Pricing Chinese floating rate bonds and swaps

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PRELIMINARY AND INCOMPLETE.

Abstract

Chinese floating rate bonds differ from American floating rate bonds in that the coupon payments are determined based on an interest rate defined by the government. While this rate is cointegrated with market determined (LIBOR equivalent) rates, it is in general not equal to the market's discount rate. As a result, Chinese floating rate bonds trade at values substantially different from par. We present empirical results concerning the relationship between the government rate and market yield curves, and develop pricing models which can be used to value Chinese floating rate bonds. We demonstrate that these models offer substantial improvements for pricing and hedging as compared to using the framework conventionally applied to American bond pricing.

The Chinese bond market has had a rather chequered past. The first half of the 20th century was characterised by very unstable governments. First the imperial government and then the nationalist government defaulted on their debt obligations. Further, loose monetary policy during the 1930s and 1940s, and the attendent hyper-inflation resulted in the decimation of most fixed income securities. As a result, even after recent market liberalisations, the Chinese investor has a healthy suspicion of the bond market.[1]

Matters are somewhat obfuscated in the floating rate and swap markets by the practice of indexing coupon payments to the one year deposit rate, which is dictated by the People's Bank of China. This differs from the Western practice of setting swap payments to be indexed to LIBOR rates. Under the Western system, the LIBOR rate reflects the actual cost of borrowing/lending for market participants. However, the one year deposit rate in China, being infrequently updated, may differ substantially from the market rates.

Pricing of floating rate debt and swaps is well understood under the Western system of pricing (see e.g. Hull (2006)). At each reset date, since the next coupon payment is set to be consistent with the rate at which future payments are discounted, a bond which pays a floating rate is worth par. Between payment dates, if interest rates change, the value may differ slightly, but since the value will again be par at the next reset date, prices are

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[1]See Huang and Zhu (2007).

generally fairly stable. A swap is also easily valued, since once the floating side of the instrument's value is known, the swap's value can be found by subtracting the value of the fixed side (which can be valued by discounting its set sequence of payments).

In contrast, a Chinese floating rate bond or swap is considerably less straightforward to value. Since, in general, the prevailing one year deposit rate at a reset date may be quite stale, it will not be equal to the market rate. Further, since this state of mismatch may well hold in the future, there is no reason to suspect that a floating rate bond's value will equal par at any reset date. Indeed, empirically, these bond's values can drift to be substantially different from par.

This paper provides a framework for valuation of Chinese floating rate bonds. We present a simple, empirically plausible model for how the government rate adjusts. We then present several models for pricing floating rate bonds and swaps which are consistent with these. Lastly, we examine the performance of these models to price and hedge several actual Chinese bonds.

The layout of the paper is as follows. Section 1 shows how we can use established short rate models in conjunction with a model for the updating of stale government rates to arrive at prices for floating rate bonds and swaps. Section 2 presents data on the behaviour of Chinese government and market rates, and posits a relationship between the two. Section 3 evaluates the efficacy of these models for pricing and hedging. Lastly, section 4 concludes.

1 Pricing floating rate bonds

We begin by assuming a risk-neutral process for the instantaneous short interest rate r_t with generator $\mathcal{L}$. The bond we are valuing pays coupons every time interval Δt, at which time, the next period's coupon yield ($\tilde{R}_t$) is set equal to the current government rate R_t. The government rate resets according to a poisson process, with intensity $\lambda(r_t, R_t)$. When the government rate updates, it will be set to $g(r_t, t)$. In the case of a fixed/floating swap, we consider the fixed side to return coupons at annual rate $\bar{R}$. All rates are quoted with continuous compounding. Hence the coupon payment associated with rate $\tilde{R}_t$ is $e^{\tilde{R}_t \delta t} - 1$, while the coupon associated with rate $\bar{R}$ is $e^{\bar{R}\delta t} - 1$.

