Inflation expectation, risk aversion and asset allocation in an equilibrium approach

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First draft: November 10, 2004
This draft: November 20, 2004

Abstract

This paper investigates the asset allocation decision of long-term investors who have different preferences about risk and have different forecasts about inflation. We employ the Black and Litterman (1992) model to investigate the asset allocation decision. Our investors have a one period power utility function. We find that when forecasted inflation increases the weight on TIPS, Treasury bonds and investment grade corporate bonds increase and the weight on the equity portfolio decreases. We also find that as investors become more risk averse they allocate more weight on less risky assets like TIPS, Treasury bonds and investment grade corporate bonds and less weight on the equity portfolio.

Keywords: *Inflation expectation, risk aversion, equilibrium approach*

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Introduction:

When a new class of asset is introduced in the financial market, the first question that comes to an investor's mind is how does it fit into their overall portfolio strategy. Treasury Inflation Protected Security (TIPS) is a new class of security which only becaome available to the U.S. investor in 1997. The single most appealing feature of TIPS is that its payoffs are inflation adjusted; as such it provides a hedge against inflation. None of the assets generally held by long-term investors effectively hedges against inflation risk (Fischer, 1975) as a result TIPS should become an invaluable addition to long term investors portfolio. In this paper we investigate how does TIPS fit in a diversified portfolio strategy of stocks, bonds (Treasury bonds and investment grade corporate bonds) and real estate. We also investigate how the change in inflation expectation and risk aversion impact asset allocation decision for the investors.

We utilize the Black and Litterman (1992) model to analyze the asset allocation decision of the long term investors. In practice, fund managers and investors focus on a small segment of the potential asset universe based on their believes or predictions. The main advantage of the Black and Litterman (1992) model over Markowitz (1952) model is that it accommodates these insights in the asset allocation decision. The goal of the Black and Litterman (1992) model is to create stable, mean-variance efficient portfolios, based on an investor's unique insights, which overcome the problem of input-sensitivity. Unlike Markowitz formulation Black-Litterman model does not unrealistically require expected returns to be specified for every component of the relevant universe. In our paper, agents (investors or fund managers) predict inflation using a naïve forecast based on historical distribution of monthly inflation. Then they use their prediction the in the Black and Litterman (1992) model to calculate their optimal portfolio weights. Another distinct advantage of this model over the Markowitz (1952) model is that it allows us to analyze the impact of risk aversion on the asset allocation decision. However, despite the more practical approach of the Black and Litterman (1992) model for institutional and individual investors, the academic literature has largely ignored this approach of asset allocation.

Kothari and Shanken (2004) and Roll (2004) employ the Markowitz (1952) framework to investigate optimal portfolio weight between stocks, Treasury bonds and TIPS. In addition to methodology, there is another key difference between our paper and the problem investigated by Kothari and Shanken (2004) and Roll (2004). Long-term investors generally hold corporate bond and real estate in addition to equity and Treasury bonds. There is some evidence that real estate provides an inflation hedge as well. Thus we investigate a problem consistent with the problem faced by long-term investors.

The rest of the study is organized as follows: section two introduces TIPS and describes different features of TIPS important to investors. Section three briefly discusses the inflation hedging capacity of different classes of assets. Section four describes the methodology. Section five analyzes the results and the last section concludes.

Introduction to indexed bonds and TIPS

The history of inflation protected security could be dated back to the Revolutionary War in 1780. According to Shiller (2003), the Commonwealth of Massachusetts first issued “depreciation notes”, to compensate U.S. soldiers for the loss of purchasing power of their money. Almost 150 years later, the Rand-Kardex Co., co-founded by the most famous economist of that time, Prof. Irving Fisher, issued inflation-indexed bonds in 1925. Twenty-years later, the inflation protected securities were available in many countries of Europe. Half a century ago in 1945 Finland introduced such a security. Investors in other developed countries also had access to government indexed bonds before U.S. investors. For example, Gilts were introduced in the UK in 1975, Real Return Bonds were introduced in Canada in 1991; Treasury Indexed Bonds were introduced in Australia in 1985 and Inflation Adjusted Bonds were introduced in New Zealand in 1977. Inflation indexed bonds have also been available in several emerging markets in Latin America such as Brazil since 1964; Chile since 1966; Columbia since 1967 and Argentina since 1972. Currently, there are 21 countries which have introduced inflation bonds over the last two decades. Table 1 summarises the details of the major inflation linked bond markets in seven countries. From 1980 to 1997, the market for inflation-protected securities was dominated by U.K. Gilts, which at that time accounted for around 80%. Heavy issuance in the U.S. and France have now reduced the UK share to around 31% and as of January 2004 the U.S. is the dominant market with around 42% share of inflation-linked bonds.

