A comparison of interest rate option models on Australian Bank Bill Futures[1]

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Abstract

This paper examines the performance of five different models in pricing options on ninety day bank bill futures traded on the Sydney Futures Exchange between 1993 and 2000. The five models analyzed are the Black, Asay, two-factor Heath, Jarrow and Morton, Extended Vasicek, and Modified Extended Vasicek. The relatively simple Asay model turns out to be the best performer, followed by the Modified Extended Vasicek. Over time, the errors produced by the models appear to fall which indicates that as time progresses; the more complicated models are becoming increasingly used by participants in the market.

1. Introduction

The valuation of interest rate contingent claims has grown increasingly important in the past decade. With growing volumes and complexity, it is important to generate accurate values. While there is much work on the theoretical side, little work tests the various models, especially for non-US data.

In this paper, we test five models of interest rate options using options on Australian bank bill futures. Two models are along the Black Scholes line, two are spot rate models and the last is a two-factor yield curve model.

We find the simple Asay model performs the best of the five, with average absolute pricing errors of less than one basis point. With average absolute pricing errors of about 1.5 basis points, the modified extended Vasicek model is second. Equal third are the extended Vasicek and two-factor Heath-Jarrow-Morton. The worst performer is Black's options on futures. All, except Black, show a dramatic decrease in errors from 1997 onwards. A possible explanation for the results is that the Sydney Futures Exchange uses Asay's model to determine margin requirements and to assess position risk.

To assess any systematic pricing errors, we regress the errors on time to expiration and moneyness. For call (put) options, all models overprice in-the-money (out-of-the-money) options and underprice out-of-the-money (in-the-money) options. The effect of time to expiration is mixed. Time to expiration is relatively more important for the extended Vasicek and Heath-Jarrow-Morton models. Moneyness is more important for Black and Asay.

The organization of the paper is as follows. The next section gives a brief overview of interest rate models. The third sections discusses past empirical tests. Section four describes our data followed by a methodology section. The penultimate section presents our findings. The final section contains our conclusions.

2. Interest Rate Modelling

Over the past 25 years, there has been a revolution in the modelling of interest rates and fixed income products. Early models adapted the Black-Scholes model to price interest rate options. The first major advance came when Vasicek (1977) directly modelled the spot interest rate and used it to evolve the entire yield curve. The most recent advance is to model the entire yield (or forward rate) curve. This section is a short overview of interest rate models.

2.1 Simple Models

Market participants use Black's (1976) options on futures contracts model to value options on bonds and interest rates. By far the simplest method available, it does provide accurate prices with little cost. As a test of more advanced models, many traders require Black prices to be recovered before using the advanced model. In Black the forward interest rate, r, is assumed to follow geometric Brownian motion with proportional volatility, σ:

$$dr = \mu r dt + \sigma r dZ \ . \qquad (1)$$

μ is the drift and dZ is a standard Weiner process. Technically, the model assumes constant interest rates over the life of the option, but by using two different (and uncorrelated) rates this problem is resolved. Another drawback is interest rates are unbounded. There is no adjustment for the long-term equilibrium level.

Asay (1982) alters Black (1976) for options with futures style settlement. When applied to American options, Lieu (1990) showed that early exercise is never optimal and that Asay's formula is correct. Thus, there is a closed form solution to the pricing problem.

2.2 Spot Rate Models

Beginning with Vasicek (1977), researchers began to develop models of the instantaneous spot interest rate and use it to compute the entire yield curve. Vasicek assumes the spot rate, r, follows a mean reverting process with absolute volatility, σ:

$$dr = a(b - r)dt + \sigma dZ \; . \tag{2}$$

Vasicek's model allows the spot interest rate to be pulled back to a mean level of rates, b, at a rate of a. By allowing for mean reversion, interest rates are pulled back to an equilibrium level if they become too high or too low.

The Vasicek model has several drawbacks. Its main problem is it allows negative interest rates with a positive probability because volatility is absolute, not proportional. A practical problem is it cannot replicate the current term structure of interest rates and volatilities. This last problem leads to arbitrage possibilities.

