OPTIMAL DIVIDEND POLICY AND CAPITAL STRUCTURE IN A MULTI-PERIOD DCF FRAMEWORK

January 2009

Martin Lally
School of Economics and Finance
Victoria University of Wellington

Abstract

This paper simultaneously analyses optimal dividend and debt policy within a conventional multi-period DCF framework, and allows for differential personal taxation on types of income, the effect of dividends and interest on the level of share issues (and hence share issue costs), the effect of dividends and interest on the level of internally financed investment (with possibly negative NPV), and the debt premium effect. In a classical tax regime, corresponding to that of the US, it is shown that the value benefits of debt are small, essentially because the debt premium effect outweighs the tax benefits, and that dividends in the form of share repurchases are desirable so as to avoid the adverse tax effects of retention. In an imputation regime, corresponding to that of New Zealand, it is shown that unimputed dividends are not desirable, that debt is not desirable in the presence of imputation credits, that dividends in the form of share repurchases have no valuation effect so long as they do not induce share issues, and that fully imputed dividends were desirable until recent changes in the corporate tax rate and the introduction of the PIE tax regime.

Acknowledgement: The comments of Andrew Smith, Kurt Hess and participants at a seminar at the University of Waikato are gratefully acknowledged.

1. Introduction

Consideration of optimal dividend policy and capital structure commences with Miller and Modigliani (1958, 1961, 1963), who show (separately) that debt is desirable and that dividends are irrelevant to a firm's value if (inter alia) all forms of personal income are equally taxed. DeAngelo and Masulis (1980) extend this analysis to recognise personal tax rates that vary over both investors and sources of income; they also simultaneously consider optimal debt policy, which is desirable because dividends and interest are alternative means for disbursing internally generated cash flow to investors. They conclude that debt is irrelevant but dividends may be relevant to a firm's value. Fung and Theobald (1984) extend this analysis to dividend imputation systems. However, both of the latter two papers assume that investors are risk neutral. They also ignore the implications of dividend and debt policy for "internally-financed" investment (i.e., that arising from internally generated cash flows that are not disbursed and which would not otherwise have been undertaken), and the level of share issues with their associated transaction costs.

Masulis and Trueman (1988) adopt the more realistic assumption that investors are risk averse and also consider the implications of dividend policy for the level of internally-financed investment, under the US tax regime. Boyle (1996) extends this analysis to tax regimes with dividend imputation. Although both of the latter two papers are principally concerned with investor preferences rather than the value maximising decisions, they show that these preferences differ across investors when internal financing is considered and that some investors prefer an increment to the level of new investment. Thus, dividend policy has implications for the level of new investment. However, the analysis in both of these latter two papers ignores debt policy. Furthermore it is single rather than multi-period, and invokes a state-preference approach rather than a risk-adjusted discount rate. These features are useful for the purpose of assessing issues at an abstract level. However the first of these features is somewhat less realistic, and the second somewhat more difficult to implement, than the multi-period DCF framework that is generally employed for capital budgeting decisions.

In view of these points, this paper seeks to analyse dividend policy taking account of all of the following points. Firstly, dividend and debt policies are simultaneously considered because dividends and interest are alternative means for disbursing internally generated cash flow to investors. Secondly, the analysis recognises that tax rates on personal income differ across both investors and forms of income, and that a dividend imputation system may operate. Thirdly, the analysis recognises that the level of dividends and interest affects the need for share issues, with their associated share issue costs (which discourages dividends and interest beyond the point at which share issues are induced). Fourthly, the analysis recognises that the level of dividends and interest affects the level of internally-financed investment, which may have negative NPV (and therefore encourages dividends and interest up to point at which such investment is avoided). Fifthly, the analysis is premised upon value maximisation. Finally, the analysis is conducted within the multi-period DCF framework, and is therefore consistent with the conventional approach to capital budgeting decisions; inter alia, it therefore allows for the debt premium effect and the fact that investors are risk averse.

2. The Model

Define S_0 as the current equity value of a firm, DIV_1 as the dividend to be paid in one year, K_1 as the share issue undertaken in one year to support the exogenously determined level of new investment at that time (i.e., justified on the basis of external financing), i as the issue cost per \$ of K_1, M_1 as additional equity investment in one year (in excess of that justified on the basis of external financing), Q_1 as the NPV per \$1 of M_1 (which will not exceed zero[1]), S_1 as the value of equity in one year if M_1 is zero, and k_e as the cost of equity capital. It follows that[2]

$$S_0 = \frac{E(DIV_1) - E(K_1)(1+i) + E(M_1)(1+Q_1) + E(S_1)}{1+k_e} \tag{1}$$

[1] A number of possible tax regimes will be considered shortly. Across these regimes, Q_1 never exceeds zero.

