THE FAMA-FRENCH MODEL, LEVERAGE AND THE MM PROPOSITIONS

Martin Lally
School of Economics and Finance
Victoria University of Wellington*

Abstract

All models for the cost of capital, and associated implementation processes, should conform to the Miller-Modigliani cost of capital propositions. Consequently any sensitivity coefficients in such models must be related to the firm's leverage. This paper applies these principles to the Fama-French model for the cost of equity and develops the relationship between its sensitivity coefficients and firm leverage. The paper then examines an empirical process developed by Fama and French (1997) to model the evolution through time of their sensitivity coefficients. It is shown that this empirical process is inconsistent with the Miller-Modigliani propositions. Separable functions are proposed for these sensitivity coefficients that are consistent with the Miller-Modigliani propositions.

JEL: G12

Keywords: Fama-French model, Leverage, Miller-Modigliani propositions

*PO Box 600, Wellington, New Zealand
Ph: +64-4-463-5233
FAX: +64-4-463-5014
Martin.lally@vuw.ac.nz

THE FAMA-FRENCH MODEL, LEVERAGE AND THE MM PROPOSITIONS

1. Introduction

Modern cost of capital theory commences with Miller and Modigliani (1958, 1963), who use arbitrage arguments to model the effect of leverage changes upon a firm's cost of equity and its weighted average cost of capital ($WACC$), in the form of MM Propositions II and III. Subsequent models deal with all factors affecting a firm's cost of equity; these models including the Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965) and Mossin (1966), the Arbitrage Pricing Theory (APT) of Ross (1976), and the Fama-French (1997) model. Each of these latter three models contains sensitivity coefficients that are asset specific. Estimates of these sensitivity coefficients should then respond immediately to asset specific events that are relevant to the cost of equity. Since one such event is a pure leverage change in a firm then these cost of equity models must be consistent with the MM propositions. Standard practice in the application of the CAPM is consistent, in that a formula like that of Hamada (1972) is invoked to effect an immediate change to the firm's equity beta, and hence its cost of equity. Parallel formulas have also been applied to the APT (see Damodaran, 1997, p. 142). However parallel formulas have not yet been presented for the Fama-French model. This paper develops these formulas, relating the Fama-French sensitivity coefficients to the firm's leverage. We then examine a process presented by Fama and French (1997) to model the evolution through time of their sensitivity coefficients, so as to test for consistency with the MM propositions.

We commence in section II by briefly describing and illustrating the process, in the CAPM context, for adjusting the cost of equity to a pure leverage change, and show that it is consistent with the MM propositions. Section III then develops the parallel process for the Fama-French model. Section IV then examines a process presented by Fama and French (1997) to model the evolution through time of their sensitivity coefficients. We show that the Fama-French process is inconsistent with the MM propositions. An alternative process is then suggested that is consistent with the MM propositions. Section V concludes.

2. The CAPM and Leverage Changes

The standard version of the CAPM specifies the cost of equity of a firm as

$$k_e = R_f + \left[E(R_m) - R_f\right]\boldsymbol{b}_e \tag{1}$$

where R_f is the riskfree rate, R_m the rate of return on the market portfolio and $\boldsymbol{b}_e$ the equity beta of the firm. As the firm's leverage changes, the equity beta changes and hence so too does the cost of equity. The relationship between leverage and $\boldsymbol{b}_e$ depends upon the tax environment, debt policy, the riskiness of debt and the types of claims present in the firm (see Hamada, 1972; Conine, 1980; Miles and Ezzell, 1985; Ehrhardt and Shrieves, 1995). To simplify the analysis we assume no personal or corporate tax, riskless debt and claims comprising only ordinary shares and straight debt (this corresponds to the MM world with no corporate tax[1]). These assumptions imply that the levered equity return R_e is related to its unlevered counterpart R_u by

$$R_e = R_u\left[1 + \frac{B}{S}\right] - k_d\frac{B}{S} \tag{2}$$

where k_d is the cost of debt, B the market value of debt and S the market value of equity (this is simply the ex post counterpart to MM Proposition II with no corporate tax). With riskless debt it follows that

$$\boldsymbol{b}_e = \boldsymbol{b}_u\left[1 + \frac{B}{S}\right] \tag{3}$$

where $\boldsymbol{b}_u$ is the equity beta in the absence of leverage (this is the Hamada, 1972, formula in the absence of corporate tax). Substituting this into (1) yields

$$k_e = R_f + \left[E(R_m) - R_f\right]\boldsymbol{b}_u\left[1 + \frac{B}{S}\right]$$

---

[1] The tax assumptions of Miller (1977) yield the same results.