Hull and White (1990, 1994) consider the Vasicek model and make the parameters time dependent, allowing the model to match current interest rates and volatilities. The model they propose is:

$$dr = [\Theta(t) + a(t)(b - r)]dt + \sigma(t)dZ \; . \tag{3}$$

Here the reversion rate and volatility parameters are functions of time. $\Theta(t)$ is an unknown function of time used to fit the initial term structure. By fitting the current yield curve and volatility structure, Hull and White eliminate the arbitrage possibilities of the original Vasicek model. Unfortunately, there is still the possibility of negative interest rates. It is also possible to use the volatility function to match the current term structure of volatilities, but this may lead to undesirable volatility curves in the future.

Cox, Ingersoll and Ross (1985) develop a model along the spirit of Vasicek that incorporates mean reversion, but allows for proportional volatility. Its dynamics are

$$dr = a(b - r)dt + \sigma\sqrt{r}dZ \tag{4}$$

Because of implementation problems, this model is not used often.

2.3 Yield Curve Modelling

While Hull and White were modifying the Vasicek model, others were considering how to start with a given yield curve and model its evolution. Ho and Lee (1986) created an arbitrage-free model of interest rates that takes the current yield curve as an input. Using a binomial process, Ho and Lee create a tree for future yield curves that can be used to price securities. Their process takes the form:

$$dr = \Theta(t)dt + \sigma dZ\ . \tag{5}$$

As in the Hull and White model, $\Theta(t)$ is a function of time used to fit the current term structure. It has a simple analytical form, $\Theta(t) = F_t(0,t) + \sigma^2 t$ with $F(0,t)$ is the instantaneous forward rate for maturity t as seen from today. The current shape of the yield curve tells the expected movements of the short rate. There are two major problems with the Ho and Lee model. The first is all interest rates have the same standard deviation because of the fixed volatility structure leading to perfect correlation. Second and more disturbing is that negative rates are possible.

A major advance came with the Heath, Jarrow and Morton (1992) model (hereafter HJM). Their model starts with the initial term structure and evolves the forward rates through time. It has been shown that all other models are special cases of the HJM model (see Chiarella and Kwon (1999)), thus it is the most general specification. Another major advantage of the HJM framework is that it permits multiple sources of uncertainties with little difficulty. The single factor stochastic process for the short rate is (from Hull (2000)):

$$\begin{aligned} dr(t) = F_t(0,t)dt + \left\{ \int_o^t \left[v(\tau,t,\Omega_T) v_{tt}(\tau,t,\Omega_T) + v_t(\tau,t,\Omega_T)^2 d\tau \right] dt \right\} dt \\ + \left\{ \int_o^t v_{tt}(\tau,t,\Omega_T) dz(\tau) \right\} dt + \left[v_t(\tau,t,\Omega_T)|_{\tau=t} \right] dz(t) \end{aligned} \quad . \qquad (6)$$

We can see that the two middle terms on the right hand side depend on the path taken by the volatility function, making the this form of HJM non-Markovian and difficult to implement. There is a substantial amount of work being done to make the HJM more user friendly. Several researchers use a Markov process with Gaussian rates, but this leads to negative rates. Others use a lognormal distribution, but have trouble finding simple solutions.

This section has briefly outlined the history of interest rate modelling. Starting from a simple Black-Scholes framework, models have evolved to highly complex forms involving the entire yield curve. The next section reviews papers that test these models on interest rate futures options.

3. Tests of Interest Rate Models

With a large number of papers addressing various issues in interest rate modeling, we choose to look at those assessing pricing errors of various models on interest rate futures options. Most previous work uses options on Eurodollar futures from the US; a few researchers have used non-US data. Most papers test the HJM framework.

Flesaker (1993) tested a single factor, constant volatility version of the HJM model, which is the continuous time limit of the Ho-Lee model. He used generalized method of moments to calculate the parameter estimates and simulation to test the results. Using options on Eurodollar futures, he found average absolute errors of 3.46 basis points for puts and 3.33 basis points for calls.

Flesaker raised an interesting point: Is mean reversion important? Chan, Karolyi, Longstaff and Sanders (1992), Li (2000), and Dempster (2000), all find the mean reversion parameters in spot rate models are indistinguishable from zero. While the parameter estimates found may be sufficiently large to include them in their respective models, the statistical insignificance raises doubts about whether interest rates are mean reversion. A time dependent reversion level may prove useful.