[2] It should be noted that if K_1 is positive, then $M_1 = 0$. Also, if M_1 is positive, then $K_1 = 0$.

Defining X_1 as the unlevered cash flow from operations in one year, *INT* as the interest payment at that time (which is subject to a corporate tax saving at rate T_c), and RET_1 as the part of X_1 that is retained to finance new investments, it follows that

$$DIV_1 = X_1 - INT_1(1 - T_c) - RET_1 \tag{2}$$

Define N_1 as the level of new investment in one year that is justified on the basis of external financing, B_0 as the debt level now, B_1 as the debt level in one year (to preserve the leverage ratio), and L_1 as follows:

$$L_1 \equiv N_1 - (B_1 - B_0) - RET_1 \tag{3}$$

If L_1 is at least zero, then $K_1 = L_1$. If L_1 does not exceed zero, then $M_1 = -L_1$. Accordingly, equation (1) can be written as follows:

$$S_0 = \frac{E(DIV_1) - E(L_1) - E(K_1)i + E(M_1)Q_1 + E(S_1)}{1 + k_e}$$

$$= \frac{E(X_1) - INT_1(1 - T_c) - E(N_1) - B_0 - E(K_1)i + E(M_1)Q_1 + E(B_1) + E(S_1)}{1 + k_e}$$

Defining k_d as the cost of debt, it follows that $INT_1 = k_d B_0$. In addition, $S_0 + B_0 = V_0$ and $S_1 + B_1 = V_1$. It follows that

$$V_0 + S_0 k_e + B_0 k_d (1 - T_c) = E(X_1) - E(K_1)i + E(M_1)Q_1 - E(N_1) + E(V_1)$$

and therefore

$$V_0 = \frac{E(X_1) - E(K_1)i + E(M_1)Q_1 - E(N_1) + E(V_1)}{1 + \frac{S_0}{V_0} k_e + \frac{B_0}{V_0} k_d (1 - T_c)}$$

The denominator term here is the weighted average cost of capital (WACC). If this is the same for all future periods, then successive substitution for terminal value yields

$$V_0 = \sum_{t=1}^{\infty} \frac{E(X_t) - E(K_t)i + E(M_t)Q_1 - E(N_t)}{(1+WACC)^t} \quad (4)$$

This is the standard valuation model for a firm or project subject to addition of the issue costs for share issues and the negative NPV arising from investment beyond that justified by recourse to external financing.[3] Adopting the usual assumption that cash flows are expected to grow at rate *g*, equation (4) reduces to the following:

$$V_0 = \frac{E(X_1) - E(K_1)i + E(M_1)Q_1 - E(N_1)}{WACC - g} \quad (5)$$

Dividend and debt policy affect the levels of $E(K_1)$ and $E(M_1)$. Prima facie, the best strategy would then be to minimise the sum of these terms in the numerator. However, dividend and debt policy also affect WACC and this must therefore also be considered. By definition

$$WACC = \frac{S_0}{V_0}k_e + \frac{B_0}{V_0}k_d(1-T_c) \quad (6)$$

The analysis should recognise that personal taxes exist, and that such tax rates vary across investors and forms of income. These considerations affect the cost of equity, and a comprehensive model that accommodates them is as follows (Lally, 1992; Cliffe and Marsden, 1992)[4]

$$k_e = R_f(1-T) + \frac{\sum_{j=1}^{n} E(DIV_{1j})T_{dj}}{S_0} + \phi\beta_e \quad (7)$$

where R_f is the risk free rate, ϕ is the market risk premium in this version of the CAPM, β_e is the equity beta, and DIV_{1j} is the time 1 dividend subject to type *j* tax

[3] This model does not directly allow for bankruptcy costs but does so indirectly through the debt risk premium.

[4] The model extends Brennan (1970) to account for dividends being taxed differently to that of interest.

treatment.[5] In addition the tax parameters T and T_{dj} are weighted averages across investors as follows

$$T = \sum_i w_i \left[\frac{t_i - t_{gi}}{1 - t_{gi}} \right] \quad \text{and} \quad T_{dj} = \sum w_i \left[\frac{t_{di}^j - t_{gi}}{1 - t_{gi}} \right] \tag{8}$$

where t_i, t_{gi} and t_{di}^j are investor i's tax rates on interest, capital gains, and cash dividends subject to type j tax treatment respectively.[6] In respect of the cost of debt k_d, this is the sum of the risk free rate R_f and the debt premium p.[7] Substitution of this result and (7) into (6) yields the following result:

$$WACC = \frac{S_0}{V_0} R_f (1 - T) + \frac{S_0}{V_0} \phi \beta_e + \frac{\sum E(DIV_{1j}) T_{dj}}{V_0} + \frac{B_0}{V_0} (R_f + p)(1 - T_c)$$

$$= R_f (1 - T) + \frac{S_0}{V_0} \phi \beta_e + \frac{\sum E(DIV_{1j}) T_{dj}}{V_0} + \frac{B_0}{V_0} R_f (T - T_c) + \frac{B_0}{V_0} p(1 - T_c) \tag{9}$$