Equation (1) also implies that

$$k_u = R_f + \left[E(R_m) - R_f\right]\beta_u$$

The last two equations then imply that

$$\begin{aligned} k_e &= R_f + \left[k_u - R_f\right]\left[1 + \frac{B}{S}\right] \\ &= k_u + \left[k_u - R_f\right]\frac{B}{S} \end{aligned}$$

which accords with MM Proposition II in the presence of riskless debt and no corporate tax. Substitution into the $WACC$ definition, with cost of debt equal to the riskfree rate and no corporate tax, yields

$$\begin{aligned} WACC &= \frac{S}{V}k_e + \frac{B}{V}R_f \\ &= \frac{S}{V}\left\{k_u + \left[k_u - R_f\right]\frac{B}{S}\right\} + \frac{B}{V}R_f \\ &= k_u \end{aligned}$$

which accords with MM Proposition III with no corporate taxes. Thus, the CAPM in conjunction with equation (3) is consistent with MM Propositions II and III.

To implement this approach one must estimate the unlevered beta. This is typically done by using time-series regression to estimate the levered beta and then using equation (3) to convert it to its unlevered counterpart. Repetition over a number of other companies in the same industry, followed by averaging over them, is typically done in order to improve the estimate of β_u. Given the resulting estimate of β_u, as the firm's leverage changes through time, equation (3) then revises the estimate for β_e and equation (1) then revises the estimate for k_e. Substitution of this into the $WACC$ definition then revises the $WACC$.

To illustrate this, suppose that the riskfree rate is .06, the market risk premium is estimated at .08 and a firm's unlevered beta has been estimated at .60. Currently the firm has B/S = .10. Following equation (3)

$$\beta_e = .60[1 + .10] = .66$$

Following equation (1)

$$k_e = .06 + .08(.66) = .113$$

Following the definition of $WACC$

$$WACC = .113(.909) + .06(.091) = .108$$

Now suppose that the firm undertakes a pure leverage change such that its B/S rises to .70[2]. Following equation (3), then (1), and then the $WACC$ definition, the revisions to β_e, k_e and $WACC$ are as follows:

$$\beta_e = .60[1 + .70] = 1.02$$

$$k_e = .06 + .08(1.02) = .142$$

$$WACC = .142(.588) + .06(.412) = .108$$

The cost of equity has then risen but $WACC$ is unchanged. These results accord with MM Propositions II and III. However, if the estimate of the equity beta is not immediately adjusted to reflect the leverage change, then the cost of equity will remain at .118 and the $WACC$ will be erroneously revised downwards by almost two percentage points to

$$WACC = .113(.588) + .06(.412) = .091$$

[2] Fama and French (1999, Figure 1) report recent market level B/S for the US at around .30. Thus a shift from .10 to .70 at the firm level represents a shift from low to high leverage.

These results violate the MM propositions. We now apply these principles to the Fama-French model.

3. The Fama-French Model and Leverage Changes

Fama and French (1992, 1993, 1995) show that firm size and the book-to-market ratio have explanatory power over expected stock returns. Subsequently Fama and French (1997) incorporate these factors into a model for determining the cost of equity and then estimate the parameters for a number of industries. By conducting estimation at the industry level they imply that all firms within the same industry (but with different leverages) warrant the same cost of equity. Clearly this is in violation of the MM propositions, and we will therefore apply their model at the individual firm level. Invoking the model, the cost of equity for a firm is

$$k_e = R_f + b\lfloor E(R_m) - R_f \rfloor + sE(R_S) + hE(R_H) \tag{4}$$

where R_S is the return on a portfolio of small stocks less that on a portfolio of large stocks, R_H is the return on a portfolio of stocks with high book-to-market equity less that on a portfolio of stocks with low book-to-market equity, and the coefficients b, s and h are the sensitivities of the firm's equity returns to R_m, R_S and R_H respectively.