Amin and Morton (1994) test six HJM class models using Eurodollar futures and options on futures data for the period from 1987 to 1992. Amin and Morton find that there are significant differences in the performance of one and two parameter HJM models. In fact, they discover that the number of parameters in the model has a larger impact on the results than the form of the models. They also find that one parameter HJM models are preferable for pricing options which have less than one year to maturity. They find that the average absolute errors range from 1.57 basis points up to 2.23 basis points, while the average error ranges from –0.13 basis points to 0.01 basis points.

For Eurodollar futures options (US rates), it is still debatable which model is to use. With regard to foreign rates, there is also uncertainty about which model to select.

Moraleda and Vorst (1996) test three single factor models: the Ho-Lee, Hull and White, and HJM. One of the features of the HJM model is that, because it is so general, many models can be captured in its framework. This is the case of the models analyzed in this paper; they are all derived in the Ritchken and Sankarasubramanian framework which is a subset of the HJM model. Moraleda and Vorst use the Spanish market and find results which make a case for the Ho-Lee model being the best[1]. While the Ho-Lee model is judged to be the best performer, Moraleda and Vorst consider that all the models tested performed relatively poorly.

Allen and Chau (1999) carry out a test of the Australian bank bill market using four models, the Asay, Black, Extended Vasicek (Hull and White), and HJM. They find that out of all the models tested, the HJM is the best performer, followed by the Extended Vasicek, Black, and Asay[2]. Allen and Chau perform their analysis for the year 1996, and similar to this study, they use Monis software (although an earlier version to the one used in this study) for the pricing of the more complex HJM and extended Vasicek models.

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[1] The actual results are not included here as the way that Moraleda and Vorst present their errors (model price – market price), where both model and market price are defined as the percentage of the face value of the underlying assets, makes them incomparable with other literature.

[2] In Allen and Chau's paper, due to the nature of the statistical analysis they do on the results, there is no mention of the overall average errors produced by the model. The closest that can be obtained are errors broken down into moneyness and time to maturity. For brevity, these results are not included.

Though several markets have been used, advanced models still produce pricing errors. Researchers do not agree on which model is best. The next section describes our data and empirical tests.

4. Data

We look at Options on 90-Day Bank Accepted Bills Futures traded on the Sydney Futures Exchange. Bank bill futures are settled by the physical delivery of bank accepted bills, unlike the cash settled Eurodollars. They are quoted as $100 - \textit{Interest Rate}$. Each contract is for AUD 1,000,000 face value. Our sample runs from 04 January 1993 to 15 November 2000.

These options are American style options with futures style settlement. This means the whole option premium is not upfront, only a fraction. An initial margin is posted and gains (losses) are added (subtracted) daily from the margin of the buyer and subtracted from (added to) the margin of the seller. In contrast, US options are paid for upfront.

The SFE trades options with strike prices on the underlying futures price set in 25 basis point intervals and the maturity of the option contracts available for trading is eight quarter months ahead. Contract expiration is in the standard quarter months of March, June, September and December. Since the futures involve physical delivery of the underlying asset the options expire on the first Friday of the delivery month. The futures expire on the second Friday of the month.

We acquired settlement prices for the options and futures from the Sydney Futures Exchange's website (www.SFE.com.au). As the data was assembled, option pairs in which one had a settlement price of zero were deleted. The final dataset includes almost 240,000 call and put pairs. The spot rate data was generously provided by Westpac Institutional Bank and consists of overnight, 1, 2, 3, 5, 6, 9 and 12 month zero coupon interest rates. The zero rates are obtained through simple linear interpolation from observed traded bond yields.

5. Methodology

We used the data to estimate theoretical values for the options and compare these to the actual prices. We calculated Black and Asay prices ourselves. We used Monis Interest Rates XL (version 6.31) to compute the HJM and Hull and White models. The procedures to derive the values follow.

