Do the New Zealand Daily Exchange Rates Behave Nonlinearl or Like Chaos

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Abstract

The last two decades have witnessed a fact that the nonlinear dynamic analysis has spread to the economi and financial field. In the mean time, following the modern physic studies, some economists and financia analysts also borrow the concept of chaos and attempt to describe the complex phenomenon of economic events and financial volatilities. Since the Bretton Wood System collapsed, the major liquid currencies have been fluctuated and characterized highly volatility. In this research, we employ statistical methods tc investigate the behaviours of New Zealand daily spot exchange rates. We found that some exchange rat es exhibit nonlinear features but not chaotic process. Further, these nonlinear structures contain compass ros pattern and consequently generate more complicated dependence.

Keywords: Exchange rates, Nonlinearity test, Chaos, Compass rose, GARCH model.

I. Introduction

Since the Bretton Woods System collapsed in the early of 1970's, most countries' exchange rate systems have switched from being fixed to being floating. Under such a regime, a deficit or a surplus in a nation's balance of payments is automatically correcte by a currency depreciation or appreciation. As a result, fluctuations of highly liquid currencies such as the US dollar, the UK pound, the Deutsche mark and the Japanese ye seem to have become quite erratic. According to traditional economic theory, tracing th behavior of exchange rate should rely on one of four models that are believed to captur some macro fundamentals: the monetary model (e.g. Frenkel (1976)), the sticky-price model (e.g. Dornbusch (1976)), the equilibrium model (e.g. Stockman (1980)), and the portfolio balance model (e.g. Branson (1984)). On the other hand, following the theory o rational expectations developed by Muth (1961), Fama (1970) proposed a parallel theor of expectation formation in finance – the efficient market hypothesis (EMH): expectations in financial markets amount to optimal forecasts using all available information. An important implication of EMH is that the stock price should approximately follow a random walk; that is, for all practical purposes, future changes i stock prices should be unpredictable. This implies that returns on stocks are subject to independent and identical distribution (i.i.d.). The random walk version of EMH was als supported by some researchers in the field of foreign exchange market. See Mussa (197 1986), and Meese and Rogoff (1983) for the evidences on the random walk properties o nominal exchange rates and real exchange rates.

The debates on which one, the complex macroeconomic model or the simple random walk model, is better for capturing exchange rate behaviour have been long-standing (se Taylor and Sarno (1998), and MacDonald and Taylor (1992) for an excellent survey). MacDonald and Taylor (1993, 1994) re-examined the monetary model and argued that although it is a valid framework for analyzing the long-run behaviour of exchange rates it can not outperform the random walk and other models in short-run dynamics. West an Cho (1995) found that for longer horizons, it is difficult to find grounds for choosing between the various models. None of the models perform well in a conventional test of forecast efficiency. Mark and Choi (1997) also found that the real exchange rate is essentially unpredictable at horizons of 3 to 12 months. They conclude that short horizo changes in real exchange rates are dominated by noise, but because real exchange rates are ultimately governed by economic fundamentals, these noisy fluctuations reverse themselves over time. Further more, Taylor and Allen (1992) investigated the behavior technical analysts (or chartist) of the Bank of England and discovered a fact that at the longest forecast horizons considered (one year or longer), nearly 85% agreed that fundamentals are more important than chart analysis at this horizon. Nevertheless, Floo and Taylor (1996) pointed out that empirical models based on pure fundamental economic theory fail to provide an adequate explanation of short-term movements in exchange rates. But the revelation that foreign exchange participants focus more on fundamentals at longer horizons suggests that more attention might fruitfully be paid to modeling the fundamental determinants of long-term exchange rates. Hence, how to accurately model and forecast the exchange rate in short run is a hot potato attracting many economists and econometricians involved in this field.

Instead of macroeconomic relations such as money demand and purchasing power parit (PPP), many researchers began deeply digging into the properties of short-run exchange rate volatilities through microstructure methods (Sarno and Taylor (2001)). Many previous statistical studies indicated that the fluctuations of exchange rates do not exhib a random walk pattern. Huizinga (1987) provided evidence that the long-run behavior o monthly real exchange rates differ from a random walk by having a notable mean reverting component. That is, there is substantial negative serial correlation of changes i real exchange rates. Hsieh (1988, 1989b) examined the statistical properties of daily exchange rates and found that exchange rate changes are not independent and identicall distributed. Actually, as observed early by Mandelbort (1963) and Fama (1965), financi returns display pronounced volatility clustering and the unconditional distributions of th price changes were typically fat-tailed or leptokurtic. The same result was also confirme for the foreign exchange rate market by Boothe and Glassman (1987). The studies by Baillie and Bollerslev (1989) and Hsieh (1989a) using daily spot exchange rate changes indicate that the conditional heteroscedasticity can be well represented by a GARCH (1,1) process. This implies that the short-term exchange rate behavior can not simply be described by a linear model (e.g. ARIMA or random walk) due to variation of the conditional variance. In contrast to the belief of Diebold and Nason (1990) and Meese and Rose (1991) that accounting for non-linearities in current exchange rate models is n a promising way to improve the measurement, many econometricians in recent years employ various nonlinear models to trace the points of short-run exchange rate movements. See, for instance, Kräger and Kugler (1993) for SETAR (self-exiting threshold autoregressive) model, Kuan and Liu (1995) for NNW (neural network) mode Sarantis (1999) for STAR (smooth transition autoregressive) model, and Wu and Chen (2001) for TVPMS (time-varying parameters and Markov-switching heteroskedasticity) model among others. In addition, Andersen et al. (1998a, 1998b, 1999, 2001) use high-frequency intradaily data to show that standard volatility models do provide accurate forecasts of exchange rates.

Since the stock market crash of October 19, 1987, interest in nonlinear complex dynamics, especially deterministic chaotic dynamics, has increased rapidly in the financ and macroeconomics professions. Hsieh (1991) defined that chaos is a nonlinear deterministic process which "looks" random. We can use the following simple populati model to illustrate the concept of chaos. Suppose that a fraction x of population has an infectious disease, so that a fraction $G = 1\text{-}x$ does not. In the simplest model of the spreading of the disease, members of the population are assumed meeting freely. The ra of increase of x is thus proportional to both x and G, i.e.

$$\frac{dx}{dt} = ax(1-x) \qquad (1.1)$$

where a is a positive constant. Then the population dynamics can be represented by the following logistic equation:

$$x_{t+1} = ax_t(1-x_t) \qquad 0<x<1 \qquad (1.2)$$

The process exhibited in Figure 1 can vary from a stable behaviour to an intermittent an then to a chaotic state by small changes in the value of a.

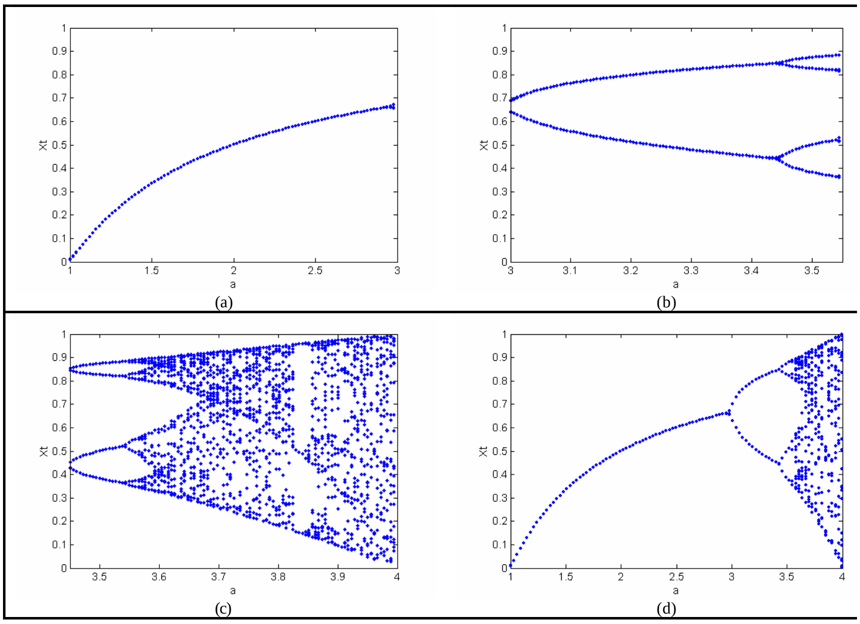


Figure 1

Figure 1(d) displays the whole process of Equation (1.2) where 1< *a* <4. In Figure 1(a), 1< *a* <3, and the system stays at an equilibrium level meaning the population reaches a size where birth and death balance out. Equation (1.2) in this stage has a single solution. However, when *a* = 3, two solutions (often called "period 2") appear in Figure 1(b). Thi event is called a pitchfork bifurcation or period doubling. As *a* further increases, the solutions proportionally appear. Finally, at approximately 3.5< *a* <4, the plot presents a chaotic pattern (see Figure 1(c)). The number of solutions tends towards infinite. Therefore, chaos can be viewed as the culmination of a sequence of changes in the cycli behavior of a nonlinear function. Meanwhile, chaotic functions are also sensitive to thei initial parameter values. Clearly, unlike random walk, the chaotic pattern at least in low dimension is forecastable.