In respect of the equity beta, the assumption that WACC is constant over time implies that the leverage ratio is constant over time. Assuming no systematic risk on debt, the appropriate formula under a classical tax regime is as follows (Miles and Ezzell, 1985):

---

[5] Different tax treatment of dividends could arise for a number of reasons. For example, in a dividend imputation regime, dividends with imputation credits are taxed differently to those without them. Alternatively, the payer may have the option of making the dividend payment in the form of share repurchases and the latter are generally taxed differently to conventional dividend payments.

[6] The weight for investor i reflects both the proportion of the value of all risky assets held by them (v_i) as well as the risk premium to variance ratio for their chosen portfolio relative to that of other investors, and the latter reflects both their degree of risk aversion and the tax rates to which they are subject on various assets (see Lally, 1992, Appendix 1). In respect of risk aversion, more risk averse investors will hold portfolios with a higher risk premium to variance ratio. In respect of tax rates, investors with a tax rate on bonds relative to equities that is high relative to other investors (being investors on high incomes) will tilt towards equities rather than bonds, which further raises their value for v_i and therefore raises T. So the value of T tends to reflect the tax rates of investors on high incomes. This demand-side effect is explicit in the analysis of DeAngelo and Masulis (1980) because they explicitly model investor behaviour rather than invoke a CAPM, which reflects investor behaviour.

[7] The premium is the excess of the promised yield to debt holders over the risk free rate and comprises compensation for expected default losses, (systematic) risk associated with default risk and the inferior liquidity of corporate bonds relative to government bonds. In respect of compensation for expected default losses, this arises from both bankruptcy costs and the default option possessed by equity holders. Since the latter is not a cost borne by capital suppliers in aggregate but merely a transfer from equity holders to debt holders, it ought to be excluded from WACC but this is not feasible.

$$\beta_e = \beta_a \left[1 + \frac{B}{S}\left(1 - \frac{R_f T_c}{1 + R_f} \right) \right]$$

where β_a is the equity beta in the absence of debt. By contrast, under a tax regime in which debt imparts no tax benefit (because the corporate tax advantage is offset by the personal tax disadvantage), the tax term disappears to yield the following result:

$$\beta_e = \beta_a \left[1 + \frac{B}{S} \right]$$

The difference between these last two formulations is slight, and therefore the simpler model (the last equation) is preferred. Substitution of the last equation into (9) then yields the following result:

$$WACC = R_f (1 - T) + \phi\beta_a + \frac{\sum E(DIV_{ij}) T_{dj}}{V_0} + \frac{B_0}{V_0} R_f (T - T_c) + \frac{B_0}{V_0} p(1 - T_c)$$

The first two terms are the cost of capital in the absence of dividends or interest payments, and are designated *k*, whilst the third term embodies the impact of paying dividends and the last two terms embody the impact of debt finance. Substitution of the last equation into equation (5), and solving for V_0, yields the following result:

$$V_0 = \frac{E(X_1) - E(N_1) - E(K_1)i + E(M_1)Q_1 - \sum E(DIV_{1j}) T_{dj} - B_0 R_f (T - T_c) - B_0 p(1 - T_c)}{k - g} \quad (10)$$

All of the effects of dividend and debt policy are now embodied in the last five terms of the numerator. The value maximising policy then maximises the sum of these five terms. The last three terms are determined by the level of dividends and interest, whilst the preceding two terms depend upon the level of retention as shown in equation (3), which in turn depends upon the level of dividends and interest as shown in equation (2). So, we solve equation (2) for RET_1 and substitute it into equation (3). The result is thus:

$$K_1 = \max[N_1 - (B_1 - B_0) - X_1 + INT_1(1 - T_c) + \sum DIV_{1j}, 0] \quad (11)$$

$$M_1 = \max[X_1 + (B_1 - B_0) - N_1 - INT_1(1 - T_c) - \sum DIV_{1j}, 0] \quad (12)$$

Our valuation model is now equations (10), (11) and (12), and allows value maximising dividend and debt policies to be determined. We now consider the results under particular tax regimes.