To compute option prices using Black and Asay, we need have an estimate of the volatility. We took the implied volatility of the at-the-money option for each maturity each day, so as to be consistent with the procedure used by the Monis software. Once this was done, we used the volatility for all options of the same maturity. The formulae for Black and Asay options are

$$c_B = e^{-rT}[F_0 N(d_1) - XN(d_2)] \qquad p_B = e^{-rT}[XN(-d_2) - F_0 N(-d_1)] \qquad (8)$$

$$c_A = F_0 N(d_1) - XN(d_2) \qquad p_A = XN(-d_2) - F_0 N(-d_1) \qquad (9)$$

$$d_1 = \frac{\ln(\frac{F_o}{X}) + \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}} \qquad d_2 = \frac{\ln(\frac{F_o}{X}) - \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}\ .$$

The subscripts, B and A, represent Black and Asay, respectively.

To find prices using the Monis software several steps are needed. Using at-the-money options for each maturity, the spot rates for maturities up to one year and the first 12 futures prices (three years), the software calibrates the parameters of the Hull and White and HJM models. To get the volatility parameters, the Monis software "minimizes the sum of squares of the percentage error between the theoretical and market prices of the underlying markets."[3] Once the volatility parameters are estimated, Monis calculates the yield curve parameters and a yield curve for each model. This yield curve is then used to value the options on the futures. The models used by Monis are two versions of the Hull and White model (the Extended Vasicek (EXV) and Modified Extended Vasicek (MEV)) and a two-factor HJM.

Under the EXV and MEV models, the evolution of the discount bond price P_T can be described by the following stochastic differential equation.

$$dP_T = \mu_T P_T dt + \sigma_{2T} P_T dZ \qquad (10)$$

Where

T is the bond maturity time

Z is the standard Brownian motion

μ_T is the investor's belief of the rate of growth of the bond price

σ_{2T} is a deterministic function given by

$$\sigma_{2T} = \frac{\sigma_2}{\alpha}(1 - e^{-\alpha(T-t)}) \qquad (11)$$

[3] Monis manual page 90

for positive constants α and σ_2. The software estimates these parameters. The associated short rate, r_t, satisfies the stochastic differential equation of the form in the Hull and White model. The MEV changes the volatility function slightly

$$\begin{aligned} \sigma_2(t,T) &= \frac{\sigma_2}{\alpha}(1-e^{-\alpha(T-t)}) \text{ for } (t < T^*) \\ \sigma_2(t,T) &= \frac{\sigma^*{}_2}{\alpha^*}(1-e^{-\alpha^*(T-t)}) \text{ for } (t \geq T^*) \end{aligned} . \tag{12}$$

This allows the volatility function to take on a richer set of possible shapes. There are now five parameters the calibration process will return (α,σ_2,α^*,σ_2^* and T^*).

For the HJM model, there are two sources of uncertainty and the evolution of the forward rate is

$$df_T = \mu_T dt + \sigma_1 dW + \sigma_2 e^{-\alpha(T-t)} dZ . \tag{13}$$

Where

σ_{2T} is the same as in the EXV model

$$\sigma_{1T} = \sigma_1(T-t) . \tag{14}$$

σ_1, σ_2, and α are the three terms returned by the calibration process.

We estimate all five models for each day in the sample period. This involved recalibrating the EXV, MEV and HJM daily. It took anywhere from 2.5 to 24 hours to compute one year's worth of values for each model on a Pentium III. There was no discernable pattern on when a fast run would occur.

With all the estimates, we calculated the actual and absolute pricing errors. These values are then used in a series of linear regressions to determine any biases in the models. The tests and results are next.

6. Empirical Results

This section presents our empirical results. We first discuss the average and average absolute pricing errors of the five models. We analyze them, by several measures, both in aggregate and broken down. We next look at the regressions of the errors on several factors to identify any systematic pricing errors.

6.1 Error Analysis

Table 1 presents the average and average absolute pricing errors. The error is defined as the market price less the model price. Excepting Asay puts, we see all models overpriced calls and puts during the sample. All models, except Black, have average errors of less than one basis point. It appears the models do well in pricing options on bank bill futures. Since the errors maybe positive or negative, the error may be biased towards zero. To adjust for this, we look at the absolute values of the errors.

With absolute errors the results are a bit different. The average absolute errors range from less than one basis point for Asay's model to greater than five for Black. MEV has the second smallest errors, at around 1.2 basis points. The EXV and HJM have errors of approximately three basis points. The standard deviations of Asay prices are much smaller than all of the others, while Black's is much larger.