These sensitivity coefficients are affected by the firm's leverage, and the relationship parallels that for the Hamada formula, i.e.,

Proposition 1: The Fama-French sensitivity coefficients b, s and h are related to the firm's leverage and its unlevered sensitivity coefficients b_u, s_u and h_u as follows[3]*:*

$$b = b_u\left[1 + \frac{B}{S}\right] \quad s = s_u\left[1 + \frac{B}{S}\right] \quad h = h_u\left[1 + \frac{B}{S}\right] \tag{5}$$

[3] The unlevered sensitivity coefficients might be viewed as industry characteristics, in the same way that unlevered betas are viewed as industry characteristics (for example, see Damodaran, 1997, Table 6.6). However, as we shall see, even the unlevered sensitivity coefficients may have firm specific elements.

The proof of this proposition is as follows. The coefficients b, s and h arise in the following regression model (see Fama and French, 1997, Table 2)

$$R_e - R_f = a + b\left[R_m - R_f\right] + sR_S + hR_H + e \tag{6}$$

Similarly the coefficients b_u, s_u and h_u arise in the following regression model

$$R_u - R_f = a_u + b_u\left[R_m - R_f\right] + s_u R_S + h_u R_H + e_u \tag{7}$$

Equation (2) links the returns R_e and R_u, and is re-written here as

$$R_e = R_u\left[1 + \frac{B}{S}\right] - k_d \frac{B}{S} \tag{8}$$

Substituting (6) and (7) into (8) yields

$$R_f + a + b\left[R_m - R_f\right] + sR_S + hR_H + e = \left\{R_f + a_u + b_u\left[R_m - R_f\right] + s_u R_S + h_u R_H + e_u\right\}\left[1 + \frac{B}{S}\right] - k_d \frac{B}{S}$$

Since each of R_m, R_S and R_H are stochastic, their coefficients on the left and right hand sides of this equation must match. Consequently

$$b = b_u\left[1 + \frac{B}{S}\right] \quad s = s_u\left[1 + \frac{B}{S}\right] \quad h = h_u\left[1 + \frac{B}{S}\right]$$

and this completes the proof of proposition 1. This leads immediately to proposition 2, as follows:

Proposition 2: If the relationships expressed in proposition 1 are employed in conjunction with the Fama-French model, then the Fama-French model will be consistent with MM propositions II and III.

The proof is as follows. Substituting (5) into the Fama-French model in equation (4) yields a cost of equity of

$$k_e = R_f + b_u\left[1+\frac{B}{S}\right]\left[E(R_m) - R_f\right] + s_u\left[1+\frac{B}{S}\right]E(R_S) + h_u\left[1+\frac{B}{S}\right]E(R_H)$$

The Fama-French equation (4) also implies that

$$k_u = R_f + b_u\left[E(R_m) - R_f\right] + s_u E(R_S) + h_u E(R_H)$$

The last two equations then imply that

$$\begin{aligned} k_e &= R_f + \left[k_u - R_f\right]\left[1+\frac{B}{S}\right] \\ &= k_u + \left[k_u - R_f\right]\frac{B}{S} \end{aligned}$$

which accords with MM Proposition II in the presence of riskless debt and no corporate tax. It then follows that MM Proposition III will also be satisfied.

To implement this approach one must first estimate the unlevered sensitivity coefficients. Paralleling the estimation of unlevered betas in a CAPM context, one could estimate the Fama-French sensitivity coefficients by the time-series regression process indicated by equation (6) above, and then use equation (5) to convert these estimates to their unlevered counterparts. Repeating this process over other firms in the same industry, and then averaging over those firms, should improve the estimates. Given the resulting estimates for b_u, s_u and h_u, as the firm's leverage changes through time, equation (5) is used to immediately revise the estimates for b, s and h, and equation (4) then revises the estimate for k_e. Substitution of this into the $WACC$ definition then revises it also.