3. Application to Particular Tax Regimes

3.1 The MM Tax Regime

We start with the simplest tax regime examined in the literature, in which debt generates a corporate tax deduction whilst personal tax rates are the same on all forms of personal income (Miller and Modigliani, 1958, 1961, 1963). Since personal tax rates are of this form, $T_{dj} = T = 0$ and therefore equation (10) reduces to the following:

$$V_0 = \frac{E(X_1) - E(N_1) - E(K_1)i + E(M_1)Q_1 + B_0 R_f T_c - B_0 p(1 - T_c)}{k - g} \quad (13)$$

Miller and Modigliani (1958) also assume that debt is risk free and therefore that $p = 0$. Miller and Modigliani (1961) also assume that $i = 0$. In respect of Q_1, they offer no explicit statement. However, consistent with the simple nature of the tax regime assumed by them, it might be assumed that dividends received by a company are not subject to corporate taxation. Thus, if a firm purchases shares in another company, the stream of dividends that it receives and passes back to its shareholders is identical to that arising to its shareholders if they directly purchased such shares. Since $1 invested directly by shareholders has market value $1, then $1 invested by a firm in the same way must add $1 to the market value of the firm. Thus, $Q_1 = 0$. Substitution of these values for Q_1, p and i into equation (13) yields the following:

$$V_0 = \frac{E(X_1) - E(N_1) + B_0 R_f T_c}{k - g}$$

In this case, dividends are irrelevant and value is maximised by maximising debt so as to maximise the corporate tax deduction on interest. These conclusions match those of Miller and Modigliani (1958, 1961, 1963).

3.2 A Classical Tax Regime

We now consider more realistic tax regimes, starting with a regime in which dividend imputation is not present (US). Both dividends and capital gains are (potentially) taxable at 15%.[8] However, capital gains are taxed only upon realisation and this deferral opportunity reduces the effective tax rate by about 50% (Protopapadakis, 1983; Green and Hollifield, 2003). In addition, defacto dividends can be paid without limit in the form of share repurchases. As shown in the Appendix, an investor's tax rate on share repurchases is approximately equal to their effective tax rate on capital gains. So, in the absence of any restriction on the level of share repurchases, share repurchases are a superior means of disbursing cash flows to shareholders and we therefore need only consider this type of dividend.[9] Furthermore, following equation (8), the value for T_d is then as follows

$$T_d = \sum_i w_i \left[\frac{t_{gi} - t_{gi}}{1 - t_{gi}} \right] = 0 \qquad (14)$$

In respect of Q_1, being the NPV arising from investments beyond the level warranted if external finance is used, one possible area in which such investments can be made is in the equity of other firms (as noted by Masulis and Trueman, 1988). In respect of the US, 20% of the dividends flowing from such an investment by a firm are subject to corporate tax, at the rate T_c. Thus, if a company purchases shares in another company, the stream of dividends that can be passed back to its shareholders is proportion $(1-.20T_c)$ of that arising from the shareholders directly purchasing such shares. Since \$1 invested directly by shareholders has market value \$1, then \$1 invested by a firm in the same way must add $\$1(1-.20T_c)$ to the market value of the firm. Thus, $Q_1 = -.20T_c$.

---

[8] Taxes may be avoided in certain cases, such as by investing via tax-exempt vehicles.

[9] The superiority of repurchases is consistent with their gradual supplanting of conventional dividends (Skinner, 2008).

Substitution of these results for Q_1 and T_d into equation (10) yields the following result:

$$V_0 = \frac{E(X_1) - E(N_1) - E(K_1)i - .20T_c E(M_1) - B_0 R_f (T - T_c) - B_0 p(1 - T_c)}{k - g} \quad (15)$$

To determine optimal dividend and debt policy, we start with dividend policy. This affects only K_1 and M_1. Following equations (11) and (12), regardless of the level of debt, the optimal level of dividends is that which ensures that $M_1 = 0$, whereupon K_1 is minimised, i.e., a residual dividend policy. So, $E(M_1) = 0$ and $E(K_1)$ is minimised. Turning now to debt policy, since dividend policy maximises the sum of the third and fourth terms in the numerator of equation (15), debt policy should be chosen to maximise the sum of the last two terms in the numerator of equation (15), which involve the tax effect of debt and the debt premium effect. Clearly, if T is greater than the corporate tax rate T_c, then both of these terms are negative at any positive debt level and therefore the optimal debt level is zero. On the other hand, if T is less than the corporate tax rate T_c, then debt may be desirable at some level.

In summary, dividends in the form of share repurchases should be strictly residual and the optimal level of debt should maximise the sum of the tax benefit net of the debt premium effect.