The value of a floating rate bond in this framework will be given by:

$$f(t, r_t, R_t, \tilde{R}_t),$$

where f is the solution of the PDE

$$0 = \frac{\partial f}{\partial t} + \mathcal{L}f + \lambda(r_t, R_t)\left(E(f(t, r_t, g(r_t), \tilde{R}_t) - f(t, r_t, R_t, \tilde{R}_t)\right).$$

On reset dates of the bond, the following jump condition must be satisfied:

$$f(t, r_t, R_t, \tilde{R}_t) = f(t^+, r_t, R_t, R_t) + \left(e^{\tilde{R}_t \delta t} - 1\right)$$

where t^+ is a time just after the coupon has been paid. For valuation of a fixed-floating swap, we replace this condition by:

$$f(t, r_t, R_t, \tilde{R}_t) = f(t^+, r_t, R_t, R_t) + e^{\tilde{R}_t \delta t} - e^{\bar{R}\delta t},$$

where $\bar{R}$ is the fixed rate for the swap.

Lastly, we need to prescribe a terminal boundary condition for the PDE. For bond pricing, this is

$$f(T, r_T, R_T, \tilde{R}_T) = e^{\tilde{R}_T \delta t},$$

while for pricing a swap, the boundary condition will be given by

$$f(T, r_T, R_T, \tilde{R}_T) = e^{\tilde{R}_T \delta t} - e^{\bar{R} \delta t}$$

Some possible choices of $\mathcal{L}$ and g are given in the following subsections.

1.1 The Cox-Ingersoll-Ross model

This model is a popular choice for an equilibrium short rate model (see Cox, Ingersoll, and Ross (1985)). Here

$$dr_t = \kappa(\theta_r - r_t)dt + \sigma_r \sqrt{r_t} dW. \tag{1}$$

In this case

$$\mathcal{L}f = \kappa(\theta_r - r_t)\frac{\partial f}{\partial r} + \frac{1}{2}\sigma_r^2 r_t \frac{\partial^2 f}{\partial r^2}.$$

If the government (on reset) sets their Δt rate to be consistent with the market, we can use the closed form solution for longer rates to obtain

$$g(r_t) = \frac{-2\kappa\theta_r}{\sigma_r^2 \delta t} \log(A(\delta t)) + \frac{r_t}{\delta t} D(\delta t)$$

where we define:

$$\begin{aligned} A(\delta t) &= \frac{2\gamma e^{\frac{(\kappa+\gamma)\delta t}{2}}}{(\kappa+\gamma)\left(e^{\gamma\delta t} - 1\right) + 2\gamma} \\ D(\delta t) &= \frac{2\left(e^{\gamma\delta t} - 1\right)}{(\kappa+\gamma)\left(e^{\gamma\delta t} - 1\right) + 2\gamma} \\ \gamma &= \sqrt{\kappa^2 + 2\sigma_r^2}. \end{aligned}$$

This approach has the advantage of providing a relatively parsimonious model for interest rates, and a simple relationship for g. However, it does not necessarily force the model to be consistent with the current yield curve, since the choice of (1) implies a particular form for yields for any given level of r_t and parameters σ_r, κ and θ_r.

1.2 The Hull-White model

One drawback of the CIR model is that it does not necessarily conform with all available interest rates. In contrast, the Hull-White model, while still a short rate model, can be calibrated to fit the observed yield curve (see Hull and White (1990)). Here, we assume

$$dr = [\theta(t) - ar]\, dt + \sigma_r dz$$

where $\theta(t)$ is a function chosen to match the initial term structure:

$$\theta(t) = \frac{\partial F_0(t)}{\partial t} + aF_0(t) + \frac{\sigma^2}{2a}(1 - e^{-2at}).$$

Hence the generator is given by:

$$\mathcal{L}f = [\theta(t) - ar_t] \frac{\partial f}{\partial r} + \frac{1}{2}\sigma_r^2 \frac{\partial^2 f}{\partial r^2}$$

In this case, an analytical formula is again available for g.