Table 2 presents the average absolute errors over time. Again excepting Black, there is a general downward trend in the pricing errors. Between 1996 and 1997 there is a dramatic reduction in pricing errors. A possible reason for the decline in errors is that market participants began adopting advanced technologies to price their options. In agreement with Table 1, Asay is still the best model.

Table 3 breaks the average absolute errors into buckets based on time to expiration and moneyness. Maturities less than 8 months are short term, 8 to 16 are medium and greater than 16 are long term. As expected, Asay generally has the smallest error in all buckets. Black's pricing errors noted in the first two tables are driven by in-the-money options, particularly those with long maturities. Recall Black prices are the present value of Asay prices, giving expensive options (long maturity and in-the-money) the largest difference from Asay prices.

With respect to the more sophisticated models, the EXV and HJM models have errors of about the same size. MEV has errors of one-third to one-half the size of the other two. This is in line with Tables 1 and 2. Interestingly, HJM and EXV have their largest errors for at-the-money options. Since the Monis software calibrates the volatility function to the options, we would expect these to have the best fit.

In general, all models price out-of-the-money options better than in-the-money options. This may occur because they have lower prices. It could turn out that out-of-the-money options have larger percentage errors than in-the-money options.

Given our large sample size, all are significantly different from zero and each other. Based on a visual examination of the data, Asay's model is clearly the best. This may not be a rejection of the other models, but more a self-fulfilling prophecy. The SFE uses Asay's model to determine initial margin payments and to set risk limits on positions. Thus, a rational trader would prefer Asay's model to meet the requirements of the exchange.

In summary, we find the Asay model provides the best fit for options on bank bill futures. The MEV is best among the sophisticated models. The next part looks at the correlation of the errors between the five models and regresses the errors on several factors.

6.2 Correlation and Regression Analysis

Table 4 presents the correlation coefficients of the actual errors between the five models. We find the highest correlation for both calls and puts is between the HJM and EXV models. Looking at the errors in the prior three tables, this is not surprising. What is surprising is the low correlation of errors between the sophisticated models (HJM, MEV, and EXV) and the simple models (Asay and Black). These are around zero. There is no apparent relationship between the errors.

To identify any systematic pricing errors, we regress the pricing errors on time to expiration, moneyness and dummy variables for the years. The equation we regressed is

$$Error = \alpha + \beta_1 Moneyness + \beta_2 Time\ to\ Expiration + \sum_{i=93}^{99} B_i D_i + \varepsilon \,. \qquad (15)$$

Where

Error	Market Price – Model Price
Moneyness	The number of basis points the option is in-the-money (negative for out-of-the-money
Time to Expiration	Maturity of the option
D_i	Dummy variables for the years 1993 to 1999

Panel A of Table 5 presents the results for calls on bank bill futures. For all five models, an increase in moneyness increases the pricing error. This effect is particularly pronounced for Black. This is not surprising given the results of earlier tables. With regard to time to expiration, the sophisticated models have smaller errors for long maturities. Black and Asay errors increase. Looking at the dummy variables, we see they become less significant (or insignificant) from 1997 onwards, except for Black. Given our large sample size, almost all of the results are significant.

To see what variables have the most influence on the pricing errors, we present the standardized regression coefficients (STB). STBs measure the relative impact of the independent variables on the dependent variable. This allows us to give a relative ranking of their importance. In this case, the dummy variables have very little impact on the pricing errors. HJM and EXV are most influenced by the time to expiration; Black and Asay are more influenced by moneyness; and MEV is influenced by neither.

Panel B shows the results for puts. For all models except Black, an increase in time to expiration and moneyness leads to a decrease in pricing errors. Black errors increase with moneyness and time to expiration. The dummy variables again show a large shift from 1996 to 1997.

The STBs are similar to those for the calls. HJM and EXV are most influenced by the time to expiration; Black and Asay are more influenced by moneyness; and MEV is not influenced by either.