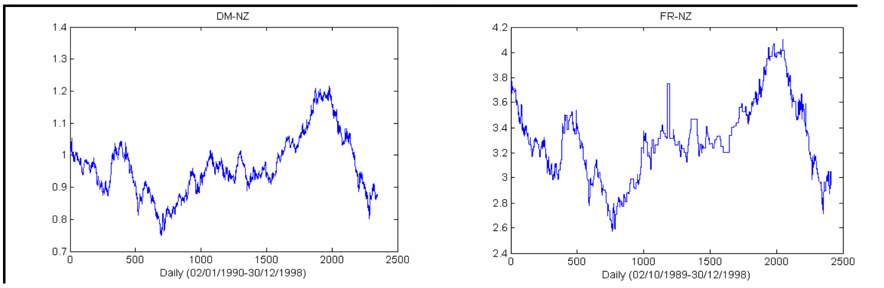
Inconsistent with Hsieh's (1991) conclusion that there is no evidence of low complexity chaotic behavior in stock returns, research results on the possibility of low-dimensional chaos in foreign exchange rates have been mixed. Bajo-Rubio et al. (1992) claimed that was possible to make short-term predictions with daily Peseta-US dollar spot and forwa exchange rates based on the presence of deterministic chaos. De Grauwe et al. (1993) presented empirical evidence of daily spot rates in some major exchange markets and found that although there are some evidences for chaos, it can not be said that they are conclusive. Further, Cecen and Erkal (1996) tested six months' hourly exchange rates a concluded that there is indeed nonlinear dependence in returns but the nature of this nonlinear dependence is far from being deterministic.

For whether or not the chaotic phenomenon exists in the foreign exchange market, it is a necessary condition to find evidence for nonlinear dependence in exchange rate returns. The reason is that an iid series is a white noise but not necessarily vice-versa. For instance, we can simulate the autocorrelations and partial autocorrelation of Equation (1.2). The results may insignificantly differ from zero implying that the series possesses the dynamic part of the white noise properties. However, x_{t+1} is obviously not iid., as it generated from a nonlinear deterministic process (see Granger (1983)).

New Zealand floated the dollar in March 1985 and the regime is characterized as a floating one to the extent that the Reserve Bank has not directly intervened in the foreig exchange market since. Nonetheless, even under this floating regime, domestic moneta policy actions have always had an influence on and have been increasingly influenced b movements in the dollar. In this case, the exchange rate level is an important monetary variable as the New Zealand economy has a large external sector. Theoretically, Dornbusch (1976)'s sticky-price model states that overshooting of the exchange rate ma occur in response to monetary policy shocks as financial markets adjust faster to disturbances than the good and labour markets. According to this, the dynamic of exchange rate thus reflects the degree of a country's monetary shocks. However, Wilkinson et al (2001) studied foreign exchange markets in New Zealand and Australia and found that monetary policy shocks do contribute to the volatility of both exchange rates, but their movements are not always consistent with theory. In particular, they fou that there is little support for the overshooting hypothesis and the exchange rates do not always move in the direction normally anticipated, especially for New Zealand. Althou Hansen and Hutchison (1997) modeled New Zealand monthly exchange rates as an erro correction process, and Rae (2000) detected that a significant time-varying risk premium is related to volatility in the NZ dollar spot market, to our knowledge no previous study has ever tested for nonlinearity in daily New Zealand spot exchange rates so far. Thus, i the evidence of nonlinearity is found, forecasts in short term may be improved by switching from a linear to a nonlinear modeling strategy. Furthermore, it will enhance tl quality of financial decision, as the detected exchange rate behaviour may provide more accurate price signals to domestic agents over time and may facilitate adjustment to a variety of external and internal shocks. Therefore, in this research we will employ statistical methods to investigate whether the New Zealand daily spot exchange rates behave nonlinearly or contain chaotic process. The remainder of this report is organized as follows. The next section involves a description of the data and gives some theoretica treatments of nonlinear dependence tests. The third section discusses the estimation results for the nonlinearity tests. In section four, we will illustrate and investigate the compass rose phenomenon. The final section offers some conclusions.

II. Research Design and Methodology

2.1 Data Description



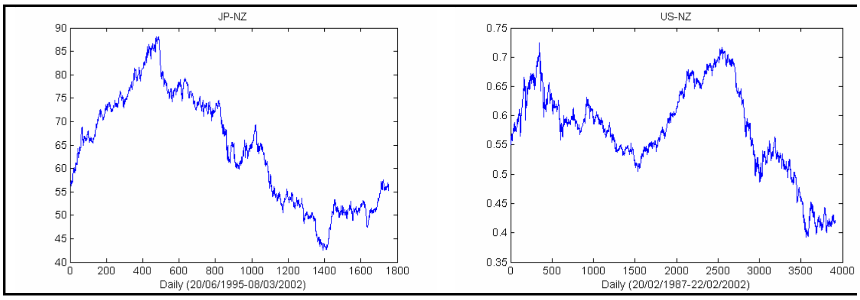


Figure 2. Daily Spot Exchange Rates

The data, taken from Datastream International, consist of the daily spot exchange rates o New Zealand dollars against four major currencies: U.S. dollars (US), Japanese yen (JP Deutsche mark (DM), and French franc (FR) (shown in Figure 2). The lengths of the fo series are unfortunately not consistent. For instance, the starting date of JP is 20 June 1995 due to unknown reason, and DM and FR are all cut off in 30 December 1998 due t the introduction of Eurodollar. We also have to use long horizon data when their frequencies are low to avoid the sensitiveness of some tests. For example, the BDS statistic requires 500 or more observations (Brock, Hsieh, and LeBaron 1991); otherwis for a small sample size, the probability distribution of the test may deviate quite substantially from the standard normal distribution. In addition, the short horizon data would also lead to noise dominating the series. Table 1 provides some information of ra data.

Table 1. The raw data

Series	Frequency	Periods	Observations
Deutsche mark (DM)	Daily	02/01/1990-30/12/1998	2347
French franc (FR)	Daily	02/10/1989-30/12/1998	2413
Japanese yen (JP)	Daily	20/06/1995-08/03/2002	1754
The US dollar (US)	Daily	20/02/1987-22/02/2002	3916

Data source: Datastream International.

The rates of change of the four exchange rates are calculated as

$$x_t = 100 \cdot \log(R_t / R_{t-1}) \qquad (2.1)$$

and displayed in Figure 3.

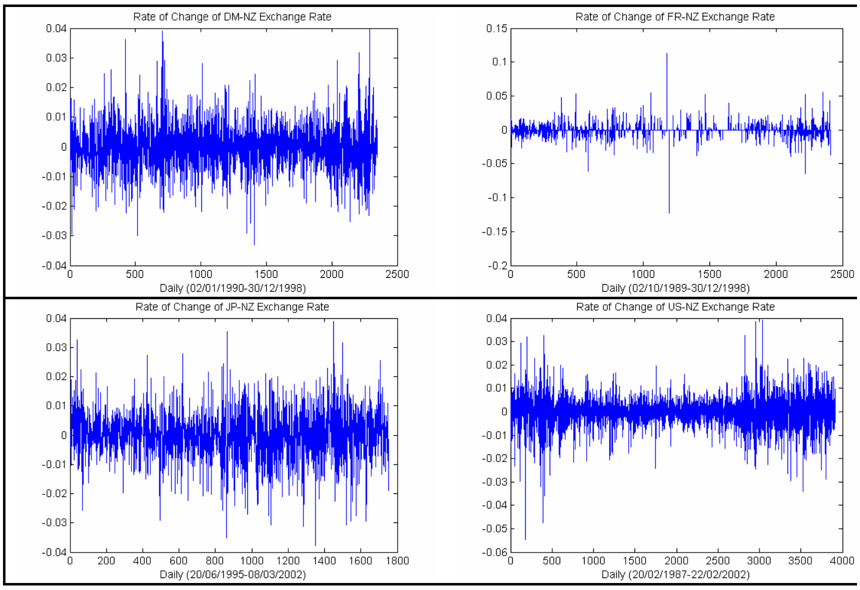


Figure 3. Rate of Change of Exchange Rates

The rate of change so obtained can be interpreted as a series of continuously compound daily returns. This transformation has become standard in the finance literature, and one possible reason for using returns rather than raw data is that the raw data are likely to be nonstationary, and non-stationarity may cause a spurious rejection of linearity.