$$g(r_t, t) = \frac{B(t, t+\delta t) - A(t, t+\delta t)}{\delta t}$$

where

$$\begin{aligned}
A(t, t+\delta t) &= -F_0(t, t+\delta t)\delta t + B(t, t+\delta t)F_0(t) \\
&\quad -\frac{1}{4a^3}\sigma^2(e^{-a(t+\delta t)} - e^{-at})^2(e^{2at} - 1) \\
B(t, \delta t) &= \frac{1 - e^{-a\delta t}}{a}
\end{aligned}$$

where $F_0(t)$ is the time zero instantaneous forward rate for time t and $F_0(t, t+\delta t)$ is the time zero forward rate from t to $t + \delta t$.

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1.3 Numerical examples

To develop some intuition for the behaviour of the model, we now present results for the CIR model with constant correction intensity, and consider the effects of varying interest rates and correction intensity on prices.

We assume parameters of $\alpha = 0.02$, $\kappa = 0.5$ and $\sigma = 0.3$, so that a long run steady-state level of r is 0.04, and shocks to interest rates take approximately two years to revert.

We first consider two scenarios. In the first, the short rate is in equilibrium ($r = 0.04$). As a result, the market yield curve is flat. We then price, for various levels of the government rate, a five year bond, paying semi-annual coupons. We vary the intensity of the government correction, and plot the results for each case. Panel A of figure 1 shows these results. We note first of all, that the bond's value is par when the government rate is consistent with the one year market rate, regardless of intensity of correction. We further note that as the intensity of the government correction increases, we move towards the Western situation, where the bond's price is worth par, regardless of the government rate. No matter how high λ is set, however, a slight difference will persist, since the first payment is being set according to the current government rate (however, with high levels of λ, this is likely to lie in a close vicinity of the current one year market rate). In contrast, as λ becomes small, the bond's behaviour is more like a fixed rate bond, since future payments are likely to be based on the current government rate.

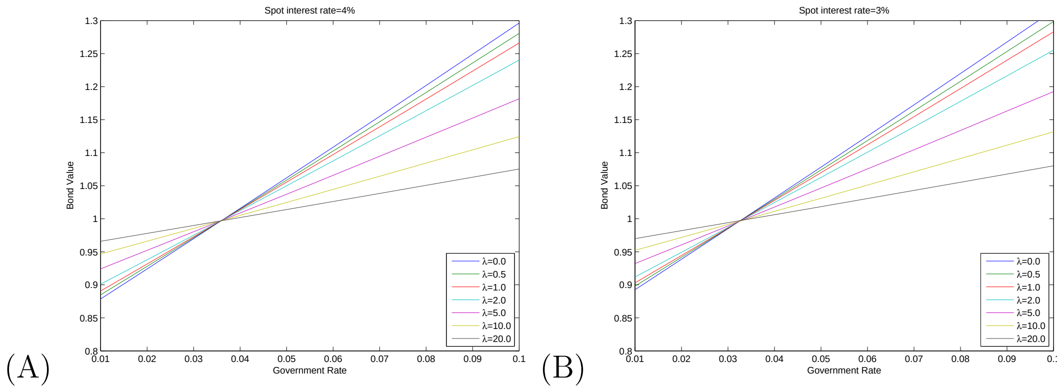

Figure 1: Floating rate bond price as a function of the government rate, assuming a flat market yield curve ($r = 0.04$), or upward sloaping yield curve ($r = 0.03$).

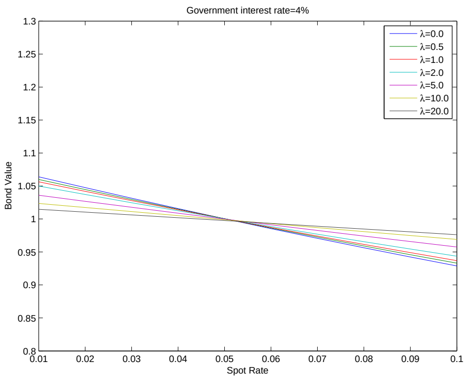

Figure 2: Floating rate bond price as a function of the market spot rate, assuming the government rate is 4%.