This section has presented the results of our paper. We find the simple Asay model produces the best fit for option on bank bill futures prices. The MEV model was next in accuracy, followed by HJM and EXV. Black's model performed worst. Using regressions to determine which factors have the greatest impact on pricing errors, we find the pricing errors produced by Black and Asay are most influenced by the degree of moneyness of the option. HJM and EXV are most influenced by the time to expiration.

7. Conclusions

In this paper we tested five models of interest rate options. We find the simple Asay model has the smallest absolute errors for options on Australian bank bill futures. More complex models using the spot rate and the yield curve have errors of approximately 1.5 to three basis points. . For call (put) options, all models overprice in-the-money (out-of-the-money) options and underprice out-of-the-money (in-the-money) options. The effect of time to expiration is mixed. Time to expiration is relatively more important for the extended Vasicek and Heath-Jarrow-Morton models. Moneyness is more important for Black and Asay.

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Table 1
Average deviation between market and model prices

This table reports the mean error and mean absolute error for all five models analyzed. Average error is defined as the average of market – model price. The standard deviations reported are the standard deviations of the average error, and average absolute error.

	Average error				*Average Absolute error*			
Model	*Calls*	*Std Dev*	*Puts*	*Std Dev*	*Calls*	*Std Dev*	*Puts*	*Std Dev*
HJM	0.0072	0.0471	0.0096	0.0521	0.0280	0.0385	0.0290	0.0443
MEV	0.0018	0.0258	0.0043	0.0339	0.0119	0.0230	0.0136	0.0314
EXV	0.0069	0.0473	0.0093	0.0524	0.0285	0.0384	0.0296	0.0442
Asay	0.0038	0.0114	-0.0023	0.0127	0.0081	0.0089	0.0079	0.0102
Black	0.0452	0.0919	0.0398	0.0778	0.0625	0.0812	0.0499	0.0718

Table 2
Average absolute errors by year

These tables report the average absolute error per year for each model analyzed. The first number in each row corresponds to the average error for the model while the number below is the standard deviation.

Absolute Calls	1993	1994	1995	1996	1997	1998	1999	2000
HJM	0.0355	0.0441	0.0341	0.0368	0.0168	0.0114	0.0226	0.0186
	0.0381	0.0585	0.0376	0.0502	0.0233	0.0163	0.0253	0.0193
MEV	0.0156	0.0174	0.0114	0.0176	0.0082	0.0068	0.0090	0.0077
	0.0237	0.0426	0.0211	0.0196	0.0177	0.0126	0.0155	0.0092
EXV	0.0355	0.0442	0.0341	0.0404	0.0168	0.0117	0.0226	0.0186
	0.0383	0.0585	0.0376	0.0483	0.0236	0.0169	0.0253	0.0193
Asay	0.0102	0.0072	0.0091	0.0133	0.0065	0.0059	0.0052	0.0067
	0.0109	0.0076	0.0088	0.0116	0.0075	0.0068	0.0053	0.0072
Black	0.0746	0.0384	0.0690	0.0799	0.0660	0.0499	0.0475	0.0891
	0.0940	0.0358	0.0810	0.0917	0.0899	0.0672	0.0635	0.1117
No. of options	30951	29246	34577	31919	29186	31872	36666	15022

Absolute Puts	1993	1994	1995	1996	1997	1998	1999	2000
HJM	0.0340	0.0460	0.0354	0.0379	0.0170	0.0126	0.0254	0.0190
	0.0468	0.0703	0.0421	0.0518	0.0302	0.0193	0.0324	0.0188
MEV	0.0148	0.0224	0.0134	0.0183	0.0085	0.0081	0.0122	0.0092
	0.0363	0.0572	0.0283	0.0210	0.0250	0.0163	0.0253	0.0112
EXV	0.0340	0.0461	0.0354	0.0417	0.0171	0.0129	0.0254	0.0190
	0.0466	0.0703	0.0421	0.0503	0.0309	0.0197	0.0324	0.0188
Asay	0.0065	0.0110	0.0092	0.0129	0.0046	0.0048	0.0068	0.0071
	0.0072	0.0117	0.0162	0.0109	0.0046	0.0055	0.0078	0.0077
Black	0.0250	0.0641	0.0530	0.0722	0.0341	0.0310	0.0541	0.0788
	0.0339	0.0849	0.0720	0.0861	0.0499	0.0452	0.0760	0.0999
No. of options	30951	29246	34577	31919	29186	31872	36666	15022

Table 3

Average absolute error analysis

These tables present average absolute errors by year broken down into time to maturity and moneyness brackets. Short maturity options are those with a time to expiration under 8 months, medium maturity options have a time to maturity between 8 and 16 months, while long maturity options have a time to maturity greater than 16 months. Out of the money options are considered to be those with moneyness less than -12.5 basis points, at the money options have moneyness between –12.5 and 12.5 basis points, while in the money options have moneyness greater than 12.5 basis points.