To illustrate, suppose that the riskfree rate is .06 and the three risk premiums are .052, .032 and .054[4]. In addition a firm's unlevered sensitivity coefficients against the three factors are estimated at .82, -.12 and -.20. Currently the firm has B/S = .10. Following equation (5) this implies levered sensitivity coefficients of[5]

$$b = .82[1+.10] = .90, \quad s = -.12[1+.10] = -.13, \quad h = -.20[1+.10] = -.22$$

Following equation (4) this implies a cost of equity under the Fama-French model of

$$k_e = .06 + .90(.052) - .13(.032) - .22(.054) = .091$$

The $WACC$ is then

$$WACC = .091(.909) + .06(.091) = .088$$

Now suppose that the firm undertakes a pure leverage change, leading to its B/S rising to .70. By definition of a pure leverage change, the unlevered coefficients b_u, s_u and h_u will not be affected. Following equation (5), then (4), and then the $WACC$ definition, the revised estimates for the levered sensitivity coefficients, the cost of equity and $WACC$ are as follows:

$$b = .82[1+.70] = 1.39, \quad s = -.12[1+.70] = -.20, \quad h = -.20[1+.70] = -.34$$

$$k_e = .06 + 1.39(.052) - .20(.032) - .34(.054) = .108$$

$$WACC = .108(.588) + .06(.412) = .088$$

So the leverage increase induces an increase in the estimated cost of equity but no change in $WACC$, in conformity with MM Propositions II and III. If the sensitivity

[4] The market risk premiums are taken from Table 1 of Fama and French (1997).
[5] The levered values are those reported in Table 2 of Fama and French (1997) for the industry labelled as "Beer". Given B/S = .10, the unlevered values were chosen to be consistent with the levered ones.

coefficients were not revised in response to the leverage change then k_e would remain at .091 and the $WACC$ would be erroneously revised downwards to

$$WACC = .091(.588) + .06(.412) = .078$$

This violates the MM propositions. We now examine a process proposed by Fama and French (1997) for revising the sensitivity coefficients s and h.

4. Other Factors Affecting Sensitivity Coefficients

The previous section has shown that each sensitivity coefficient is a separable function of leverage and the unlevered sensitivity coefficient. This implies that any other factors that affect a sensitivity coefficient must do so via the unlevered sensitivity coefficient. Thus each sensitivity coefficient is a separable function of leverage and other factors. In contrast with this, in their Table 4, Fama and French (1997) characterise the sensitivity coefficient s as a function of only firm "size", and they define "size" as equity value. Consistent with this they model s as a function of the firm's market equity value (ME), i.e.,

$$s = s_1 + s_2 Ln(ME) \tag{9}$$

In addition they characterise the sensitivity coefficient h as a function of only the firm's "book-to-market" ratio, and define this in terms of equity rather than total firm value. Consistent with this they model h as a function of the firm's book equity to market equity, i.e.,

$$h = h_1 + h_2 Ln\left(\frac{ME}{BE}\right) \tag{10}$$

These formulations lack the separability property and therefore conflict with equation (5). A simple illustration of this conflict arises by considering the particular case of a firm with s = 0 and h = 0. Invoking equation (5), this implies that $s_u = h_u = 0$. A pure leverage change cannot affect s_u or h_u, and therefore cannot affect s or h when the unlevered values are zero. However a pure leverage change will affect ME, and may affect ME/BE. Consequently the use of equations (9) and (10) will lead to revised estimates for s and h. Thus the results from equations (9) and (10) will conflict with those from equation (5).

Since the Fama-French equations (9) and (10) conflict with equation (5), and the latter is consistent with the MM propositions, this implies that the Fama-French equations will violate the MM propositions. To illustrate this, consider the following example. A firm has B/S = .10 and b = .90. It also has the following values for s_1, s_2, h_1 and h_2:[6]

$$s_1 = .10, \quad s_2 = -.15, \quad h_1 = .27, \quad h_2 = .73$$

The firm also has a market equity value of \$4.6b and a book-to-market equity ratio of .51. Substitution of these values into equations (9) and (10) yields[7]

$$s = .10 - .15Ln(4.6) = -.13, \quad h = .27 + .73Ln(.51) = -.22$$

Substitution of these values for s and h, along with the above value for b of .90, a riskfree rate of .06, and market risk premia of .052, .032 and .054, into equation (4) yields a cost of equity under the Fama-French model of

$$k_e = .06 + .90(.052) - .13(.032) - .22(.054) = .091$$

With B/S = .10, it follows that the $WACC$ will be

$$WACC = .091(.909) + .06(.091) = .088$$

These values for k_e and $WACC$ are identical to those obtained earlier in section III in illustrating the application of equation (5). Consistent with the above market equity value of \$4.6b, and the assumed B/S = .10, the market value of debt is \$460m (which

[6] The values are taken from Table 4 of Fama and French (1997) for the industry labelled as "Beer".
[7] These values for s and h match those presented by Fama-French in their earlier Table 2 for that industry because the market value for equity and the book-to-market equity ratio were chosen to produce this conformity.

will also be attributed to the book value of debt). Consistent with the above book-to-market equity ratio of .51, the book value of equity will be \$2.35b.