Table 2. Summary Statistics of Log Price Changes

Statistics	DM	FR	JP	US
Mean	-0.005649	-0.01021	-0.001924	-0.007053
Median	0.000	0.000	0.000	0.000
Standard Deviation	0.7737	0.8294	0.8622	0.6281
Kurtosis	4.8315	46.5210	4.4605	9.1618
Skewness	0.1582	-0.1299	-0.1701	-0.4260
Maximum	3.956	11.280	3.891	3.904
Minimum	-3.302	-12.270	-3.781	-5.450
Ljung-Box-Pierce Q-Test				
Raw Data $Q_x(50)$	83.2772*	56.2505	33.9979	78.4577*
Squared Data $Q_{xx}(50)$	313.4267*	348.5543*	171.8912*	756.5550*

Note: * indicates that the results are significant where the critical value is 67.5048 at lag 50 (0.05).

Table 2 provides summary statistics of the returns. The kurtosis statistics are all greater than that of standard normal distribution, especially for FR, indicating heavy tails. The statistics of Ljung-Box-Pierce Q-test are also reported in the bottom of Table 2. The $Q_{xx}(50)$ statistics of the squared data are significantly larger than the $Q_x(50)$ statistics o the raw data, suggesting strong conditional heteroscedasticity.

2.2 The Statistic Test

Barnett et al. (1997) argued that some of the "competing" nonlinearity tests could be viewed as complementary rather than competing. So using all of them jointly can produ a deeper insight into the nature of the nonlinearity that may exist in the data. They suggested that the BDS test and the Kaplan test are omnibus tests that test nonlinearity against all possible alternatives. If linearity is rejected by the BDS and Kaplan tests, it becomes reasonable to use more focused tests to try to distinguish among the possible forms of nonlinearity. In this research we only implement two nonparametric tests (the BDS test and White's ANN test) and two parametric tests (Tsay's F test and RESET tes to detect the nonlinearity. Meanwhile, Hsieh's (1989b) third order moment test is also considered.

2.2.1 The BDS Test

The BDS test developed by Brock et al (1996)[1] can be applied to the residuals of a linea autoregression to check whether they are generated by an independent and identical distribution (iid) process. It takes the concept of the correlation integral (the idea of Grassberger and Procaccia) and transforms it into a formal statistic test which is asymptotically distributed as a standard normal variable. Given the *m*-history of the filtered data:

1-history: $x_t^1 = x_t$

2-history: $x_t^2 = (x_t, x_{t+1})$

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m-history: $x_t^m = (x_t, x_{t+1}, ..., x_{t+m-1})$ (2.2)

[1] The first version of the BDS test was developed in 1987 as unpublished manuscript by Brock, W.A., Dechert, W.D., and Scheinkman, J.A.

An m-history is thus a point in m-dimensional space and m is known as the "embedding dimension". Next choose an arbitrary size of dimensional distance e of a pair vectors. The only condition is that it must not exceed the spread of the series:

$$0 < e < \max(x_t) - \min(x_t) \tag{2.3}$$

Now consider the probability of any pair of observations (of length T) x_i^m, x_j^m lying within e of each other:

$$P_1 \equiv P(\|x_i^m - x_j^m\| \leq e) \qquad \text{for any integers } i \neq j. \tag{2.4}$$

$\|\cdot\|$ denotes the sup -norm for distance measurement.

A similar relationship in dimension 2 is defined by any two observations and their respective neighbors directly preceding them. The probability of "closeness" here mean that the probability of any two observations is "close" to each other as well as their two predecessors being "close" to each other, i.e. the probability of a history of two observations being within e of each other:

$$P_2 \equiv P(\|x_{i-1}^m - x_{j-1}^m\| \leq e, \|x_i^m - x_j^m\| \leq e) \qquad \text{for any integers } i \neq j. \tag{2.5}$$

Hence, the probabilities for the two dimensions P_1 and P_2 are obviously different. Nevertheless, if the series is i.i.d., there is a well defined relationship between the two: the probability of a two-observation history being "close" is equal to the square of the probability of any two observations being "close":

$$P_2 = P_1^2 \qquad \text{if } x_t \sim i.i.d. \tag{2.6}$$

For any dimension, the BDS test for embedding dimension m can be described as a test the null hypothesis that the probabilities for dimension 1 and for dimension m are equal

$$H_0 : P_m = P_1^m \qquad H_1 : P_m \neq P_1^m \tag{2.7}$$

Therefore, the above null hypothesis is equivalent to the test of "iidness" against all oth alternatives:

$$H_0 : x_t \sim i.i.d \tag{2.8}$$

To obtain the test statistic, probability P_m is calculated by the correlation integral $C_{m,T}(e)$ in finite space on a sample of T observations:

$$C_{m,T} = \frac{2}{(T-m+1)(T-m)} \sum_{1<i<j<T} I_e(x_i^m, x_j^m) \qquad (2.9)$$

where I_e is an indicator (Heavside) function that equals 1 if $\|x_i^m - x_j^m\| \leq e$ and 0 otherwise.

The BDS test statistic is therefore defined as follows:

$$W_{m,T}(e) = \sqrt{T}\left(\frac{C_{m,T}(e) - C_{1,T}(e)^m}{s_{m,T}(e)}\right) \qquad (2.10)$$

where the estimated variance is given by:

$$s_{m,T}^2(e) = 4\left[K^m + 2\sum_{l=1}^{m-1} K^{m-l}C^{2l} + (m-1)^2 C^{2m} - m^2 K C^{2m-2}\right] \qquad (2.11)$$

Parameter *C* is the first-dimensional correlation integral:

$$C = C_{1,T}(e) \qquad (2.12)$$

and *K* is the probability of any triple of observations lying within distance e of each other and is thus to be computed as:

$$K = K_T(e) = \frac{6\sum_{i<j<k} h_e(x_i^m, x_j^m, x_k^m)}{T_m(T_m-1)(T_m-2)} \qquad (2.13)$$

where $h_e(u,v,w) = \frac{1}{3}\left[I_e(u,v)I_e(v,w) + I_e(u,w)I_e(w,v) + I_e(v,u)I_e(u,w)\right]$ (2.14)

Then the BDS statistic follows the standard normal distribution asymptotically:

$$\lim_{n\to\infty} W_{m,T}(e) \sim N(0,1) \qquad \text{for any } m, e \qquad (2.15)$$

In general, the BDS test is a two-side test so that the rejection of the null of iid occurs when the estimated value of the *W*-statistic is more extreme (in either tail) than the corresponding statistic from the normal tables. Meanwhile, the BDS statistic is more complicated due to the determination of the embedding dimension *m* and the distance e (in number of standard deviation s of the data). Brock et al. (1991) states that if *m* is to large relative to the sample size, the BDS statistic will be very ill-behaved, because ther are too few independent (nonoverlapping) points. If e/s is too small or too large, the BDS statistic will also be ill-behaved, because there are too few or too many points "close" to any given *m*-vector. Depending on the Monte Carlo simulations, they

recommend that the dimension should be between 2 to 5, and e / s should be from 0.5 t
2. It also necessary to note that Brock et al. (1991) warn that the number of nonoverlapping data points T / m should not be less 200 for the asymptotic distribution hold. Otherwise, one must bootstrap the size of the BDS statistic.

The framework of the BDS test designed for this research mainly follows the procedure of Hsieh (1989b) and Brock et al. (1991). For the aim of BDS test, the calculations mus be in terms of estimated residuals of linear fit. We then employ the following autoregression:

$$x_t = b_0 + b_{Mo} D_{Mo,t} + b_{Tu} D_{Tu,t} + b_{We} D_{We,t} + b_{Th} D_{Th,t} + b_H HOL_t + \sum_{i=1}^{k} b_i x_{t-i} + x_t \qquad (2.1$$

where $D_{Mo,t}$, $D_{Tu,t}$, $D_{We,t}$, and $D_{Th,t}$ are dummy variables for Monday, Tuesday, Wednesday, and Thursday respectively, and HOL_t is the number of holidays (excluding weekends) between two successive trading days. Here, the denoted holidays are: New Year's Day (1 January and 2 January), Waitangi Day (6 February), Good Friday, Easter Monday, ANZAC Day (25 April), Queen's Birthday (6 June), Labour Day (28 October) Christmas Day (25 December), and Boxing Day (26 December). These dummy variable are incorporated in order to remove any deterministic calendar effects in the data which may arise from the market microstructure or the non-homogeneous arrival of informatio through the week. Failure to remove seasonality from the data may lead to a spurious rejection of whiteness. Meanwhile, the choice of autoregressive lag length k can be mad on various grounds, including Box-Jenkins interactive procedure, or the minimization of Schwarz's Bayesian Information Criterion and Akaike's Information Criterion, or the maximization of $\bar{R}^2$. Hsieh (1989b) proposed an alternative method that the k is determined by the minimum number of lags of Ljung-Box-Pierce statistic $Q_x(50)$ which is insignificant at the 10% level. Accordingly, the identified models are k = 0, 10, 10, an 1 for DM, FR, JP, and US respectively. Although the t-ratio of dummy variable for Monday in DM is slightly high, the regression results basically indicate that four series do not exhibit daily seasonality (see Table 3).