<Please insert Table 1 here>

The U.S. Treasury started issuing inflation-linked securities in January 1997. Since 1997, the U.S. Treasury's Inflation-linked bond market has grown to over $160 billion as of December 2003[1]. Market capitalization for TIPS is still small compared to the $3.1 trillion nominal Treasury bond market. Treasury has so far issued fourteen

[1] Barclays Capital Research (2004).

inflation-linked bonds[2] with the size of the issues and coupon rates different from each other. Treasury issues inflation-linked bonds of three different maturities; these are 5years, 10 years and 30 years. These securities are widely known as Treasury Inflation Protected Security (TIPS). Table 2 provides the coupon rate, issue date, and market value (as of December 2003) for all TIPS issues. We see that the coupon rate varies from a maximum of 4.25% to a minimum of 1.875%. The size of issue also varies from a low of $5 billion to a high of $23 billion.

<Please insert Table 2 here>

TIPS offers investors protection against inflation. It pays a fixed real coupon rate on a semi-annual basis on an inflation-adjusted principal. To illustrate how TIPS work, consider the first TIPS issued on July 16, 1997 with five years of maturity, par value of $1000, and a coupon rate of 3.625%. We adjust the principal semi-annually for inflation calculated by using the consumer price index for all urban consumers (CPI-U) set at a lag of three months. Table 3 presents how the bonds cash flow will be calculated. The first payment comes in January 1998. Annualized inflation was 2.26% in October 1997, so the par value of the bond increases from $1000 to $1022.56. Since the coupon rate is 3.625%, the coupon payment on this amount is $18.54. The principal value increases with inflation and because the interest payment is 3.625% of the principle, they also increase proportional to the general price level.

<Please insert Table 3 here>

Two empirical features of TIPS need to be mentioned; these are liquidity and cliental of TIPS. TIPS is relatively illiquid compared to nominal Treasury bonds. In 1998 the trading volume of TIPS was less then a billion dollar daily. But the liquidity

[2] The first issue was of five year maturity, and matured in July 2002.

has gone up. Since October 2002, the daily trading volume of TIPS has increased to over 3 billion dollars (see Figure 1). Nevertheless, TIPS will likely never achieve the same liquidity as nominal Treasury bonds, largely because of the different roles that the two types of securities play in financial markets. The other important empirical feature of TIPS is that it is primarily held by end users such as depository institutions, insurance companies, mutual funds, hedge funds, individuals, non-financial companies and others. We see from Figure 2 that irrespective of maturity, higher proportions of TIPS are held by end users compared to nominal Treasury bonds. This feature of TIPS can be reconciled with the illiquidity of TIPS; if a higher proportion of investors are is by-and-hold investors or end-users, then we should expect less liquidity for TIPS compared to nominal Treasury bonds.

Coupon payments of TIPS like any treasury bond are exempt from state and local tax. For federal tax purposes semi-annual interest payments are taxable in the year that they are received. Inflation adjustments to principle are taxed as interest in the year in which the adjustment occurs. However, the adjustment of principal is not received until the maturity. Consequently, investors who have tax deferred account will be most interested in TIPS.

Inflation hedging capacity of different classes of assets:

Inflation is a real concern for long term investors. Traditionally long term investors hold equity, corporate bonds, treasury bonds and real estate as a part of their investment. However, neither equity nor nominal bonds can provide a hedge against inflation. In classical economic theory equity is assumed to be a hedge against inflation but empirical literature (Bodie, 1976, Jaffe and Mandelker, 1976, and

Nelson, 1976) find that equity is a perverse hedge against inflation. Similarly, nominal bonds also can not hedge against inflation (Campbell and Shiller, 1996, Campbell and Ammer, 1993).

Real estate literature arrives at two different conclusions regarding the inflation hedging capacity of real estate. All studies (Brueggeman, Chen, Thibodeau, 1984, 1992, Sirmans and Sirmans, 1987, Hoag, 1980, Miles and McCue, 1984 and Hartzell, Hekman and Miles, 1987) that use commingled real estate funds (CREF) to represent real estate finds that real estate provides an inflation hedging benefit. On the other hand, the conclusion is unclear for studies which use real estate investment trust (REIT) index data. Titman and Warga (1987) find that market indexs and REITs have significant explanatory power above benchmark inflation forecasts. They provide more accurate forecast of inflation than returns on the market index. Chan, Hendershott and Sanders (1990) show, on the other hand, that REITs are not a hedge against expected inflation and a perverse hedge against unexpected inflation. Glascock, Lu and So (2002), conclude that the observed REIT returns as a perverse inflation hedge are spurions.