3.3 A Dividend Imputation Regime

We now turn to a tax regime in which dividend imputation operates, and focus upon a regime of the New Zealand type. If dividends are paid, and imputation credits are available, then they should be attached to the maximum possible extent. So, leaving aside share repurchases, dividends are effectively of two types: those with full imputation credits attached (paid first) and those without imputation credits.[10] For fully imputed dividends (type 1), Lally (2000) shows that

[10] If a dividend is paid in excess of the level that can be fully imputed, this dividend can be considered to be two dividends, one of which has full imputation credits attached and the other without imputation credits.

$$T_{d1} = T - (1-T)U\left[\frac{T_c}{1-T_c}\right] \tag{16}$$

where U is the across-investor average utilisation rate for imputation credits (1 for investors who can fully utilise the credits and zero for those who cannot use them at all). Since this version of the CAPM assumes that national equity markets are fully segmented, and therefore closed to foreign investors, foreign investors (who cannot fully utilise the credits) should be ignored in choosing an estimate for U. The only other class of investors with a utilisation rate less than 1 are tax exempt investors, who are small in New Zealand. This implies a value for U close to 1. Using the approximation of $U = 1$, equation (16) then reduces to the following:

$$T_{d1} = \frac{T - T_c}{1 - T_c}$$

For unimputed dividends (type 2), $T_{d2} = T$. A third class of dividends is also available: defacto dividends can be paid in the form of on-market share repurchases (type 3 dividends).[11] In respect of these share repurchases, and by contrast with the US case, the statutory rate applicable to the cash proceeds in New Zealand would be zero. Furthermore, and again by contrast with the US case, most New Zealand investors are exempt or effectively exempt from capital gains tax (with the remainder only subject to capital gains because the "dividend" was received by an intermediary entity that was subject to capital gains tax). Despite these differences, the analysis in the Appendix is still valid; the fact that all recipients of the cash proceeds would be exempt from capital gains tax and that some investors in general would be exempt from capital gains tax simply gives rise to particular numerical value for their statutory and effective tax rates rather than invalidating the analysis in the Appendix. So, the relevant tax rate for all investors is still their effective capital gains tax rate. Accordingly, as shown in the previous section, $T_{d3} = 0$.

[11] The only restrictions on the level of these share repurchases relates to preserving the solvency of the company. However, on-market repurchases beyond the level of the company's "subscribed capital" do induce a reduction in the company's imputation credits whilst those below this threshold are exempt from this effect (see section CD24 of the Income Tax Act 2007). In respect of off-market share repurchases, these are subject to certain restrictions (as detailed in section CD22 of the Act) that seem to be designed to preclude defacto dividends. So, we can ignore these.

In respect of Q_1, New Zealand firms can invest unlimited sums in the shares of other firms paying fully imputed dividends. Receipt of a fully imputed dividend implies that the recipient corporation pays no company tax on it. Thus, the stream of dividends that it passes back to its shareholders is identical to that arising from the shareholders directly purchasing such shares. Since $1 invested directly by shareholders has market value $1, then $1 invested by a firm in the same way must add $1 to the market value of the firm. Thus, $Q_1 = 0$.

Substitution of these values for T_{d1}, T_{d2}, T_{d3} and Q_1 into equation (10) yields the following result:

$$V_0 = \frac{E(X_1) - E(N_1) - E(K_1)i - E(DIV_{11})\left[\frac{T - T_c}{1 - T_c}\right] - E(DIV_{12})T - B_0 R_f (T - T_c) - B_0 p(1 - T_c)}{k - g} \tag{17}$$

We now seek to determine optimal dividend and debt policy. If T exceeds T_c, then neither conventional dividends nor debt will be desirable because the tax effects shown in equation (17) are negative in both cases, $E(K_1)$ may be raised (which is undesirable), and the adverse debt premium effect arises in the case of using debt. In addition, dividends in the form of share repurchases have no valuation effect so long as they are not so high as to induce share issues (i.e., induce a positive value for K_1) and are undesirable beyond that point.

By contrast, if T is less than T_c, then fully imputed (type 1) dividends are likely to be desirable because the tax effect shown in equation (17) would be positive and likely to outweigh any upward effect on $E(K_1)$. Unimputed dividends would be undesirable because the tax effect shown in equation (17) would be undesirable (the coefficient T on these dividends would be positive). Dividends in the form of share repurchases would again have no valuation impact so long as they are not so high as to increase the level of share issues (and are undesirable beyond that point). In respect of debt, this might be optimal because the tax effect shown in equation (17) would be positive and this might outweigh the debt premium effect and any adverse effect upon $E(K_1)$.

However, if fully imputed dividends were paid, the tax benefits from interest would simply come at the expense of those from fully imputed dividends.[12] The other two effects of debt are negative. So, it could not be optimal to simultaneously pay fully imputed dividends and interest. Accordingly the optimal policy would be the better of the following: no debt coupled with the maximum level of fully imputed dividends or no dividends coupled with the debt level that maximised the sum of the debt sensitive terms in the numerator of equation (17).

In summary, If T exceeds T_c, then neither conventional dividends nor debt will be desirable, because the tax effect is negative in either case and dividends in the form of share repurchases have no valuation effect so long as they are not so high as to induce share issues. By contrast, if T is less than T_c, then the optimal policy would be the better of the following: no debt coupled with the maximum level of fully imputed dividends or no dividends coupled with the debt level that maximised the sum of the debt sensitive terms in the numerator of equation (17). In addition, unimputed dividends would be undesirable and dividends in the form of share repurchases would have no valuation effect so long as they were not so high as to induce share issues.