Calls All years

Maturity	Moneyness	Asay	Black	HJM	MEV	EXV	N
Short	Out	0.0043	0.0064	0.0148	0.0072	0.0150	20708
Short	At	0.0017	0.0015	0.0234	0.0089	0.0239	4593
Short	In	0.0088	0.0343	0.0200	0.0120	0.0201	22236
Medium	Out	0.0052	0.0168	0.0272	0.0093	0.0277	45941
Medium	At	0.0025	0.0047	0.0466	0.0140	0.0478	4676
Medium	In	0.0112	0.0996	0.0322	0.0144	0.0329	46014
Long	Out	0.0053	0.0308	0.0299	0.0114	0.0306	44201
Long	At	0.0021	0.0112	0.0439	0.0159	0.0446	4788
Long	In	0.0137	0.1569	0.0294	0.0142	0.0300	46282

Puts All years

Maturity	Moneyness	Asay	Black	HJM	MEV	EXV	N
Short	Out	0.0064	0.0064	0.0175	0.0094	0.0176	22236
Short	At	0.0017	0.0026	0.0250	0.0128	0.0253	4593
Short	In	0.0069	0.0244	0.0209	0.0141	0.0211	20708
Medium	Out	0.0047	0.0093	0.0291	0.0109	0.0298	46014
Medium	At	0.0025	0.0081	0.0538	0.0196	0.0550	4676
Medium	In	0.0121	0.0863	0.0323	0.0167	0.0329	45941
Long	Out	0.0060	0.0211	0.0269	0.0115	0.0274	46282
Long	At	0.0021	0.0170	0.0443	0.0187	0.0449	4788
Long	In	0.0121	0.1309	0.0334	0.0162	0.0341	44201

Table 4

Pricing error correlation of models

This table reports the correlation of pricing errors between models.

Calls	HJM	MEV	EXV	Asay	Black
HJM	1				
MEV	0.586	1			
EXV	0.960	0.609	1		
Asay	0.029	0.197	0.021	1	
Black	0.016	0.121	0.006	0.805	1

Puts	HJM	MEV	EXV	Asay	Black
HJM	1				
MEV	0.680	1			
EXV	0.968	0.696	1		
Asay	0.106	0.226	0.098	1	
Black	-0.119	-0.128	-0.130	-0.545	1

Table 5

Regression results for pricing errors.

Where Deviation is defined as the market price of the option – the model price of the option. D93, D94, D95, D96, D97, D98, and D99 are dummies for the years, 1993, 1994, 1995, 1996, 1997, 1998 and 1999. Moneyness is defined as Futures price – strike price for calls and Strike price-Futures price for puts. HJM is the Heath, Jarrow and Morton model, MEV is the Modified Extended Vasicek, and EXV is the Extended Vasicek model, these three models prices are obtained from the Monis software. Asay is the Asay modification of Black Scholes, while Black is Black's modification of Black Scholes. For each model there are 239439 observations. The numbers in the same row as the model name are the parameter estimates, while the number below these is the t-statistic for that figure. The t-statistics are unreliable due to the large number of observations. The last two columns contain the R^2 and adjusted R^2 for each model. Stb are the standardised regression coefficients.