Table 3. The Regression Results of Dummy Variables

Series	*k*	$D_{Mo,t}$	$D_{Tu,t}$	$D_{We,t}$	$D_{Th,t}$	HOL_t
DM	0	-0.10601 (-2.10)	-0.06695 (-1.32)	-0.06048 (-1.20)	-0.06797 (-1.34)	-0.01588 (-0.17)
FR	10	-0.08455 (-1.58)	-0.03460 (-0.65)	-0.03788 (-0.71)	-0.04375 (-0.82)	-0.18773 (-1.91)
JP	10	0.02031 (0.31)	-0.10310 (-1.57)	-0.08901 (-1.36)	-0.05909 (-0.90)	0.1218 (0.99)
US	1	-0.03593 (-1.13)	-0.01095 (-0.34)	-0.02331 (-0.73)	0.01237 (0.39)	-0.01649 (-0.27)

Note: figures in parentheses are t-statistics.

Apart from the BDS test of the filtered returns, we also perform the BDS test of standardized residuals of the GARCH (1, 1) model for these exchange rates. This is because that the asymptotic distribution does not approximate very well when the BDS statistic applied to standardized residuals of ARCH, GARCH, and EGARCH models (se Brock et al. (1991) and Hsieh (1991)). A simplest GARCH (1, 1) model is specified as follows:

$$h_t = d_0 + d_{Mo} D_{Mo,t} + d_{Tu} D_{Tu,t} + d_{We} D_{We,t} + d_{Th} D_{Th,t} + d_H HOL_t + d h_{t-1} + q x_{t-1}^2 \qquad (2.17)$$

where x_{t-1} is estimated from Equation (2.16).

The standardized residuals are obtained from:

$$z_t = \hat{x}_t / \hat{h}_t^{1/2} \qquad (2.18)$$

The purpose of this diagnostic test is that if the GARCH (1, 1) model is correctly specified, the standardized residuals z_t should be i.i.d. random variables in large samples.

The critical values of BDS test are given in Brock et al (1991) (Appendix C and Appendix F), and Hsieh (1991) (Table XIII). Although these Monte Carlo simulation results are generated from 100 to 2500 observations, we can roughly infer that the null hypothesis can be rejected with 95% confidence when *W* exceeds 2.0 and with 99% confidence when *W* exceeds 3.0 in a larger distance e / s . As a result, both significant positive and negative BDS statistic values indicate non-i.i.d. behavior.

2.2.2 White's Neural Network Test

The main idea of White's (1998a, 1998b) test involves modeling the elements of the generating process as an augmented single hidden-layer feed forward neural network (SHLFFN). A sketch of the structure of SHLFFN is given in the following figure:

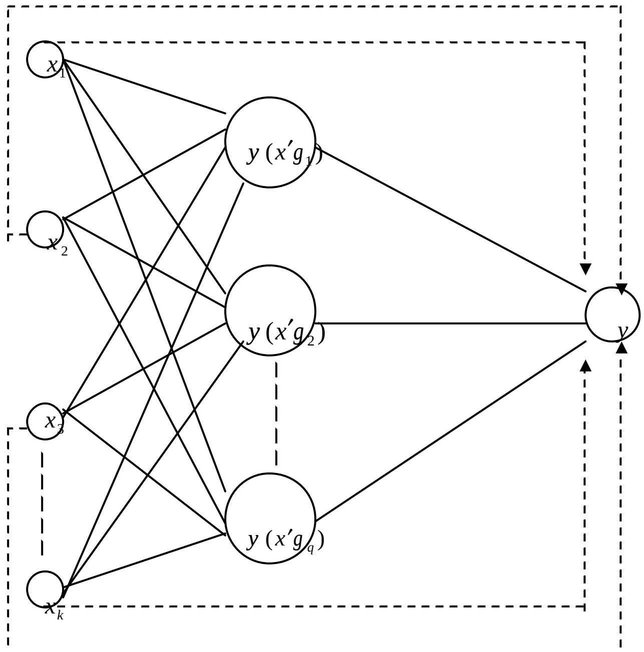


Figure 4. SHLFFN

In the left of Figure 4, x_l (l = 0, 1, 2,……, k) denotes the column vector of messages ser by the neurons (or input units): $x = (x_0, x_1, x_2, \ldots\ldots, x_k)'$. In network lingua, each unit is said to be arranged in a hidden layer. In the first step the incoming information is summed up and the signal is received by the layer as $g_{jl} x_l$, where $g_{jl} = (g_{j0}, g_{j1}, g_{j2}, \ldots\ldots, g_{jk})'$ (j = 1, 2,……, q) denotes pseudo random numbers as connection strength associated with hidden layer unit j, then the value of the activation function can be represented as $y(x'g_j)$. Hence, the hidden-layer unit transmits the filtered new information $y(x'g_j)$ and produces the output y. The SHLFFN thus perform as:

$$y = x'q + [y(x'g_1), y(x'g_2), \ldots\ldots, y(x'g_q)]b \qquad (2.19)$$

where y is filtered residuals and $x'q$ is the augmented component. In the case of b = 0, the model reduces to an affine-type network, a structure resembling a linear regression model.

Carrying out White's test for neglected nonlinearity under the hypothesis of linearity in the mean, the activation function $y(x'g_j)$ is specified as logistic function:

$$y(x'g_j) = (1 + e^{-x'g_j})^{-1} \qquad (2.20)$$

The steps of implementation are:

(1) Regress y_t linearly on X_t as $y_t = X_t'q + e_t$ and save the estimated residuals $\hat{e}_t$.

(2) Generate the phantom hidden unit activations

$\Psi_t = [y(x'g_1), y(x'g_2), \ldots\ldots, y(x'g_q)]'$ and regress $\hat{e}_t$ on X_t and Ψ_t as

$$\hat{e}_t = X_t'd + \Psi_t'l + e_t \qquad (2.21)$$

The complex test statistic is given by

$$M_T = \left(T^{-1/2}\sum_{t=1}^{T}\Psi_t\hat{e}_t\right)'\hat{W}_T^{-1}\left(T^{-1/2}\sum_{t=1}^{T}\Psi_t\hat{e}_t\right) \qquad (2.22)$$

where T is sample size and $\hat{W}_T$ is a consistent estimator of $W^* = \text{var}\left(T^{-1/2}\sum_{t=1}^{T}\Psi_t e_t^*\right)$

($e_t^* = y_t - X_t'q^*, q^*$ is the parameter vector of the optimal linear least squares approximation to E($y_t \mid X_t$)).

$$M_T \xrightarrow{d} c^2_{(1-a)}(q) \text{ as } n \to \infty \text{ under } H_0 \qquad (2.23)$$

To avoid implicit computation of $\hat{W}_T$, M_T can be computed as

$$nR^2 \xrightarrow{d} c^2_{(1-a)}(q) \qquad (2.24)$$

where R^2 is the uncentered squared multiple correlation from Equation (2.21).
If $nR^2 > c^2_{(1-a)}(q)$ then reject H_0 of linearity in the mean.

2.2.3 Tsay's Nonlinearity Test

Tsay's (1986) F test for nonlinearity is a powerful parametric test derived from Keenan' test but involves cross-product terms at different lag lengths in the auxiliary regression, such as $x_{t-i}x_{t-j}$. The first step is to regress y_t on the vector $X_t = (1, x_{t-1}, ..., x_{t-M})'$ by least squares $x_t = X_t'q + e_t$ and obtain the residuals $\hat{e}_t$. Second, compose the vector Z_t defined by $Z_t' = vech(U_t'U_t)$ where $U_t = (x_{t-1}, x_{t-2}, ..., x_{t-M})$. The $vech(\cdot)$ denotes the half-stacking vector operator Z_t that consists of a $m = M(M+1)/2$ dimensional vector containing the components of the lower triangular part of $U_t'U_t$. That is $Z_t' = (x_{t-1}^2, x_{t-1}x_{t-2}, ..., x_{t-M}^2)$. The third step then regress Z_t on X_t and save the residual $\hat{e}_t$. Next regress the estimated residuals $\hat{e}_t$ on $\hat{e}_t$: $\hat{e}_t = \hat{e}_t'd + u_t$ and save the residuals $\hat{u}_t$ as well. The Tsay's F test statistic is defined

$$\hat{F} = \frac{\left[(\sum \hat{e}_t \hat{e}_t)(\hat{e}_t' \hat{e}_t)^{-1}\right] \cdot \left[(\sum \hat{e}_t' \hat{e}_t)/m\right]}{\left[\sum \hat{u}_t^2 /(T - M - m - 1)\right]} \quad (2.25)$$

for the null hypothesis $H_0 : d = 0$.