There is wide spread agreement among academics that indexed bonds are essential for long-term investors. Fisher (1975) argues that index bonds would allow investors to fully hedge against inflation. Roll (1998) claims that a well designed indexed bond will generate strong investor demand. Lamm (1998) argues[3] that TIPS should dominate nominal bonds and drive them out of diversified portfolios entirely.

[3] Based on the simulation result. His simulations are based on the assumptions that TIPS have higher Sharpe ratios then nominal bonds and have a similar correlation to other classes of assets.

Campbell and Viceira (2001) argue that in the presence of significant inflation risk[4] nominal bonds are risky and are not good substitutes for inflation-indexed bonds.

Data and Methodology

Data Description

TIPS data is provided by Barclay Capital. We use all the available TIPS issues to construct a TIPS index return. We only concentrate on TIPS issues with 7-10 years maturity because they represent the majority of US TIPS. Treasury bonds are represented by the Datastream 7-10 year government bond index. For investment grade corporate bonds, we use the Lehman Brother US Corporate investment grade bond from Datastream. The National Real Estate Investment Trusts (NAREITs) and the S&P 500 Composite index represent real estate and equity in our data.

Table 4 Panel A presents the summary statistics of these five classes of assets. During the sample period, TIPS return is exceptional. Mean monthly return of TIPS, Treasury bonds and corporate bonds are 0.64%, 0.61% and 0.65%; which are quite comparable. Compared to bonds, real estate has a higher mean return (0.87%) but the performance of the equity market was very poor. The mean return of the S&P 500 composite index is only 0.50%. On the other hand, TIPS has the lowest volatility are all classes of assets during this period. The only assets which have a comparable volatility are investment grade corporate bonds. The equity market as represented by the S&P 500 index had the highest volatility followed by real estate during this period.

<Please insert Table 4 here>

[4] For the type of inflation risk they estimated in the US from 1926-1996.

Table 4 Panel B shows the correlation of returns between TIPS, Treasury bond, corporate bond, real estate and the S&P 500 composite index. The correlations between TIPS, Treasury bonds and corporate bonds generally positive, while the return correlation between TIPS and real estate is almost close to zero. The equity index has a negative correlation with TIPS return. A similar correlation structure is also reported by Roll (2004) for daily returns[5]. This low correlation between TIPS and other classes of assets implies that TIPS should offer a hedge benefit against poor performance of other classes of assets.

Methodology

We assume that investors have power utility function over a one-period terminal wealth and the portfolio returns follow a log normal distribution. The investor's expected utility maximization could be described in terms of asset returns as follows (See Campbell and Viciera, 2001):

$$\underset{\{w\}}{Max} \ \log E_t\left(1 + R_{p,t+1}\right) - \frac{\gamma}{2}\sigma_{pt}^2$$

$$\text{subject to } \sum_{i=1}^{N} w_i = 1 \text{ and } w_i \geq 0 \text{ for any } i \qquad (1)$$

where w is an N-vector of asset weights; $E\left(R_{p,t+1}\right)$ is the expected portfolio return; σ_{pt}^2 is the conditional variance of the log portfolio return; γ is the coefficient of absolute risk aversion.

It is well known that asset allocation optimization is highly sensitive to the expected returns estimation. The expected return based on historical average mean returns could yield an unreasonable and extremely volatile weight. Best and Grauer

[5] Page 34, table 3

(1991) show that under a short selling constraint positively weighted mean-variance efficient portfolio's weights are extremely sensitive to changes in asset means. A small increase in the mean of just one asset drives half the securities from the portfolio, but the portfolio's expected return and standard deviation are virtually unchanged. In addition, the historical mean estimation does not result in the equilibrium of demand and supply. Black and Litterman (1992) point out that a starting point of expected returns should be the equilibrium expected return because it results in market capitalization portfolio weights being optimal for investors who use mean-variance asset allocation. They further derive the equilibrium posterior expected returns by combining the prior view, market equilibrium weight, and the subjective views of investor on the assets' expected returns. In the Black and Litterman (1992) model expected return could be described as follows:

$$\mu = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right] \qquad (2)$$

where $\boldsymbol{\Pi}$ is the vector of prior expected return or the implied equilibrium return from market portfolio[6, 7]; $\boldsymbol{\Pi = \gamma\Sigma W_{MKT}}$ where $\boldsymbol{W}_{MKT}$ is the market portfolio weight; $\boldsymbol{\tau}$ is a scaling parameter in the covariance matrix which is inversely proportional to the relative weight given to the implied equilibrium return; $\boldsymbol{P}$ and $\boldsymbol{Q}$ capture investor subjective views, and they could be expressed as follows.