We next consider the effect of an upward sloping yield curve, by changing r to 3%. This is presented in panel B of figure 1. Results are qualitatively similar. We note, however, that since the short rate is lower, the bond is relatively more valuable. The bond is also *more* sensitive to variations in the intensity of changes in the government rate.

Allowing the yield curve shape to vary completely, we next consider fixing the government rate at 4%, and varying the spot rate. We are thus considering a full spectrum of market yields. Again, we vary λ to obtain multiple graphs of the bond's price. These results are contained in figure 2. Here, we see again that as λ becomes small, the bond behaves more like a fixed rate bond, while as λ becomes very large, its price becomes far flatter. Interestingly, we again see a single spot rate at which the bond trades at par – in this case approximately 5%.

Lastly, we fix the market rate at 4%, and consider the effect of varying λ for varying levels of the government rate (see figure 3). Our intuition from the previous graphs is maintained – the government rate has a huge effect on bond prices for low levels of λ, but

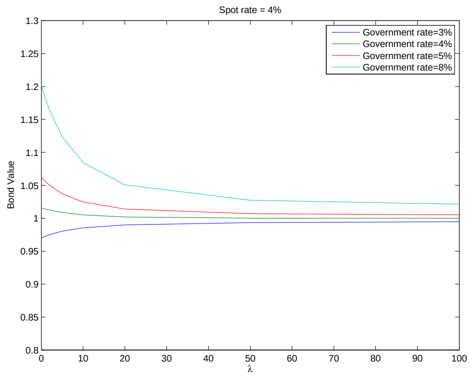

Figure 3: Floating rate bond price as a function of the intensity of government corrections. We assume that the government rate is currently consistent with the market spot rate (4%), and that the yield curve is flat.

for higher levels the difference is less pronounced. As noted before, even as λ tends to infinity, differences persist, since the initial government rate is fixed for each graph.

2 The relationship between government and market rates

For subsequent calculations, we require a model for the intensity with which the Chinese government updates their one year rate. We posit a relationship of the form:

$$\lambda(r_t, R_t) = b_0 + b_1|r_t(1) - R_t| + b_2(r_t(1) - R_t)^2.$$

where $r_t(1)$ is the one year market rate observed at time t. We can estimate this relationship, if we observe the dates on which updates occur. The log likelihood function is:

$$\log(L) = \sum_t \log(h(r_t, R_t))$$

where

$$h(r_t, R_t) = \begin{cases} \lambda(r_t, R_t)\Delta t & \text{if } R_t \neq R_{t-1} \\ 1 - \lambda(r_t, R_t)\Delta t & \text{if } R_t = R_{t-1} \end{cases}.$$

where Δt is the length (in years) of one time step. Maximising the log likelihood function using data from 10 December 2004 until 13 November 2008 leads to the estimates given in table 1. We note that both b_1 and b_2 are economically insignificant, and therefore posit a parsimonious model where $\lambda = 2.2083$: the government's update intensity is constant.

We make use of a selection of fixed rate bonds to calibrate interest rate processes. Figure 2 contains information on the securities used.

Assuming that the short rate (r_t) is equal to the 3 month market rate, we proceed to find the combination of parameters (κ, θ and σ_r) which yields the best least-squares fit

Parameter	Estimate
b_0	2.2083
b_1	0.0000
b_2	0.0246

Table 1: Estimates for the interest rate update intensity function.