In practice, Tsay prefers to use the usual partial F statistic for testing $H_0 : b = 0$ in the linear least squares regression:

$$x_t = X_t' a + Z_t' b + w_t \quad (2.26)$$

Therefore, under the assumption that x_t is a linear AR(*M*) process, the partial F statistic follows an F distribution with degrees of freedom *m* and *T*-*M*-*m*-1 (*T* is sample size).

2.2.4 The RESET Test

Ramsey's (1969) RESET (regression error specification test) is a general test for misspecification of functional form which could aga in be viewed as a more general formulation of Keenan's test, which forces functions of the fitted values of a higher orde than two. The statistic is organized via the following steps. In the first, regress x_t linear on

$$\mathrm{X}_{t-1} = (1, x_{t-1}, \ldots\ldots, x_{t-p})' \quad (2.27)$$

as $x_t = X_{t-1} f + e_t$ and save the estimated residuals $\hat{e}_t$ and fitted values $\hat{x}_t$. In the mean time, compute sum of squared residuals $SSR_0 = \sum_{t=p+1}^{T} e_t^2$. Next, regress $\hat{e}_t$ on *s* powers of the fitted values $\hat{x}_t$ (where $s \geq 2$ can be set at any level) and X_{t-1}' as well:

$$\hat{e}_t = X_{t-1}' a + M_{t-1}' b + u_t \quad (2.28)$$

where $\mathrm{M}_{t-1} = (\hat{x}_t^2, \ldots\ldots, \hat{x}_t^{s+1})'$ for some $s \geq 1$. Then obtain $SSR_1 = \sum_{t=p+1}^{T} u_t^2$ from the regression.

If the linear AR(*p*) model in Equation (2.27) is adequate, then $a = b = 0$ in Equation (2.28). Thus, a normal *F* statistic of Equation (2.28) can be calculated as:

$$\tilde{F} = \frac{(SSR_0 - SSR_1)/(s + p + 1)}{SSR_1 /(T - s - 2p - 1)} \quad (2.29) \quad (T \text{ is sample size})$$

which under the linearity and normality assumption follows an *F*(0.05, *s* + *p* +1, *T*-*s*-2*p*-1) distribution.

Another form of nonlinearity test proposed by Keenan (1985) is that to use $\hat{x}_t^2$ only and modify the second step of the RESET test to eliminate the multicolinearity between $\hat{x}_t^2$ and X_{t-1} by fitting the regression

$$\hat{x}_t^2 = X'_{t-1} j + w_t \quad (2.30)$$

and saving the residuals w_t. Then the sum of squared residuals $SSR_1 = \sum_{t=p+1}^{T} u_t^2$ is obtaine via:

$$\hat{e}_t = \hat{w}_t g + u_t \quad (2.31)$$

If $\tilde{F} > F(0.05, s + p + 1, T - s - 2p - 1)$, reject the null hypothesis of linearity.

2.2.5 Hsieh's Third-Order Moments Test

After the nonlinearities detected by the BDS test, it is necessary to distinguish between two types of nonlinear dependence in the filtered returns:

Additive Dependence $x_t = w_t + f(x_{t-1},......,x_{t-k}, x_{t-1},......,x_{t-k})$; (2.32)

and *Multiplicative Dependence* $x_t = w_t f(x_{t-1},......,x_{t-k}, x_{t-1},......,x_{t-k})$ (2.33

where x_t is the residual from the Equation 2.17 and w_t is an iid random variable with zero mean and independent of past x_t's and x_t's. *f*( ) here denotes nonlinear functio of past x_t's and x_t's. The additive dependence indicates that nonlinearity enters only through the mean of the process whereas the multiplicative dependence is only through the variance of the process, which is basically the general form of conditional heteroskedasticity. Hsieh's (1989b) third-order moments test designed to diagnose whether the filtered return is nonlinearity in mean or variance. The test first define

$$r_{xxx}(i, j) = E(x_t x_{t-i} x_{t-j}) / r_x^3 \qquad (2.34)$$

Under the null hypothesis of multiplicative dependence, which implies that:

$$E(x_t | x_{t-1}, \ldots\ldots, x_{t-k}, x_{t-1}, \ldots\ldots, x_{t-k}) = 0 \qquad (2.35)$$

Hence $r_{xxx}(i, j) = 0 \quad \forall\, i, j > 0$.

$r_{xxx}(i, j)$ is then computed as:

$$g_{xxx}(i, j) = \left(T^{-1} \sum x_t x_{t-i} x_{t-j}\right) \Big/ \left(T^{-1} \sum x_t^2\right)^{0.5} \qquad (2.36)$$

where T is the sample size. The t-statistic for the test is:

$$t - stat = \frac{T^{-1.5} g_{xxx}(i, j) \left[\sum x_t^2\right]^3}{\sum x_t^2 x_{t-i}^2 x_{t-j}^2} \qquad (2.37)$$

The test is rejected only in the presence of additive nonlinearity but not multiplicative nonlinearity. The rejection is registered if the absolute value of t statistic is greater than 2.576 constituting a two-sided test at the 1% significance level. According to the simulations carried out by Hsieh (1989b), the third-order moments test has higher powe against nonlinear moving average, threshold autoregressive models, and tent map for i, j = 1, 2. If one finds evidence consistent with nonlinearity in mean, the process is possibl chaotic dynamic.