We now consider the effect of the firm undertaking a pure leverage change. Consistent with the earlier illustrations, B/S is raised to .70. Accordingly the firm must borrow \$1.62b and pay it out to shareholders. Book debt therefore rises by \$1.62b and book equity falls by the same amount. With riskless debt the market value of debt will rise by this amount. Riskless debt and no taxes also imply that firm value is invariant to pure leverage changes. Consequently the market value of equity must fall by \$1.62b. Thus ME falls to \$2.98b and BE to \$730m, implying that BE/ME falls to .24. Substitution of these values into equations (9) and (10) yields new values for s and h of

$$s = .10 - .15Ln(2.98) = -.06, \quad h = .27 + .73Ln(.24) = -.76$$

Substitution of these into equation (4), along with the unchanged value for b and the market risk premia, yields a revised cost of equity of[8]

$$k_e = .06 + .90(.052) - .06(.032) - .76(.054) = .064$$

With the B/S ratio now equal to .70, it follows that the $WACC$ will be revised to

$$WACC = .064(.588) + .06(.412) = .062$$

By contrast, when B/S was equal to .10, the values for k_e and $WACC$ were .091 and .088 respectively. Thus, the use of equations (9) and (10) implies that a pure leverage change leads to both the cost of equity and $WACC$ falling, the latter by 2.6 percentage points. These are substantial violations of MM Propositions II and III. Moreover the result is not unusual. Table 1 shows the results of this type of calculation for the first ten industries listed in Table 2 of Fama and French (1997). The average change in $WACC$, in response to the pure leverage shift, is a decline of 2.7 percentage points. One industry even sees its $WACC$ decline by so much as to become negative! By

[8] The value for b is not adjusted because Fama and French provide only for changes in s and h.

contrast, as illustrated earlier, the use of equation (5) yields results that are fully consistent with the MM cost of capital propositions.

The problem here is not the Fama-French model, as specified in equation (4). The problem lies with equations (9) and (10), which lack the separability property. If these equations expressed s_u as a function of firm "size" and h_u as a function of the firm's "book-to-market" ratio, separability would be preserved. However, since s_u and h_u are unlevered coefficients, then both "size" and "book-to-market" would have to be defined in terms of total firm value instead of equity value. This leads to proposition 3, as follows:

Proposition 3: If the relationships expressed in proposition 1 are employed in conjunction with the Fama-French model, and s_u is modeled as a function of firm size defined in total value terms, and h_u is modeled as a function of the ratio of the firm's total book to total market value, then the Fama-French model will be consistent with MM propositions II and III.

The proof is as follows. Under the first condition specified here, which is reflected in equation (5), the Fama-French model in equation (4) becomes

$$k_e = R_f + b_u\left[1+\frac{B}{S}\right]\left[E(R_m) - R_f\right] + s_u\left[1+\frac{B}{S}\right]E(R_S) + h_u\left[1+\frac{B}{S}\right]E(R_H) \qquad (11)$$

The second and third conditions specified in the proposition are

$$s_u = s_{u1} + s_{u2}Ln(MV), \qquad h_u = h_{u1} + h_{u2}Ln\left(\frac{BV}{MV}\right) \qquad (12)$$

where MV is the market value of the firm and BV its book value. With these formulations for s_u and h_u, they are (naturally) invariant to pure leverage changes. Following equation (4) the unlevered cost of equity would then be

$$k_u = R_f + b_u\left[E(R_m) - R_f\right] + s_u E(R_S) + h_u E(R_H)$$

This equation and equation (11) then implies that

$$k_e = R_f + \left[k_u - R_f\right]\left[1 + \frac{B}{S}\right]$$
$$= k_u + \left[k_u - R_f\right]\frac{B}{S}$$

and this is MM proposition II. The MM proposition III then follows.