Maturity	Issue date	Coupon	Frequency
25 Sep 2011	25 Sep 2001	2.95	Annual
18 Dec 2008	18 Dec 2001	3.00	Annual
18 Apr 2012	18 Apr 2002	2.54	Annual
12 Jun 2009	12 Jun 2002	2.93	Annual
20 Sep 2017	20 Sep 2002	2.60	Semi-annual
19 Nov 2010	19 Nov 2003	3.50	Annual
20 Apr 2009	20 Apr 2004	4.42	Annual
25 May 2011	25 May 2004	4.89	Annual
28 Feb 2015	28 Feb 2005	4.40	Semi-annual
25 May 2012	25 May 2005	3.37	Annual
15 Nov 2020	15 Nov 2005	3.65	Semi-annual
27 Feb 2013	27 Feb 2006	2.51	Annual
27 Mar 2016	27 Mar 2006	2.80	Semi-annual
17 Apr 2009	17 Apr 2008	2.12	Annual

Table 2: Fixed coupon bonds used for interest rate process calibration.

Parameter	Estimate
θ	0.0000
κ	0.0000
σ_r	0.1533

Table 3: Estimates for the interest rate process.

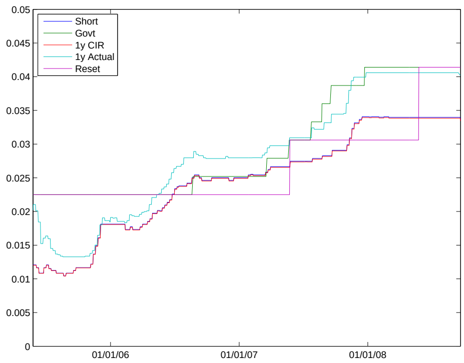

Figure 4: Miscellaneous interest rates for the pricing example.

to bond prices from 10 December 2004 until 13 November 2008. We obtain the estimates displayed in table 3.

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3 Pricing and hedging performance

We now consider pricing of a floating rate bond: specifically, the annual coupon bond maturing on May 25, 2010. This bond was issued in 2000, and has its coupon set to the one year government rate.

Figure 4 plots the short rate proxy, one year government rate, one year market rate and the floating rate bond's reset rate ($\tilde{R}_t$) for the period 23 May 2005 until 13 November 2008.

We then implement the model outlined in section 1, using the parameter estimates presented in section 2. The resulting prices are displayed in figure 5.

To provide a frame of reference, we also consider a model in which no account is made for the staleness of government rates. In this case, we calculate:

$$f(t) = (1 + \tilde{R}_t)e^{-r_t(\tau - t)(\tau - t)}$$

where τ represents the next coupon payment time. We assume, consistent with the stale price model, that interest rates follow a CIR process, as estimated in section 2.

Examining the results, we observe that the model which does not account for stale prices has difficulty explaining the extreme variability in bond prices actually observed in

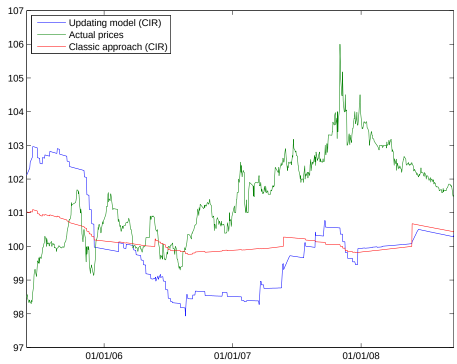

Figure 5: Theoretical and observed prices for the pricing example. Prices are clean - i.e. with accrued interest removed.

the market. In contrast, the stale-prices model, while capturing some variability, has a fairly poor fit (in most cases). Two possible explanations for this exist. First, the interest rate model, being quite simplistic, cannot capture the actual shape of the yield curve very effectively. Examining figure 4, we see that the model predicts a one year rate almost identical to the short rate, whereas the actual one year rate is considerably higher.

Second (and equally importantly) the model for the government rate updating assumes a constant intensity. Again examining figure 4, we see that although the government was slow to update their rates in the first part of the sample, during the second half, they were far more rapid. Investors might therefore have predicated their pricing on a relatively low λ. In particular, as interest rates rose, they might well have hoped that their floating rate bonds would become "locked in" at a high rate of interest, even after the market rates fell.

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4 Conclusion

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