4. Some Examples

4.1 A Classical Tax Regime

We now consider an example based upon the US tax regime discussed in section 3.2. In particular, dividends and capital gains are taxed at 15%, the top personal rate on interest is 35%, and the corporate tax rate is 35% (Berk and DeMarzo, 2007, Table 15.3). However capital gains taxes are paid only on realisation and, as noted earlier, this reduces the effective tax rate by about 50%. Furthermore, in view of the range in tax rates on interest and the fact that T tends to reflect the tax rates of investors with high incomes (see footnote 5), we adopt an average rate on interest of 30%.[13] Consistent with this, the value for T is as follows:

---

[12] Raising INT_1 by \$1 reduces the available imputation credits by $\$1T_c$, and therefore reduces the level of fully imputed dividends that can be paid by $\$1T_c(1 - T_c)/T_c = \$1(1 - T_c)$. Consequently, if fully imputed dividends are being paid, the effect of raising interest payments by \$1 is to raise the fourth term, and lower the sixth term, in the numerator of equation (17) by matching amounts.

[13] Alternative values will be considered later. Interestingly, Graham (2000, p. 1913) also estimates the average tax rate on interest at 30% but this reflects the 1993 situation when the top marginal rate on interest was slightly higher than it is currently (Berk and DeMarzo, 2007, Table 15.3).

$$T = \sum_i w_i \left[\frac{t_i - t_{gi}}{1 - t_{gi}} \right] = \frac{.30 - .075}{1 - .075} = .24$$

Suppose also that X_1 is uniformly distributed on [\$2m, \$8m], N_1 = \$1.8m for certain, the issue costs post-company tax are i = .05, $Q_1 = -.20T_c = -0.07$, R_f = .065, g = .04 and k = .10. Substitution into equation (15) yields the following result:

$$V_0 = \frac{\$3.2m - .05E(K_1) - .07E(M_1) - .065B_0(.24 - .35) - B_0 p(0.65)}{.10 - .04} \qquad (18)$$

We start with $DIV_1 = B_0 = 0$. In this case, and following equation (11), K_1 is always zero and therefore $E(K_1) = 0$. Also, following equation (12), M_1 is positive for all possible values of X_1. It follows from equation (12) that

$$E(M_1) = E(X_1) - N_1 = \$5m - \$1.8m = \$3.2m$$

Substitution of these results into equation (18) yields the following result:

$$V_0 = \frac{\$3.2m - .07(\$3.2m)}{.10 - .04} = \$49.6m \qquad (19)$$

Turning now to optimal policy, and as shown in section 3.2, the optimal debt level maximises the sum of the last two terms in the numerator of equation (18) whilst dividends are paid to ensure that $M_1 = 0$ thereby minimising the sum of the second and third terms in the numerator of equation (18). For values of p less than .011, the sum of the last two terms in the numerator of equation (18) is positive, and values for p within this range are possible for at least moderate leverage levels (so long as the recent sharp rise in corporate debt premiums is ignored). Thus, to explore the issue further, it is necessary to specify a relationship between p and B_0. Almeida and Philippon (2007, Table I) give risk premiums for bonds of various credit ratings, for a range of maturities, and we use the results for five year bonds. They also give estimates of leverages associated with those credit ratings from a number of studies (ibid, Table V) and we use the last of the results cited there.[14] This permits a relationship between p and leverage to be specified. Clearly the relationship is non-linear and therefore $Ln(p)$ is regressed on leverage to yield

$$Ln(p) = -5.79 + 4.42\frac{B_0}{V_0}$$

which implies that p rises from .003 at leverage of zero to .018 at leverage of 40% and .043 at leverage of 60%.[15] With this non-linear relationship, the problem requires simultaneous solution of B_0 and V_0. However, as we will see, V_0 is not particularly sensitive to leverage and therefore an acceptable approximation is obtained by using V_0 = \$50m. So, the last equation is simplified to

$$Ln(p) = -5.79 + 4.42\left[\frac{B_0}{\$50m}\right] \tag{20}$$

With this relationship, the level of debt maximising the sum of the last two terms in the numerator of equation (18) is B_0 = \$8.27m, at which p = .006. In addition, dividends are chosen to ensure that $M_1 = 0$. Following equation (12), this implies that

$$\begin{aligned} 0 &= \max[X_1 + (B_1 - B_0) - N_1 - INT_1(1 - T_c) - DIV_1, 0] \\ &= \max[X_1 + .04B_0 - \$1.8m - .071B_0(1 - .35) - DIV_1, 0] \\ &= \max[X_1 + .04(\$8.27m) - \$1.8m - .071(\$8.27m)(1 - .35) - DIV_1, 0] \\ &= \max[X_1 - DIV_1 - \$1.85m, 0] \end{aligned}$$