$$\begin{bmatrix} P_{1,1} & \cdots & P_{1,n} \\ \vdots & \ddots & \vdots \\ P_{k,1} & \cdots & P_{k,n} \end{bmatrix} \begin{bmatrix} \mu_1^* \\ \vdots \\ \mu_N^* \end{bmatrix} = \begin{bmatrix} Q_1 \\ \vdots \\ Q_k \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{bmatrix} \qquad (3)$$

where k is the number of views; $\boldsymbol{P}$ is the weights of the portfolio representing the view; $\boldsymbol{\mu}^*$ is the expected return according to the view. $\boldsymbol{P}$ and $\boldsymbol{\mu}^*$ could be jointly

[6] The market portfolio weight is the proportion of an asset's market capitalization relative to total market capitalization of all assets in portfolio.
[7] When there is no view, $\boldsymbol{P=0}$, the posterior return equals to the implied equilibrium return.

specified either by the views on the absolute returns level of assets or by the views on the relative performance of assets. $\boldsymbol{Q}$ refers to portfolio returns according the view. This study assumes that investors have limited forecasting skill and their major concern is the level of inflation. As a results, the vector $\boldsymbol{\mu}^*$ is expressed in terms of an absolute return, estimated from the regression between asset returns, expected inflation and unexpected inflation. The row vector $\boldsymbol{P}$ is based on asset equilibrium weights. Naturally, vector $\boldsymbol{Q}$ is the multiplication of $\boldsymbol{P}$ and $\boldsymbol{\mu}^*$. The uncertainty of the view, ε, is normally distributed with variance Ω.

$$\Omega = \begin{bmatrix} \omega_1 & 0 & 0 & 0 \\ 0 & \omega_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \omega_k \end{bmatrix} \tag{4}$$

where ω_i is the variance of the uncertainties in expressed view; $1/\omega_i$ representing the confidence level in each view. He and Litterman (2002) suggest that the confidence level on a view should be calibrated so that the ratio between $\boldsymbol{\Omega}$ and $\boldsymbol{\tau}$ is equal to the variance of the portfolio in the view, $P\Sigma P^T$. Mathematically, this could be shown as follows.

$$\frac{\overline{\omega}}{\tau} = P\Sigma P^T$$

$$\overline{\omega} = \frac{\sum_{i=1}^{k}\left(\frac{CF}{LC_i}\right)}{k} \tag{5}$$

where CF is a calibration factor and LC_i is the prior confidence level of the views.

We consider the portfolio allocation of three investors who have different views about expected future inflation. All investors form their views of expected inflation based on the realized inflation distribution from January 1960 to September

2004. Three expected inflation views are the percentile 10, 50, and 90 of realized inflation, which are 0.39%, 3.46%, and 9.24% per annum respectively. Since each investor has only one view, the prior confidence level (LC) attached to each view is 100%.

Inflation impacts asset return. Fama and Schwert (1977) argue that asset returns could be determined by expected inflation and unexpected inflation. To keep the model simple, we propose that investors employ a naïve strategy by forecasting current expected inflation based on the previous period realized inflation. Specifically, asset returns and inflation can be modelled as follows:

$$\tilde{R}_{jt} = \alpha_j + \beta_j B_t + \gamma_j (\Delta_t - B_t) + \tilde{\eta}_{jt} \tag{6}$$

where Δ_t is the inflation at time t; B_t is the expected inflation rate at time t, which is estimated from the inflation at time $t-1$; $\Delta_t - B_t$ refers to the unexpected inflation; β_j is the sensitivity of asset returns to the expected inflation rate while γ_j refers to a sensitivity of asset returns to the unexpected inflation.

Inflation expectation, risk aversion and optimal asset allocation:

Table 5 reports the optimal weight allocation according to the Black and Litterman (1992) model from three investors who have three distinct views regarding expected inflation. One of the oldest and unsolved puzzle of modern finance is the *equity-premium puzzle.* Cochrane (2001) argues that in any model, a high equity premium must come from large *risk* or large *risk aversion.* He shows[8] that most of the empirical inconsistencies associated with the puzzle can be avoided if a high risk

[8] See figure 21.1, page 480.

aversion rather than a large risk is assumed to derive the empirically consistent equity premium.