Formulas (12) allow the cost of equity to respond, as Fama and French seek, to changes in the firm's size and its book-to-market ratio. However size and book-to-market must be defined in total value rather than equity value terms, so that the parameters s_u and h_u are invariant to pure leverage changes. Pure leverage changes are dealt with in equation (5). Thus the parameter s is a separable function of leverage and size, and the parameter h is a separable function of leverage and the book-to-market ratio, i.e.,

$$s = \left[1 + \frac{B}{S}\right]\left[s_{u1} + s_{u2}Ln(MV)\right], \quad h = \left[1 + \frac{B}{S}\right]\left[h_{u1} + h_{u2}Ln\left(\frac{BV}{MV}\right)\right] \tag{13}$$

All of this demonstrates that purely empirical processes should not be used to estimate the sensitivity coefficients in any cost of equity model. Any such process must be consistent with the MM cost of capital propositions. Accordingly equations like (5) must be employed to deal with leverage changes, otherwise violations of the MM cost of capital propositions will occur. Equations like (12) can be used to model other factors that affect the sensitivity coefficients. Thus each sensitivity coefficient in a cost of equity model must be a separable function of leverage and other factors, as in equations (13).

5. Conclusion

This paper shows that the sensitivity coefficients in the Fama-French model are related to the firm's leverage in a fashion like that of the Hamada formula.

Furthermore such a formula (or a variant to reflect variation in factors such as the taxation regime) must be used to deal with leverage changes by a firm, and failure to do so leads to violations of the MM cost of capital propositions. It is also shown that the empirical formulas developed and used by Fama and French (1997) to model the evolution through time in the cost of equity are in conflict with these MM propositions. Substitute formulas are suggested that are consistent with the MM propositions. These substitute formulas exhibit separability in leverage and other factors affecting the sensitivity coefficients. Furthermore size must be defined in terms of total firm value rather than merely equity value.

REFERENCES

Conine, T., “Corporate Debt and Corporate Taxes: An Extension”. *The Journal of Finance*, 35 (1980), 1033-36.

Damodaran, A. *Corporate Finance: Theory and Practice*. New York: John Wiley & Sons, Inc (1997).

Ehrhardt, M., and E. Shrieves. “The Impact of Warrants and Convertible Securities on the Systematic Risk of Common Equity.” *The Financial Review*, 30 (1995), 843-856.

Fama, E., and K. French. “The Cross-Section of Expected Returns.” *The Journal of Finance*, 47 (1992), 427-65.

____________________ “Common Risk Factors in the Returns on Stocks and *Journal of Financial Economics*, 33 (1993), 3-56.

____________________ “Size and Book-to-Market Factors in Earnings and Returns.” *The Journal of Finance*, 50 (1995), 131-155.

____________________ “Industry Costs of Equity.” *Journal of Financial Economics*, 43 (1997), 153-93.

____________________ “The Corporate Cost of Capital and the Return on Corporate *The Journal of Finance*, 54 (1999), 1939-1967.

Hamada, R. “The Effect of the Firm’s Capital Structure on the Systematic Risk of *The Journal of Finance*, 27 (1972), 435-52.

Lintner, J. “The Valuation of Risky Assets and the Selection of Investments in Stock Portfolios and Capital Budgets.” *Review of Economics and Statistics*, 47 (1965), 13-37.

Miles, J., and J. Ezzell. “Reformulating Tax Shield Valuation: A Note.” *The Journal of Finance*, 40 (1985), 1485-92.

Miller, M. “Debt and Taxes.” *The Journal of Finance*, 32 (1977), 261-76.

Modigliani, F., and M. Miller. “The Cost of Capital, Corporation Finance and the *American Economic Review*, 48 (1958), 261-97.

________________________ “Corporate Income Taxes and the Cost of Capital: A Correction.” *American Economic Review*, 53 (1963), 433-43.

Mossin, J. “Equilibrium in a Capital Asset Market.” *Econometrica*, 24 (1966), 768-83.

Ross, S. “The Arbitrage Theory of Capital Asset Pricing.” *Journal of Economic Theory*, 13 (1976), 341-60.

Sharpe, W. "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions
The Journal of Finance, 19 (1964), 425-42.