So, if X_1 = \$2m, then DIV_1 = \$0.15m whilst if X_1 = \$8m then DIV_1 = \$6.15m. Following this residual policy, K_1 will always be zero and therefore $E(K_1) = 0$. Substitution of these results for B_0, $E(M_1)$ and $E(K_1)$ into equation (18) yields V_0 = \$53.8m. This compares with V_0 = \$49.6m with no debt or dividends as shown in equation (19). So, the value increment from the optimal debt and dividend level is

[14] The data predate the recent "credit crunch". Furthermore, the data involve averaging over firms in a range of industries and therefore are merely illustrative of the appropriate relationship for a particular industry.
[15] This transformation appears to generate a linear relationship.

about 8%. However, if the debt level were set at \$8.27m and dividends remained zero, then M_1 would always be positive and equation (12) then implies that

$$
\begin{aligned}
E(M_1) &= \$5m + E(B_1 - B_0) - \$1.8m - INT_1(1-.35) \\
&= \$3.2m + .04B_0 - .071B_0(1-.35) \\
&= \$3.2m + .04(\$8.27m) - .071(\$8.27m)(1-.35) \\
&= \$3.15m
\end{aligned}
$$

Substitution into equation (18) along with B_0 = \$8.27m yields V_0 = \$50.1m. So, optimal debt policy affects firm value only trivially. It is the adoption of a residual dividend policy that largely explains the 8% value gain.

These results are not particularly sensitive to the values of parameters for which there is considerable uncertainty, most particularly p and T. Since higher values for these two parameters further reduce the already low level of debt, we consider lower values for these two parameters. In respect of p, we consider the result from reducing the second coefficient in equation (20) to p_c = 3.80, which would lower the debt premium at leverage of 40% from .018 to .014. In respect of T, there is considerable room for doubt over the appropriate value particularly due to the extent to which various tax-reducing schemes operate.[16] For example, suppose that the existence of these schemes reduces the average tax rate on both interest income and capital gains by 30%. Accordingly the value for T shown earlier in this section falls from .24 to .17. Table 1 shows the results from these four possible combinations of values for p_c and T.[17] Averaged across the four cases, the increase in value from optimal dividend and debt policies is 9.3% with only 2% of this coming from debt policy.

---

[16] A prominent example is a Roth IRA scheme in which employees allocate part of their post-tax income to an account, which could be invested in a variety of assets including bonds or equities, and whose subsequent payouts are not taxable under certain conditions. However, there are limits on the amounts that can be invested and there may be penalties in the event of withdrawal before the investor reaches age 60 (Reilly and Brown, 2006, pp. 52-54).

[17] The reduction in T may also raise the value of the firm in the absence of debt and dividends, via a reduction in the discount rate k. However, since our primary concern is with the value gains from optimal debt and dividend policy, we do not attempt to estimate this effect.

In summary, within the US tax regime, debt generates a tax benefit and some level of it is optimal (depending upon the size of the debt premium and the tax benefit) in conjunction with residual dividends (paid in the form of share repurchases). Across a range of possible values for the debt premium and tax benefit, and relative to zero debt and dividends, the valuation gain from optimal debt and dividends is about 9% with the great majority of this coming from dividend rather than debt policy.

4.2 A Dividend Imputation Tax Regime

We now consider an example from a tax regime featuring dividend imputation and focus upon the New Zealand tax regime discussed in section 3.3. As discussed in that section, the optimal course of action depends inter alia upon whether the tax parameter T is less than the corporate tax rate T_c and whether imputation credits are available. So, we consider four possible cases, corresponding to the presence and absence of imputation credits coupled with the situation before and after certain recent changes in the tax regime (the recent reduction in the corporate tax rate from 1.4.2008 and the introduction of the PIE tax regime from 1.10.2007).

The first case involves the earlier tax regime coupled with a firm having imputation credits. Under the earlier tax regime, Lally and Marsden (2004) estimate the tax parameter T at .27 whilst the corporate tax rate was T_c = .33. In addition, suppose that X_1^u is uniformly distributed on [$2m, $8m], $IC_1^u = .4X_1^u$, N_1 will be $1.8m for certain, the expected growth rate in cash flows is g = .04, the issue costs post-company tax are i = .05, k = .10, and R_f = .065. Substitution into equation (17) yields the following:[18]

$$V_0 = \frac{\$3.2m - .05E(K_1) + .0896E(DIV_{11}) + .0039B_0 - B_0 p(0.67)}{.10 - .04} \qquad (21)$$

With zero dividends and debt, K_1 is always equal to zero and therefore V_0 is as follows.

$$V_0 = \frac{\$3.2m}{.10 - .04} = \$53.3m \qquad (22)$$

[18] Unimputed dividends are omitted because they should never be paid, as discussed in section 3.3.