<Please insert Table 5 here>

We examine asset allocation when investors' absolute degree of risk aversion assumes a value of 25. When investors forecast a low inflation rate (0.39% par annum), we expect them to invest the majority of their wealth in equity. However, under a moderate or high inflation expectation, investors should put more weight in indexed bonds. Consistent with the prediction, we see that as expected inflation increases, the model assigns more weight to indexed bonds and nominal bonds and less weight on the equity index.

On the other hand, investors with higher degree of risk aversion are expected to assign more weight on assets that have lower risk. So we expect to see more weight on indexed bonds, nominal Treasury bonds and investment grade corporate bonds for higher risk aversion and less weight on equity and real estate (which has higher sample variance) for a given level of inflation expectation. For example, when γ increases from 50 to 250 we see that for a moderate inflation expectation; the weight on TIPS increases from 4.45% to 12.75%, the weight on Treasury bond increases from 6.48% to 11.16% while the weight on the equity index decreases from 89.07% to 70.96%.

Conclusion:

Inflation plays a critical role in the asset allocation decision of long term investors. In the presence of inflation risk the asset allocation decision between long term and myopic investors are different. Over the last few decades economists have developed theories showing that show that there is no good close substitute for indexed bonds for long term investors. In 1997 the US Treasury started issuing indexed bonds. The goal of this paper is to investigate the asset allocation decision of long term investors between equity, nominal Treasury bond, investment grade corporate bond, real estate and indexed bonds across different levels of inflation forecasts and risk preference.

Main stream literature employs the Markowitz (1952) framework to analyze asset allocation in the mean-variance framework. Conventional wisdom suggests that as inflation expectations or risk aversion change, asset allocation will also change. The Markowitz (1952) framework does not allow us to examine the impact of change of inflation expectation or risk aversion on asset allocation. We employ The Black and Litterman (1992) model to analyze the portfolio allocation decision of long-term investors. In our study mean-variance optimizing investors form their prediction about asset return and inflation using the Fama and Schwert (1977) model. Investors make a naïve forecast about inflation using a historical monthly inflation distribution. In our paper three types of investors forecast three levels of inflation; they are 10 percentile, 50 percentile and 90 percentile values of the historical monthly inflation distribution. We find as the forecasted value of inflation increases, the weight of TIPS, Treasury bonds and investment grade corporate bonds increase while the weight on equity portfolio decreases. However, the rate of growth of weight is higher for TIPS compared to Treasury bonds and investment grade corporate bonds.

In this paper we also investigate how asset allocation changes when investors attitude towards risk changes. The more risk averse the investors the more weight they allocate on TIPS, Treasury bonds, and investment grade corporate bonds for any level of inflation forecast and the less weight they allocate on equity portfolio. In addition we also observe that the rate of increase in weight is higher for TIPS compared to nominal Treasury bonds.

The Black and Litterman (1992) model is a more practical model compared to Markowitz (1952) and yet simple to apply. In this paper we showed how this model can be used to come up with useful results consistent with more rigorous theories of finance. We expect that this model will find its popularity among academics and we will see further useful application of this model for investors.

References

Barclays Capital Research, 2004, "Barclays Inflation-Linked Bond Indices: January 2004", *Barclays Bank PLC.*

Best, M., and Grauer, R., 1991, "On the Sensitivity of Mean-Variance-Efficient Portfolios to Changes in Asset Means: Some Analytical and Computational Results," *Review of Financial Studies* 4, 315-342.

Black, F., and Litterman, R., 1992, "Global portfolio optimisation," *Financial Analysts Journal* 48, 28-43.

Bodies, Z., 1976, "Common Stocks As Hedge Against Inflation," *Journal of Finance* 31, 459-70.

Brueggeman, Chen, and Thibodeau, 1984, "Real Estate Investment Funds: Performance and Portfolio Considerations," *AREUEA Journal*, 333-354.

Brueggeman, Chen, and Thibodeau, 1984, "Some Additional Evidence on the Performance of Commingled Real Estate Investment Funds: 1972-1991," *Journal of Real Estate Research* 7, 433-448.

Sirmans, G. S., and Sirmans, C. F., 1987, "The Historical Perspective of Real Estate Returns," *Journal of Portfolio Management* 13, 22-31.

Campbell, John Y., and J. Ammer, 1993, "What Moves the Stock and Bond Markets? A Variance Decomposition for Long Term Asset Returns," *Journal of Finance* 48, 3-37.