Table 1

WACC and leverage

Industry	$B/S = .10$ *WACC*	$B/S = .70$ *WACC*	Change
Drugs	.062	-.003	-.065
MedEq	.085	.066	-.019
Hlth	.117	.100	-.017
Comps[a]			
Chips	.117	.090	-.027
BusSv	.120	.098	-.022
LabEq	.114	.095	-.019
Hshld	.090	.072	-.018
Meals	.123	.094	-.029
Beer	.088	.062	-.026

This table shows the change in *WACC* for various industries arising from a pure leverage change that raises B/S from .10 to .70. I doing so the cost of equity is calculated from the Fama-French model along with their suggested process for calculating the sensitivity coefficients.

[a] Not calculated because Book Equity would become negative.

THE FAMA-FRENCH MODEL, LEVERAGE AND THE MM PROPOSITIONS

MM III with no taxes: $WACC = k_u$

$\Rightarrow$ leverage changes do not affect $WACC$

Consistent with use of the CAPM, providing that the estimate of $\boldsymbol{b}_e$ adjusts immediately to a leverage change, i.e.,

$$\boldsymbol{b}_e = \boldsymbol{b}_u \left[1 + \frac{B}{S} \right]$$

$$\Rightarrow k_e = R_f + \left[E(R_m) - R_f \right] \boldsymbol{b}_u \left[1 + \frac{B}{S} \right]$$

$$WACC \equiv k_e \left(\frac{S}{V} \right) + R_f \left(\frac{B}{V} \right)$$

$$\Rightarrow WACC = R_f + \left[E(R_m) - R_f \right] \boldsymbol{b}_u = k_u$$

Is the Fama-French model consistent with MM III?

$$k_e = R_f + b\left[E(R_m) - R_f\right] + sE(R_S) + hE(R_H)$$

where R_S is the return on a portfolio of small stocks less that on a portfolio of large stocks, R_H is the return on a portfolio of stocks with high book-to-market equity less that on a portfolio of stocks with low book-to-market equity, and the coefficients b, s and h are the sensitivities of the firm's equity returns to R_m, R_S and R_H respectively.

Proposition 1: The Fama-French sensitivity coefficients b, s and h are related to the firm's leverage and its unlevered sensitivity coefficients b_u, s_u and h_u as follows:

$$b = b_u\left[1 + \frac{B}{S}\right] \quad s = s_u\left[1 + \frac{B}{S}\right] \quad h = h_u\left[1 + \frac{B}{S}\right]$$

Proposition 2: If the relationships expressed in proposition 1 are employed in conjunction with the Fama-French model, then the Fama-French model will be consistent with MM propositions II and III.

Proof: $$k_e = R_f + b_u\left[1 + \frac{B}{S}\right]\left[E(R_m) - R_f\right] + s_u\left[1 + \frac{B}{S}\right]E(R_S) + h_u\left[1 + \frac{B}{S}\right]E(R_H)$$

$$WACC \equiv k_e\left(\frac{S}{V}\right) + R_f\left(\frac{B}{V}\right)$$

$$\Rightarrow WACC = R_f + b_u\left[E(R_m) - R_f\right] + s_u E(R_S) + h_u E(R_H) = k_u$$

Fama and French (1997, Table 4) characterise s as a function of only firm "size", and they define "size" as market equity value, i.e.,

$$s = s_1 + s_2 Ln(ME)$$

They also characterise h as a function of only the firm's "book-to-market" ratio, and define this in terms of equity rather than total firm value, i.e.,

$$h = h_1 + h_2 Ln\left(\frac{ME}{BE}\right)$$

These formulations lack the separability property of proposition 1, and therefore proposition 2 also fails.

Eg: $s = 0$ and $h = 0$

$\Rightarrow s_u = h_u = 0$

A pure leverage change cannot affect s_u or h_u

$\Rightarrow$ a pure leverage change should not affect s or h

But a pure leverage change affects ME, and may affect ME/BE

$\Rightarrow$ a pure leverage change affects s and h through the Fama-French functions

$\Rightarrow$ the Fama-French functions violate proposition 1

Solution: $s_u = s_{u1} + s_{u2} Ln(MV), \quad h_u = h_{u1} + h_{u2} Ln\left(\frac{BV}{MV}\right)$

General Principle: Estimation processes for sensitivity coefficients in cost of equity models must be checked for conformity with the MM propsitions.