III. Empirical Results

Table 4. The BDS Test Results of Nonlinear Dependence

m	*e / s*	*DM*			*FR*			*JP*			*US*		
		Raw Data	*Filtered Return*	*GARCH (1, 1)*	*Raw Data*	*Filtered Return*	*GARCH (1, 1)*	*Raw Data*	*Filtered Return*	*GARCH (1, 1)*	*Raw Data*	*Filtered Return*	*GARCH (1, 1)*
2	0.50	**4.79**	**4.50**	0.37	**4.19**	**3.09**	**2.99***	**4.29**	**3.75**	-0.04	**9.95**	**9.81**	**2.78***
3	0.50	**5.93**	**5.55**	0.73	**5.81**	**4.73**	**4.67**	**5.21**	**4.89**	-0.03	**14.37**	**14.38**	**4.54**
4	0.50	**5.89**	**5.37**	0.09	**6.81**	**5.67**	**5.62**	**6.39**	**5.91**	0.18	**18.40**	**18.50**	**5.39**
5	0.50	**6.67**	**6.18**	0.10	**7.66**	**6.79**	**6.77**	**7.44**	**6.74**	0.30	**23.41**	**23.52**	**6.46**
6	0.50	**6.98**	**6.47**	-0.28	**7.88**	**7.25**	**7.24**	**8.40**	**7.66**	0.83	**30.02**	**30.19**	**7.88**
7	0.50	**8.32**	**7.37**	-0.37	**8.11**	**7.61**	**7.61**	**9.55**	**8.83**	1.27	**38.45**	**38.48**	**9.22**
8	0.50	**10.10**	**8.72**	-0.48	**8.70**	**8.20**	**8.20**	**11.15**	**9.97**	1.16	**50.34**	**50.37**	**11.37**
9	0.50	**11.90**	**9.93**	-0.59	**9.42**	**8.86**	**8.87**	**13.32**	**10.58**	0.79	**65.81**	**66.10**	**13.63**
10	0.50	**13.65**	**11.43**	0.56	**10.36**	**9.78**	**9.79**	**12.61**	**7.87**	-0.18	**86.25**	**87.25**	**14.97**
2	1.00	**4.47**	**4.34**	-0.14	0.91	0.49	0.32	**4.18**	**3.64**	-0.38	**10.34**	**10.38**	**2.95***
3	1.00	**5.55**	**5.36**	-0.19	0.80	0.50	0.38	**5.40**	**5.01**	-0.23	**13.41**	**13.55**	**3.84**
4	1.00	**6.02**	**5.69**	-0.54	0.71	0.60	0.49	**6.62**	**6.29**	0.14	**16.24**	**16.37**	**4.33**
5	1.00	**6.68**	**6.26**	-0.57	1.03	1.01	0.91	**7.86**	**7.64**	0.74	**19.26**	**19.34**	**4.98**
6	1.00	**7.29**	**6.88**	-0.46	1.14	1.24	1.15	**9.18**	**9.16**	1.44	**22.83**	**22.94**	**5.90**
7	1.00	**8.13**	**7.77**	-0.08	1.28	1.33	1.26	**10.41**	**10.58**	1.96	**26.42**	**26.52**	**6.48**
8	1.00	**8.70**	**8.40**	0.00	1.35	1.37	1.30	**11.64**	**12.02**	**2.34***	**30.56**	**30.66**	**7.04**
9	1.00	**9.30**	**9.05**	0.06	1.68	1.71	1.64	**13.27**	**13.63**	**2.65***	**35.40**	**35.51**	**7.56**
10	1.00	**10.18**	**9.98**	0.27	1.98	2.04	1.98	**15.11**	**15.39**	**2.93***	**40.93**	**41.18**	**8.09**
2	1.50	**4.81**	**4.67**	-0.49	1.28	0.38	-0.26	**4.04**	**3.53**	-0.33	**10.16**	**10.12**	**3.11**
3	1.50	**6.09**	**5.94**	-0.46	1.07	0.50	0.02	**5.05**	**4.71**	-0.37	**12.44**	**12.42**	**3.60**
4	1.50	**6.83**	**6.64**	-0.66	1.02	0.58	0.15	**6.23**	**5.86**	-0.08	**14.08**	**14.04**	**3.86**
5	1.50	**7.42**	**7.18**	-0.75	1.16	0.87	0.48	**7.35**	**6.96**	0.43	**15.81**	**15.73**	**4.36**
6	1.50	**7.97**	**7.71**	-0.65	1.36	1.23	0.88	**8.35**	**8.09**	1.01	**17.77**	**17.70**	**5.01**
7	1.50	**8.65**	**8.38**	-0.36	1.66	1.49	1.17	**9.19**	**9.00**	1.46	**19.44**	**19.36**	**5.40**
8	1.50	**9.05**	**8.76**	-0.32	1.88	1.69	1.39	**9.80**	**9.80**	1.77	**21.23**	**21.12**	**5.75**
9	1.50	**9.44**	**9.16**	-0.29	**2.14***	1.90	1.62	**10.54**	**10.51**	1.96	**23.08**	**22.90**	**5.99**
10	1.50	**9.97**	**9.68**	-0.18	**2.36***	2.05	1.79	**11.33**	**11.29**	**2.15***	**24.93**	**24.87**	**6.22**
2	2.00	**4.86**	**4.72**	-0.99	0.17	-0.32	-1.02	**3.82**	**3.39**	-0.07	**9.88**	**9.79**	**3.85**
3	2.00	**6.35**	**6.23**	-0.89	0.19	-0.06	-0.59	**4.62**	**4.21**	-0.30	**11.76**	**11.67**	**3.85**
4	2.00	**7.27**	**7.09**	-1.02	-0.05	-0.23	-0.69	**5.64**	**5.15**	-0.14	**12.67**	**12.56**	**3.87**
5	2.00	**7.86**	**7.66**	-1.12	0.20	-0.04	-0.46	**6.61**	**6.02**	0.21	**13.75**	**13.62**	**4.22**
6	2.00	**8.30**	**8.09**	-1.12	0.69	0.45	0.07	**7.42**	**6.90**	0.69	**14.89**	**14.78**	**4.61**
7	2.00	**8.83**	**8.61**	-0.85	1.05	0.81	0.46	**8.04**	**7.58**	1.05	**15.79**	**15.66**	**4.82**
8	2.00	**9.11**	**8.89**	-0.82	1.54	1.32	0.97	**8.31**	**8.10**	1.28	**16.72**	**16.57**	**5.03**
9	2.00	**9.42**	**9.19**	-0.77	1.91	1.69	1.36	**8.65**	**8.40**	1.35	**17.59**	**17.33**	**5.05**
10	2.00	**9.77**	**9.53**	-0.70	**2.15***	1.94	1.63	**8.98**	**8.73**	1.43	**18.38**	**18.25**	**5.21**

Table 5. The Statistic of White's Neural Network

Series	Input Layer	$\tilde{W}_n$ $(n \times R^2)$	
		3 Hidden Layers	*5 Hidden Layers*
DM	5	14.08*	14.08*
FR	15	2.41	9.65**
JP	15	3.51	8.77
US	6	7.83*	15.66*

Note: The input layer is decided by five dummy variables outlined in the text together with the number of lags of the dependent variables chosen by $Q_x(50)$.

*, ** indicate significant at 5% and 10% respectively.

Table 6. Tsay's F Test for Nonlinearity on Returns

Series	DM	FR	JP	US
$M = 2$ F(3, 3910)	2.96*	0.54	0.66	11.53*
$M = 3$ F(6, 3906)	2.23*	0.29	0.44	6.13*
$M = 4$ F(10, 3901)	1.39	0.89	0.90	4.59*
$M = 5$ F(15, 3895)	1.88*	0.75	0.80	3.52*
$M = 6$ F(21, 3888)	1.61*	0.71	0.92	3.29*

Note: * indicates significant at 5%, where the F critical values are as follows:

F(3, ∞) = 2.60, F(6, ∞) = 2.10, F(10, ∞) = 1.83, F(15, ∞) = 1.67, F(21, ∞) = 1.57.

Table 7. The RESET Test for Nonlinearity on Returns

Series	RESET	Keenan
DM (s = 4, p = 0)	0.883	0.077
FR (s = 4, p = 10)	0.083	0.003
JP (s = 4, p = 10)	0.39	0.004
US (s = 4, p = 1)	2.23*	1.33

Note: * indicates significant at 5%, where the F critical values are as follows:

F(5, ∞) = 2.21, F(6, ∞) = 2.10, F(15, ∞) = 1.67.

Table 8. Third Order Moments Test Statistics for Filtered Returns

Lag					
i	*j*	DM	FR	JP	US
1	1	-0.0519	-0.1609	-0.0509	-0.1315
		(-0.3122)	(-0.1644)	(-0.2797)	(-0.2225)
2	1	0.0505	-0.0128	0.0185	0.0779
		(0.9456)	(-0.6122)	(0.3529)	(0.6428)
2	2	0.0425	0.0197	-0.0250	0.1109
		(0.2200)	(0.1183)	(-0.1799)	(0.3616)
3	1	0.0184	-0.0091	-0.0096	-0.0001
		(0.3329)	(-0.4852)	(-0.1892)	(-0.0012)
3	2	0.0254	0.0008	0.0256	-0.0336
		(0.4107)	(0.0313)	(0.6407)	(-0.5038)
3	3	-0.0421	0.0300	-0.0172	-0.0152
		(-0.2286)	(0.2343)	(-0.0877)	(-0.0347)
4	1	0.0118	-0.0004	0.0153	-0.0027
		(0.2064)	(-0.0218)	(0.2820)	(-0.0322)
4	2	0.0145	0.0454	0.0155	0.0273
		(0.1893)	(1.8949)	(0.3764)	(0.4198)
4	3	-0.0108	-0.0040	0.0159	-0.0496
		(-0.1723)	(-0.1960)	(0.2427)	(-0.6369)
4	4	0.0102	-0.0046	-0.0985	-0.0319
		(0.0493)	(-0.0055)	(-0.5054)	(-0.0802)
5	1	-0.0298	0.0070	0.0213	0.0255
		(-0.6399)	(0.3781)	(0.5014)	(0.4568)
5	2	0.0085	0.0174	0.0217	-0.0125
		(0.1691)	(1.1767)	(0.4138)	(-0.2506)
5	3	-0.0554	-0.0075	0.0196	0.0156
		(-1.0953)	(-0.7899)	(0.2914)	(0.3635)
5	4	0.0463	0.0382	0.0212	0.0433
		(1.1966)	(0.5944)	(0.3204)	(0.6299)
5	5	-0.0785	-0.0482	0.0099	0.0621
		(-0.4708)	(-0.1114)	(0.0499)	(0.4081)

Note: t statistics are in parentheses.