Campbell, John Y. and Robert J. Shiller, 1996, "A Scorecard for Indexed Government Debt," NBER Working Papers 5587, *National Bureau of Economic Research, Inc.*

Campbell, John Y. and Luis M. Viceira, 2001, "Strategic asset allocation: Portfolio choice for long-term investors," Oxford University Press, Oxford.

Cochrane J., 2001, "Asset Pricing," *Princeton University Press*.

Fama, E. F., and Schwert, G., 1977, "Asset Returns and Inflation," *Journal of Financial Economics* 5, 115-146.

Fischer, Stanley, 1975, "The Demand for Indexed Bonds," *Journal of Political Economy* 83, 509-534.

Glascock, J. L., C. Lu and W. S. Raymond, 2002, "REIT Returns and Inflation: Perverse or Reverse Cauaslity Effects?," *Journal of Real Estate Economics and Finance* 24, 301-317.

Hartzell, D., Hekman, J.S., and Miles, M.E., 1987, "Real Estate Returns and Inflation," *AREUEA Journal* 15, 617-637.

He, G. and Litterman, R., 1999, "The Intuition Behind Black-Litterman Model Portfolios," *Investment Management Research. Goldman, Sachs & Company* (December).

Hendershott and Sanders (1990)

Hoag, J. W., 1980, "Towards Indices of Real Estate Value and Return," *Journal of Finance* 35, 569-580.

Jaffe, I and G. Mandelker (1976): 'The 'Fisher Effect' for Risky Assets: An Empirical Investigation," *Journal of Finance* 31, 447-458.

Kothari, S. P. and Jay Shanken, 2004, "Asset Allocation with Inflation-Protected Bonds," *Financial Analysts Journal* Jan/Feb, 54-70.

Lamm, R. M., 1998, "Asset Allocation Implications of Inflation Protection Securities," *Journal of Portfolio Management* 24, 93-100.

Markowitz, H., 1952, "Portfolio Selection," *Journal of Finance* 7, 77-91.

Miles, M. and McCue, T. 1984, "Commercial Real Estate Returns," *AREUEA Journal* 12, 355-377.

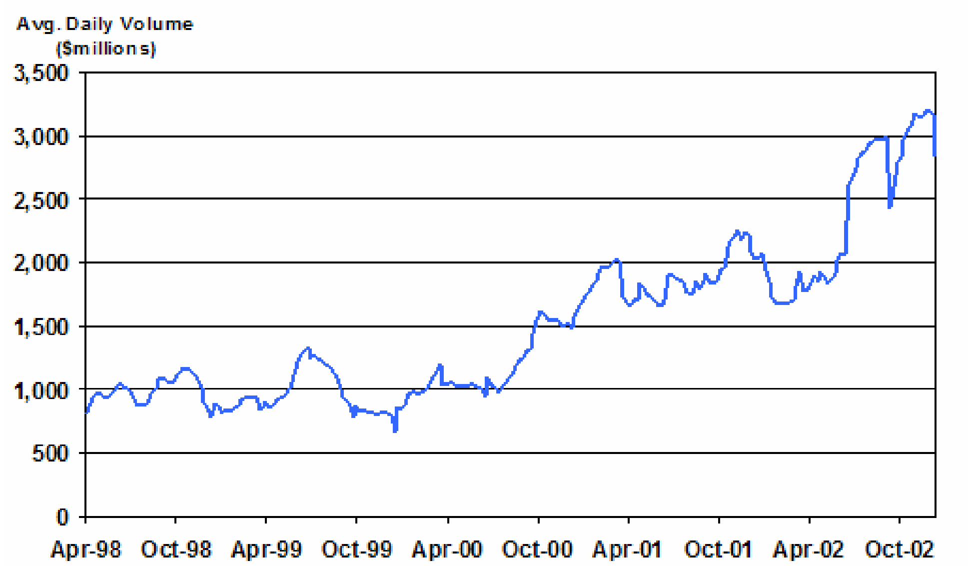
Nelson, C. R., "Inflation and Rates of Return on Common Stocks," *Journal of Finance* 31, 471-483.

Roll, Richard, 2004, "Empirical TIPS," *Financial Analysts Journal* Jan/Feb, 31-53.

Shiller, Robert J., 2003, The Invention of Inflation Indexed Bonds in Early America. NBER Working Paper No. 10183. <http://papers.nber.org/papers/w10183>.

