1027	12.4%
6	$ 0.1097	$ 0.1027	6.4%

4.3 Vesting period

In Table 5, for our 6-year maturity razorback option, the valuation bias decreases from 25.4 percent to 6.4 percent as the vesting period increases from 0 to 6 years. A decreasing valuation bias is apparent since an American option becomes a European option when the vesting period lengthens to equal the option maturity. In that case, the remaining difference between the European razorback option value and the Black-Scholes value is due to stochastic dividends.[17]

Table 6: Effects of cost of equity on the Black-Scholes valuation bias			
Cost of Equity	*LSM value*	*B-S value*	*Bias*
10%	$0.1544	$ 0.1315	14.8%
11%	$0.1425	$ 0.1166	18.2%
12%	$0.1320	$ 0.1027	22.2%
13%	$0.1198	$ 0.0899	25.0%
14%	$0.1102	$ 0.0782	29.0%
15%	$0.1008	$ 0.0676	32.9%
16%	$0.0933	$ 0.0580	37.8%
17%	$0.0842	$ 0.0495	41.2%
18%	$0.0774	$ 0.0419	45.9%
19%	$0.0705	$ 0.0352	50.1%
20%	$0.0647	$ 0.0294	54.6%

4.4 Cost of equity qua growth rate of strike price

The discussion in section 2.2 suggested that early exercise might be optimal when the cost of equity is higher than the riskless rate. Indeed, Table 6 reveals that the higher the cost of equity determining the growth rate of the strike price, the larger is the valuation bias. Notice that the valuation bias increases from 14.8 percent to 54.6 percent as the cost of equity increases from 10 percent to 20 percent. Most of the

[17] See Table 2 in Section 3.2 above.

valuation bias in this case is due to the early exercise premium.[18] Thus it appears that the valuation bias and the value of the option are both highly sensitive to changes in the cost of equity.

Table 7: Effects of dividend yield on the Black-Scholes valuation bias

Dividend Yield	*LSM value*	*Simulated European*	*% Early exercise premium*	*B-S value*	*Bias*
0%	$ 0.1453	$ 0.1227	15.6%	$ 0.1230	15.4%
1%	$ 0.1402	$ 0.1169	16.6%	$ 0.1158	17.4%
2%	$ 0.1361	$ 0.1141	16.2%	$ 0.1091	19.8%
3%	$ 0.1320	$ 0.1091	17.4%	$ 0.1027	22.2%
4%	$ 0.1253	$ 0.1034	17.5%	$ 0.0968	22.8%
5%	$ 0.1238	$ 0.1019	17.7%	$ 0.0911	26.4%
6%	$ 0.1210	$ 0.0979	19.1%	$ 0.0858	29.1%
7%	$ 0.1166	$ 0.0969	16.9%	$ 0.0808	30.7%
8%	$ 0.1135	$ 0.0940	17.2%	$ 0.0761	33.0%
9%	$ 0.1109	$ 0.0912	17.8%	$ 0.0717	35.4%
10%	$ 0.1083	$ 0.0901	16.8%	$ 0.0675	37.7%

4.5 Dividend yield

Table 7 reveals that the valuation bias increases from 15.4 percent to 37.7 percent as the dividend yield rises from 0 to 10 percent. This results appears surprising since the discussion in section 2.1 suggested that early exercise might have a minimal impact on option value for dividend protected calls.

However, in this case the valuation bias is driven almost entirely by path dependency of cumulative dividends. Notice that European razorback option values reported in the 'Simulated European' column in Table 7 show that the percentage early exercise premium is quite insensitive to variation in dividend yields, rising only from 15.6 percent to 16.8 percent as the dividend yield increases from 0 percent to 10 percent.[19] Consequently, variation in the valuation bias is almost entirely due to differences between the simulated European values and the Black-Scholes values. This result is due to the fact that path-dependent cumulative dividends add an extra measure of volatility to option payoffs at maturity and hence increase the option value.

This demonstrates that a 'dividend-protected' razorback call option does not provide complete protection, as its value varies significantly with varying dividend payments. For example, Table 7 shows that as the dividend yield rises from 0 percent to 10 percent razorback option falls by 25.5 percent (from 14.53 cents to 10.83 cents).

[18] We have also calculated the relative early exercise premium using the European values of the razorback options. The percent early exercise premium increases from 8.42 percent to 52.86 percent as the cost of equity increases from 10 percent to 20 percent.

[19] The small difference is mostly likely 'noise'. We have run the simulations at a zero cost of equity and the early exercise premium is virtually zero.

Table 8: Summary of effects on size of Black-Scholes valuation bias	
Factor	*Direction of impact*
Volatility	Negative
Maturity	Positive
Vesting period	Negative
Cost of equity	Positive
Dividend yield	Positive

4.6 Summary of factors affecting the valuation bias

The qualitative effects on the valuation bias stemming from the five factors considered are summarised in Table 8. In general, the higher the cost of equity or the dividend yield, or the longer the maturity of the razorback option, then the larger is the valuation bias. On the other hand, the higher the stock volatility or the longer the vesting period, then the lower is the valuation bias. The magnitude of the valuation bias from the Black-Scholes formula can be substantial, particularly so when the cost of equity and the dividend yield are high.

6. Conclusion and summary

Razorback options issued by Australian and New Zealand companies provide an interesting example of an innovative design for executive stock options that explicitly takes into account the cost of capital in setting rewards for managers In this paper, we have examined problems associated with valuing razorback executive stock options. Valuation difficulties stem from the stock-price path dependency of option value and the possibility of optimal early exercise by razorback option holders. To overcome these valuation difficulties we adopt a least squares Monte Carlo approach recently developed by Longstaff and Schwartz (2001).

Our analysis reveals that use of the Black-Scholes formula to value razorback options leads to potentially significant valuation biases. This result is important as the Black-Scholes formula features prominently in recognised accounting standards and is routinely used to value executive stock options. Results from numerical experiments suggest that the Black-Scholes formula undervalues a representative razorback option by more than 22 percent. The magnitude of the bias can be larger or smaller depending on particular parameter values. The most significant factors affecting the magnitude of the bias are the cost of equity used to set the growth rate for a strike price and the dividend yield of the underlying shares. Security analysts and other practitioners should be aware of these potential biases and exercise caution in using the Black-Scholes formula to value razorback executive stock options.

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