Calls

Model	Intercept	Callmoney	Time2Ex	Dum93	Dum94	Dum95	Dum96	Dum97	Dum98	Dum99	R^2	Adjusted R^2
HJM	0.0352	0.0013	-0.0274	0.0075	0.0078	0.0051	0.0088	-0.0001	-0.0026	0.0003	0.0977	0.0976
t statistic	74.12	22.87	-142.85	16.4	16.82	11.35	19.68	-0.24	-5.69	0.6		
Stb	0	0.0453	-0.2875	0.0533	0.0543	0.038	0.0638	-0.0008	-0.0185	0.002		
MEV	0.0028	0.0015	-0.0021	0.0037	0.0017	0.0033	-0.0019	0.0014	0.0008	0.0012	0.0156	0.0156
t statistic	10.28	44.86	-18.73	14.28	6.43	12.85	-7.18	5.21	3.03	4.56		
Stb	0	0.0928	-0.0394	0.0485	0.0217	0.045	-0.0243	0.0172	0.0103	0.016		
EXV	0.0377	0.0014	-0.029	0.0067	0.0069	0.0043	0.006	-0.0007	-0.0032	-0.0003	0.1047	0.1047
t statistic	79.19	23.4	-150.84	14.58	14.79	9.52	13.33	-1.52	-7.16	-0.73		
Stb	0	0.0462	-0.3024	0.0472	0.0475	0.0318	0.0431	-0.0048	-0.0232	-0.0025		
Asay	0.0013	0.0049	0.002	0.0005	-0.0014	0.001	0.0019	-0.0006	-0.0005	-0.0002	0.5009	0.5009
t statistic	14.67	468.27	56.67	6.08	-16.93	12.87	22.89	-6.91	-6.58	-2.91		
Stb	0	0.6898	0.0848	0.0147	-0.0406	0.0321	0.0552	-0.0163	-0.0159	-0.0073		
Black	0.014	0.0468	0.0363	-0.0193	-0.0235	-0.0094	0.0176	-0.0168	-0.0233	-0.0159	0.7298	0.7298
t statistic	27.59	750.89	176.77	-39.52	-47.46	-19.68	36.57	-34.43	-48.23	-33.89		
Stb	0	0.8138	0.1946	-0.0703	-0.0837	-0.0361	0.0649	-0.0597	-0.0859	-0.0624		

Puts

Model	Intercept	Putmoney	Time2Ex	Dum93	Dum94	Dum95	Dum96	Dum97	Dum98	Dum99	R^2	Adjusted R^2
HJM	0.0404	-0.0014	-0.0304	0.0078	0.0065	0.0044	0.0103	0.0016	-0.0008	0.0013	0.0937	0.0937
t statistic	76.81	-21.28	-142.82	15.47	12.7	8.9	20.76	3.14	-1.6	2.63		
Stb	0	-0.0422	-0.2881	0.0504	0.041	0.0299	0.0675	0.01	-0.0052	0.0089		
MEV	0.008	-0.0015	-0.005	0.0041	0.0005	0.0026	-0.0004	0.0031	0.0026	0.0022	0.0136	0.0136
t statistic	22.37	-35.06	-34.63	11.87	1.3	7.82	-1.04	8.92	7.51	6.58		
Stb	0	-0.0726	-0.0729	0.0404	0.0044	0.0274	-0.0035	0.0295	0.0256	0.0231		
EXV	0.0429	-0.0014	-0.032	0.007	0.0056	0.0036	0.0075	0.001	-0.0015	0.0007	0.0999	0.0999
t statistic	81.31	-21.74	-149.9	13.82	10.86	7.24	15.01	1.98	-2.93	1.42		
Stb	0	-0.043	-0.3013	0.0449	0.035	0.0242	0.0486	0.0063	-0.0096	0.0048		
Asay	-0.0025	-0.0049	-0.0008	0.0008	-0.001	0.0011	0.0035	0.0012	0.0011	0.0003	0.3962	0.3961
t statistic	-23.4	-378.59	-19.79	7.74	-9.82	11.34	34.95	11.94	10.88	3.34		
Stb	0	-0.6134	-0.0326	0.0206	-0.0259	0.0311	0.0927	0.0309	0.029	0.0092		
Black	0.0116	0.0374	0.0329	-0.0159	-0.0104	-0.0078	0.0213	-0.0162	-0.0193	-0.0148	0.6712	0.6711
t statistic	24.52	643.49	171.67	-34.93	-22.51	-17.5	47.5	-35.55	-42.77	-33.58		
Stb	0	0.7695	0.2086	-0.0686	-0.0438	-0.0354	0.093	-0.068	-0.0841	-0.0682		