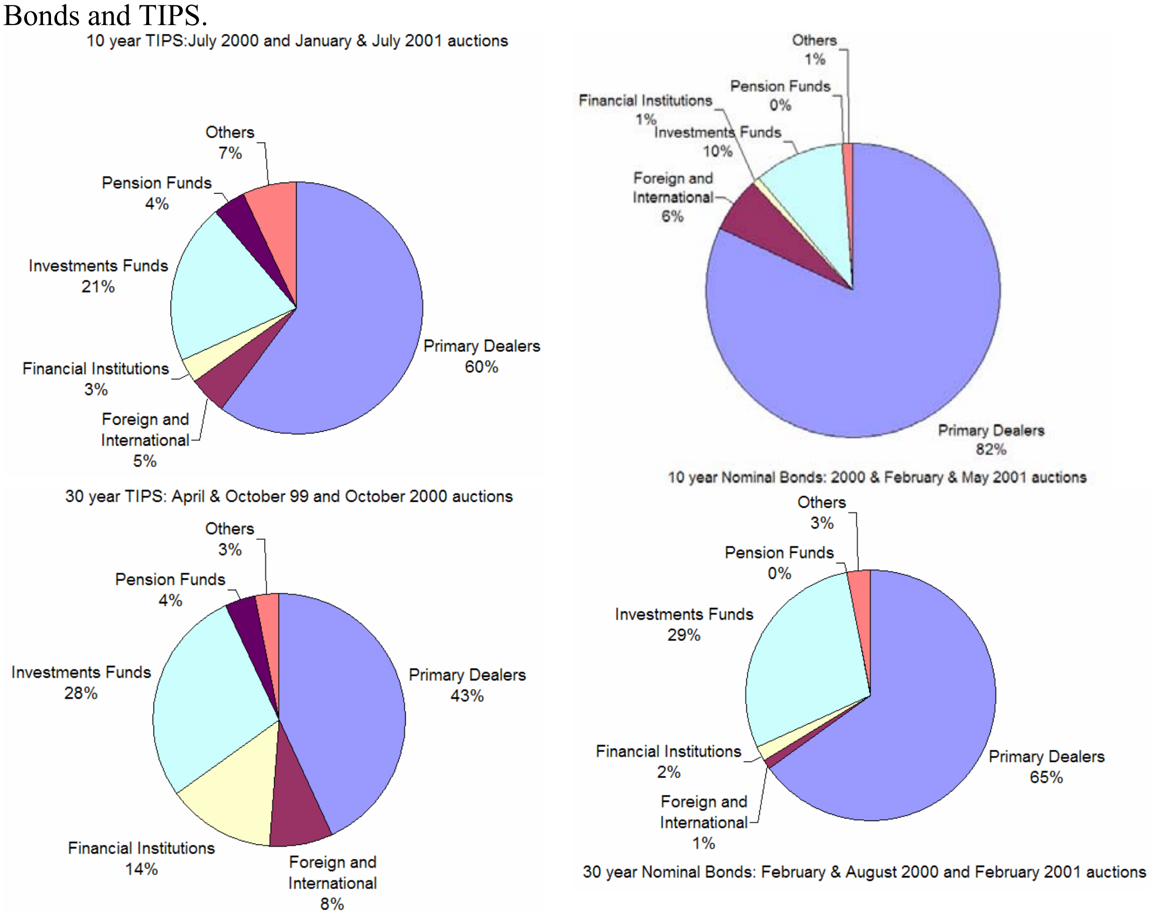
Titman, S., and Warga, A. 1989, "Stock Returns as Predictors of Interest Rates and Inflation," *Journal of Financial and Quantitative Analysis* 24, 47-58.

Figure 1: Liquidity of TIPS:
Dealer Transactions in TIPS, 3-month Moving Average of Daily Volume



Source: Federal Reserve Bank of New York

Figure 2: Distribution of Auction Awards: A comparison between nominal Treasury Bonds and TIPS.



Source: Department of Treasury, Office of Market Finance (July 30, 2001)

Table 1: Major sovereign inflation linked bond market as of January 2004

Issuer's Govt	Market Value ($mn)	% of Overall	No. of Issues	Average Real Yield	Average life
World	479,850	100	39	2.09	12.5
Australia	6,885	1.4	4	3.48	11.6
Canada	19,182	4	4	2.76	23.7
France	64,160	13.4	5	2.19	11.8
Italy	12,998	2.7	1	1.54	4.7
Sweden	24,769	5.2	5	2.75	14
UK	149,055	31.1	9	1.92	14.8
US	202,801	42.3	11	1.98	11.3

Source: Barclays Capital Research (2004)

Table 2: History of US TIPS.