The results of the BDS test are given in Table 4[2]. As expected, there is only slight chang in each filtered returns series further confirming the presence of nonlinearity in the raw data. Except French franc FR, the other three filtered returns DM, JP, and US demonstrate significant nonlinear dependence in the data, especially for US. The BDS test results of filtered returns for FR, however, seem a bit more complicated. That FR is non-i.i.d. only holds for the small distance $e/s = 0.5$. It is clear that the larger the distance, the more significant the independence is. Theoretically, when sample size is getting larger, the distribution for $e/s = 2.0$ should start to move closer to the normal i

[2] There are five computational programs can be considered for the BDS test so far: Dechert's MS-DOS, LeBaron's Matlab and C, Trapletti's R, Eviews 4.0, and Kanzler's Matlab. Details please see Franch and Contreras (2002). In this research we use Kanzler's Matlab code.
Note that in Table 4, bold figures with * indicate significant at 5% and bold figures without * indicate significant at 1%.

both lower and upper tails. Meanwhile, there is a large range of dimensions for which th respective sizes of the error are not significantly different between e / s = 1.5 and e / s 2.0. This implies that either or both of e / s = 1.5 and e / s = 2.0 will yield satisfactory results. Therefore, we can conclude that FR exhibits i.i.d. pattern in both e / s = 1.5 and e / s = 2.0. However, the evidence from the standardized residuals of GARCH (1, 1) is mixed. The residuals of three series DM, FR, and JP tend towards linearity (except e / s = 0.5 for FR) but US still consistently behaves as non-i.i.d. The GARCH (1, 1) model fo US has the worst fit failing the BDS diagnostic test every time. It may be either that the GARCH models used in various studies are misspecified and hence do not capture all o the dependence in the squared residuals, or more likely, that there are further complex patterns within the data.

The results of White's test in Table 5 do not seem to depart considerably from the BDS test results except for JP. The regression result of JP shows serious multicolinearity due to the low frequency of the data. This symptom may directly bias the statistic of the test Teräsvirta et al (1993) suggest that if the neural network test is implemented, the way o allotting values to the intercept need careful consideration. They also point out that the neural network test using randomly selected parameters is much less powerful than the procedure based on the dual representation of the Volterra expansion. Interestingly, in this empirical research we did not find the significant difference between regression wit and without intercept. But we find that White's test is sensitive to select the number of hidden layer. More hidden layer may lead to increase the power of the test.

Table 6 represents the results of Tsay's F test for nonlinearity on returns. We arbitrarily extend the lag to $M = 6$. The results are consisted with those of White's test. The values of partial F test of DM and US are significantly over the critical values reflecting nonlinearity. The multicolinearity problem occurs again in JP series, and FR still exhibi linearity.

The RESET test results in Table 7 look less powerful than Tsay's F test. We set s from to 4 but each regression suffers multicolinearity. Even Keenan's method does not improve the situation. The final result is that only US rejects the null. It should be noted that both Tsay's test and RESET test are sensitive to departures from linearity in the mean.

Finally, the absolute values of *t* statistic for Hsieh's third order moments test in Table 8 are all less than 1, which indicates that the four series are not characterized as a chaotic process.

IV. Compass Rose

In this section, we employ visual method to test whether the "compass rose" pattern exi in NZ foreign exchange market. As we will discuss soon, once such pattern appears, the complex dynamics would bias the BDS test and the estimation of (G)ARCH models.

Recently, some economists borrowed ideas from the physical sciences to uncover the essence of nonlinearities in financial data. A utilized method is phase portrait. It is a we known instrument of the dynamical complex system such as Henon attractor map. This method forces embedded data in more than two dimensional space and inspects the resu from the portrait (the simplest version is to plot x_{t+1} against x_t). The basic point of this that a non-uniform distribution of the plotted dots indicates the presence of some underlying structure. In consequence, the inference is that data are forecastable. However, the characteristic of the non-continuity of price quotations in financial time series is not consistent with the conclusion. Crack and Ledoit (1996) plotted the return o a share in a period R_{t+1} against that in the previous period R_t, and found that the notabl rays in the picture clearly shooting in the major directions of the compass are the thicke They thus named such a pattern a "compass rose". The details of the methodology are a follows:

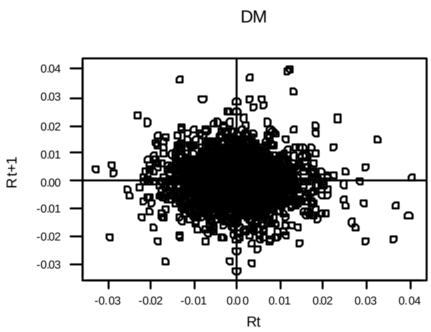
Let R_t and P_t be the return and the closing price of a stock at time *t* respectively, and *h* be the tick size. R_t is defined as

$$R_t = (P_t - P_{t-1})/P_{t-1} \qquad (4.1)$$

Suppose that daily price changes are small relative to the price level, then

$$\frac{R_{t+1}}{R_t} = \frac{(P_{t+1} - P_t)/P_t}{(P_t - P_{t-1})/P_{t-1}} \approx \frac{(P_{t+1} - P_t)/P_{t-1}}{(P_t - P_{t-1})/P_{t-1}} = \frac{n_{t+1}}{n_t} \qquad (4.2)$$

where the integer $n_t \equiv (P_t - P_{t-1})/h$ is the price change in ticks at time *t*. The authors demonstrated that if stock prices with finite ticks were to remain relatively stable throughout the period, data points would cluster at discrete distances from the origin, as the result, the pattern would be a grid. If stock prices vary, data points are spread centrifugally and evenly on the ray, generating a compass rose pattern. Crack and Ledoi (1996) summarized that the cause of the compass rose phenomenon in common stock returns can be described through the existence of three necessary conditions: (1) The price level has to be large relative to the price change from the last to the current period (2) The price changes occur in discrete jumps of a small number of ticks. This conditio requires that the asset is liquid and trades under orderly market conditions. (3) The volatility or intensity stock prices movements vary over a relatively wide range. They conclude that the compass rose can not be used to make abnormal profits: it is structure without predictability (This conclusion is controversial. Chen (1997) argued that ARM GARCH models contained in compass rose can improve forecasting performance). Meantime, among other consequences, they conjecture that the compass rose may bias the BDS test and the estimation of (G)ARCH models. Figure 5 reports the return patter represented by circles for four exchange rate series.



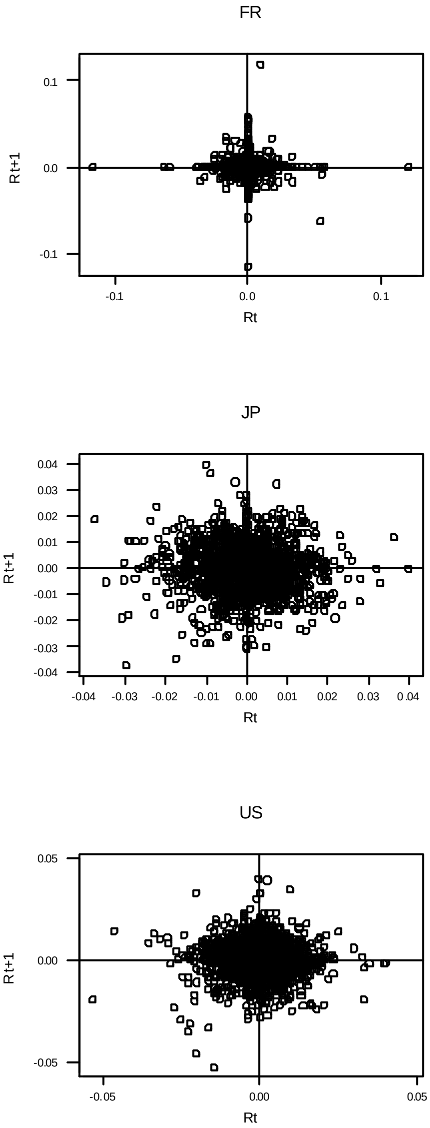
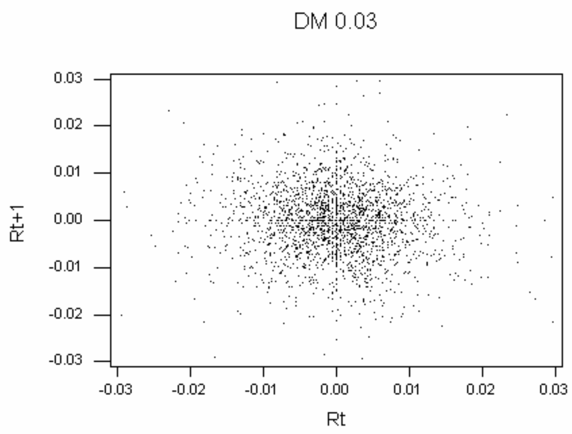
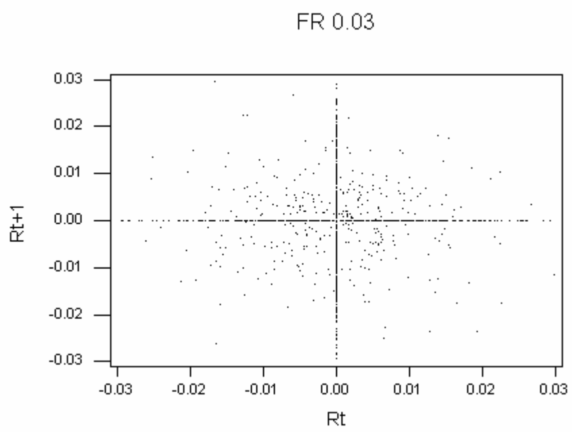