Assets	Date of issue	Maturity Date	Coupon rate	Par Amount USD	Marke Value USD*
5 years maturity TIPS	July 16 1997	July 11 2002	3.625		
10 years maturity TIPS	January 22 1997	January 15 2007	3.375	15758	20225.1
	January 15 1998	January 15 2008	3.625	16811	21605.7
	January 15 1999	January 15 2009	3.875	15899	20574.1
	January 19 2000	January 15 2010	4.25	11319	14708.7
	January 17 2001	January 15 2011	3.5	11000	13328.8
	January 15 2002	January 15 2012	3.375	6000	7093.9
	July 15 2002	July 15 2012	3	23011	26138.3
	July 10 2003	July 12 2013	1.875	20000	20174.3
30 years maturity TIPS	April 15 1998	April 15 2028	3.625	16783	23912.9
	April 16 1999	April 15 2029	3.875	19494	28569.5
	October 15 2001	April 15 2032	3.375	5000	6469.2

Source: Barclays Capital
* As of December 2003.

Table 3: Principal and coupon payment for TIPS

Time	Inflation in last quarter		Par Value	Coupon Payment	Principal Payment	Total payment
Jul-97			1000			
Jan-98	2.26%	(October, 1997)	1022.56	18.5339	0	18.5339
Jul-98	1.49%	(April, 1998)	1037.82	18.8104	0	18.810426
Jan-99	2.98%	(October, 1998)	1068.7	19.3702	0	19.370224
Jul-99	8.31%	(April, 1999)	1157.51	20.9799	0	20.97989
Jan-00	2.17%	(October, 1999)	1182.59	21.4345	0	21.434524
Jul-00	-0.70%	(April, 2000)	1174.32	21.2845	0	21.284482
Jan-01	2.09%	(October, 2000)	1198.91	21.7302	0	21.730179
Jul-01	3.46%	(April, 2001)	1240.42	22.4827	0	22.482695
Jan-02	-3.32%	(October, 2001)	1199.28	21.7369	0	21.736944
Jul-02	6.22%	(April, 2002)	1273.89	23.0892	1273.89	1296.9761

Table 3: Summery Statistics

Panel A shows the summary statistics of five asset classes: Treasury Inflation Protected Securities (TIPS7-10), Nominal Treasury Bonds (TBOND7-10), Investment Grade Corporate Bonds (LHCCORP), Real Estate Investment Trusts (NAREIT), and Standard and Poors 500 Composite Index (S&P500). Panel B reports the Spearman rank correlation of monthly asset returns. Panel B shows Spearman correlation of these assets.

	TIPS7-10	TBOND7-10	LHCCORP	NAREIT	S&P500
			Panel A: Summary		
Mean	0.64%	0.61%	0.65%	0.87%	0.50%
Median	0.63%	0.67%	0.74%	1.44%	0.52%
Std.Dev.	1.33%	1.88%	1.41%	4.15%	4.91%
Maximum	3.86%	5.54%	3.66%	9.27%	10.41%
Minimum	-4.41%	-6.14%	-4.34%	-16.56%	-11.45%
			Panel B: Spearman Correlation		
TIPS7-10					
TBOND7-10	0.21				
LHCCORP	0.55	0.06			
NAREIT	0.01	-0.06	0.19		
S&P500	-0.20	-0.18	-0.01	0.22	

Table 5: Black-Litterman Asset Allocation

This table shows the Black-Litterman asset allocation in five asset classes under the short sale constraints. Panel A to Panel C reflects the impact of investors' risk aversion on the asset allocation.

	Low Inflation	Moderate Inflation	High Inflation
Panel A: Coefficient of absolute risk aversion = 25			
TIPS7-10	0.00%	0.02%	9.13%
TBOND7-10	0.00%	5.31%	9.24%
LHCCORP	0.00%	0.00%	0.19%
NAREIT	0.00%	0.00%	0.00%
S&P500	100.00%	94.68%	81.45%
Panel B: Coefficient of absolute risk aversion = 50			
TIPS7-10	2.35%	4.45%	8.04%
TBOND7-10	5.56%	6.48%	8.19%
LHCCORP	0.00%	0.00%	0.57%
NAREIT	0.00%	0.00%	0.00%
S&P500	92.08%	89.07%	83.20%
Panel C: Coefficient of absolute risk aversion = 250			
TIPS7-10	12.55%	12.75%	13.01%
TBOND7-10	11.06%	11.16%	11.33%
LHCCORP	5.11%	5.13%	5.33%
NAREIT	0.00%	0.00%	0.00%
S&P500	71.29%	70.96%	70.33%