FR
0.1
0.0
-0.1
Rt+1
-0.1
0.0
0.1
Rt
JP
0.04
0.03
0.02
0.01
0.00
-0.01
-0.02
-0.03
-0.04
Rt+1
-0.04
-0.03
-0.02
-0.01
0.00
0.01
0.02
0.03
0.04
Rt
US
0.05
0.00
-0.05
Rt+1
-0.05
0.00
0.05
Rt

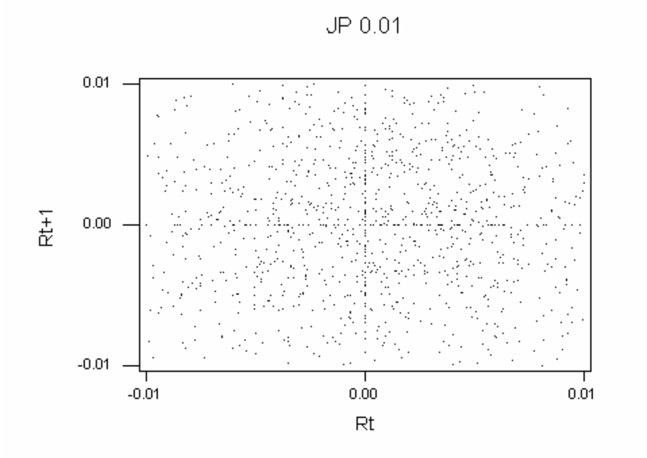
Figure 5


DM 0.03
0.03
0.02
0.01
0.00
-0.01
-0.02
-0.03
Rt+1
-0.03
-0.02
-0.01
0.00
0.01
0.02
0.03
Rt

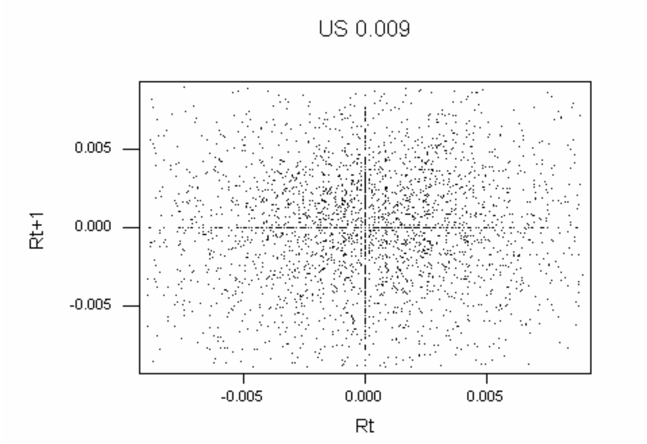
(1)


FR 0.03
0.03
0.02
0.01
0.00
-0.01
-0.02
-0.03
Rt+1
-0.03
-0.02
-0.01
0.00
0.01
0.02
0.03
Rt

(2)

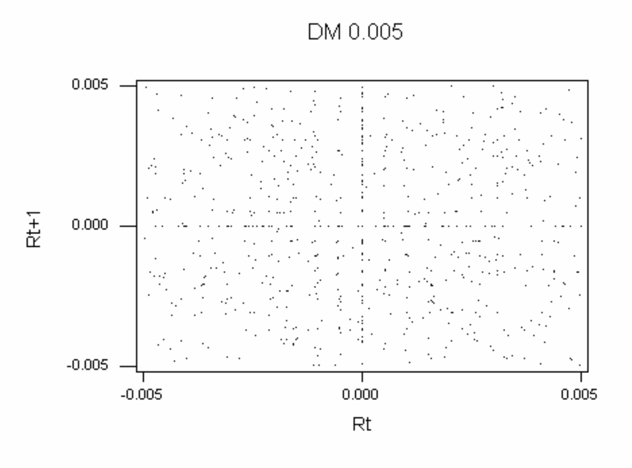

JP 0.01
0.01
0.00
-0.01
Rt+1
-0.01
0.00
0.01
Rt

(3)

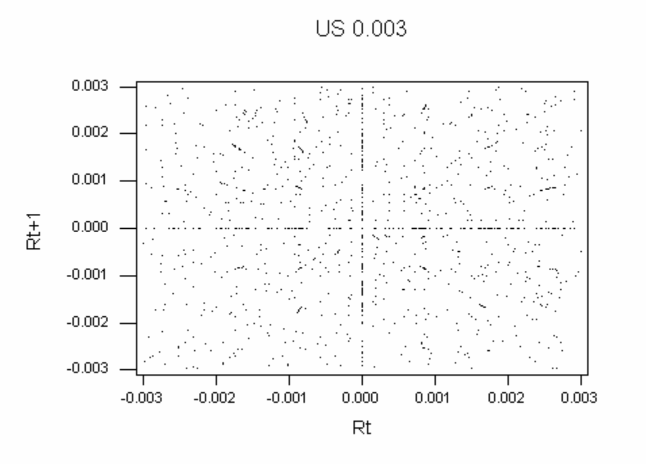

US 0.009
0.005
0.000
-0.005
Rt+1
-0.005
0.000
0.005
Rt

(4)

Figure 6a



(1)



(2)

Figure 6b

When all available observations are used in the plot of the diagram, the resulting graph does not resemble the compass pattern. After zooming in returns in 3% magnitude for DM and FR, 1% for JP, and 0.9% for US, the notable rays clearly point to the east, wes south and north in Figure 6a(1)-(4) but each graph does not illustrate the compass rose pattern. The elusiveness compass rose is only unmistakably observed for DM and US in Figure 6b(1) and Figure 6b(2) after further restricting the returns to a 0.5% limit and a 0.3% limit respectively. Data points significantly spread centrifugally and evenly on the ray. How could this happen? The possible reason could be the high embedding dimensions of these two series. Such high embedding dimensions are generated by the fact that the exchange rates are determined by a large number of real and nominal variables, for instance, money supply, interest rates, price level, and national income etc Meanwhile, these two rounded series are not time-invariant. They are represented by th feature of discrete jumps which is driven by the nonlinear stochastic system. The consequence thus creates new complicated dependence.

V. Conclusion

This research has emphasized on two major questions: Do the NZ daily spot exchange rates contain nonlinear structure? And, if so, is there any evidence that the process may be chaotic? Our findings are summarized as follows.

First, we have found that the nonlinear structure do exist in the New Zealand foreign exchange market. This finding may in a way shed new light on solving the puzzle of the elusive movements of the NZ exchange rates mentioned in Wilkinson et al (2001). On t other hand, financial analysts may base on this information to predict the future movements of exchange rates via the approximate nonlinear models and obtain surprise earnings. Therefore, the EMH in NZ foreign exchange market can also be challenged. A the results of nonlinearity tests indicate, the nonlinearity is highly significant for DM ar US. For FR, the nonlinearity hypothesis is unanimously rejected by all tests, as it posse i.i.d characteristics. The results of White's ANN test, Tsay's F test and RESET test on J disagree with the BDS test result which demonstrates that JP is not an i.i.d. pattern. Thi disagreement could be due to the presence of multilinearity. However, if one takes into account the conclusion of Barnett et al (1997), which states that the BDS or Kaplan test a powerful test to rule out the narrowest null of exact linearity, then in this case, the BD test is most telling that JP is non-i.i.d.

Second, Hsieh's third order moments test provides strong evidence of multiplicative dependence implying that nonlinearity enters through the variance of the process rather than through means. Therefore, we can conclude that the process is unlikely to be chaotic.

Third, the results of the BDS test for the GARCH (1, 1) standardized residuals of DM a JP no longer provide support for nonlinearity demonstrating that the GARCH (1, 1) model appears to fit the data pretty well. The failure of the BDS test for GARCH (1, 1) standardized residuals of US prompts us to consider an alternative method, such as stochastic volatility (SV) model suggested by Kim, Shephard, and Chib (1998), to capture the volatility.

Finally, we have explored the compass rose patterns in DM and US. As the patterns reveal a discrete jump-diffusion process, the finding may help to price currency option more accurately, for ignoring the jump component could induce serious mis pricing of currency option (see Jorion (1988)). Moreover, according to previous studies, the presence of compass rose may bias the BDS statistics and the estimation of the (G)ARC model. If this is true, then the influence based on the BDS test or (G)ARCH model enta a Monte Carlo simulation. This is also an area that can be explored in further research.

In short, some NZ daily spot exchange rates do behave nonlinearly but not as a chaotic process. Although the long run movements of these exchange rates are driven by fundamentals, their short-run variations may still be traced via complex nonlinear dependence.

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