Since T is less than T_c then, as discussed in section 3.3, the optimal course of action would be the better of the following: no debt coupled with the maximum level of fully imputed dividends or no dividends coupled with the debt level that maximised the sum of the debt sensitive terms in the numerator of equation (21). The maximum level of fully imputed dividends is as follows:

$$DIV_{11} = IC_1^u \left[\frac{1-.33}{.33} \right] = .4X_1 \left[\frac{1-.33}{.33} \right] = .812X_1 \tag{23}$$

Since X_1 is uniformly distributed on [\$2m, \$8m], then DIV_{11} is uniformly distributed on [\$1.62m, \$6.50m]. It follows that $E(DIV_{11})$ = \$4.06m. In addition, following equations (11) and (23):

$$\begin{aligned} K_1 &= \max(\$1.8m - X_1 + DIV_{11}, 0) \\ &= \max(\$1.8m - X_1 + .812X_1, 0) \\ &= \max(\$1.8m - 0.188X_1, 0) \end{aligned}$$

Since X_1 is uniformly distributed on [\$2m, \$8m], it follows that K_1 is always positive (share issues are always undertaken) and therefore

$$E(K_1) = \$1.8m - 0.188E(X_1) = \$0.86m$$

Substitution of these values for $E(K_1)$ and $E(DIV_{11})$ along with $B_0 = 0$ into equation (21) yields the following result:

$$V_0 = \frac{\$3.2m - .05(\$0.86m) + .0896(\$4.06m)}{.10 - .04} = \$58.7m \tag{24}$$

Turning now to debt, conditional upon the absence of dividends, the optimal level of debt maximises the sum of the debt sensitive terms in the numerator of equation (21). Substituting equation (20) into equation (21), the optimal debt level is \$3.91m at which $p = .004$. At this debt level, K_1 is always zero and therefore $E(K_1) = 0$. So, following equation (21) with zero dividends, V_0 = \$53.4m. This is inferior to the result in equation (24) and only fractionally greater than the result in equation (22).

So, the optimal policy is zero debt and fully imputed dividends, yielding a firm value of \$58.7m as shown in equation (24). This exceeds the \$53.3m shown in equation (22) above when no dividends are paid by 10%, and reflects the tax benefits from paying fully imputed dividends (rather than retaining the funds and thereby generating capital gains) net of the transaction costs on the share issues resulting from doing so.

The second case we consider differs from the first only in that the firm has no imputation credits. Following equation (17):

$$V_0 = \frac{\$3.2m - .05E(K_1) - .27E(DIV_1) + .0039B_0 - B_0 p(0.67)}{k - g} \tag{25}$$

Conventional dividends are now undesirable because of the adverse tax effect (and dividends in the form of share repurchases are neutral so long as they do not raise K_1). Unlike the first case, debt is now desirable because it no longer reduces the level of imputation credits. The optimal level of debt maximises the sum of the last two terms in the numerator of equation (25). Substituting equation (20) into equation (21), the optimal debt level is B_0 = \$3.91m at which p = .004. At this debt level, K_1 is always zero and therefore $E(K_1) = 0$. So, following equation (25) with zero dividends, V_0 = \$53.4m. This is only marginally greater than the result in equation (22).

The third case recognises the recent changes in the tax regime (the reduction in the corporate tax rate to 30% and the introduction of the PIE tax regime) coupled with a firm that has imputation credits. The effect of the PIE regime is to largely eliminate capital gains tax and lower the average tax rate on ordinary income (interest and gross dividends) to 30%.[19] So, $T = T_c = .30$. Following equation (17), the tax benefits from

[19] As discussed in footnote 5, the value for T is primarily determined by investors with high incomes. An investor in an investment fund that has elected to become a PIE would be taxed at 30% on interest and gross dividends if their income was above \$38,000 per annum, and 19.5% if their income was less. Otherwise investors would be taxed at 19.5%, 33% or 39% depending upon their income level. Thus, if there is substantial use of PIEs by investors with high incomes, a good estimate of the average investor tax rate on interest and gross dividends would then be 30%. Accordingly, a good estimate of T would be .30.

both debt and fully imputed dividends now collapse to zero. In addition, as before, suppose that X_1^u is uniformly distributed on [$2m, $8m], $IC_i^u = .4X_1^u$, N_1 will be $1.8m for certain, the expected growth rate in cash flows is g = .04, the issue costs post-company tax are i = .05, k = .10, and R_f = .065.[20] Substitution into equation (17), with omission of unimputed (type 2) dividends, yields the following:

$$V_0 = \frac{\$3.2m - .05E(K_1) - B_0 p(0.67)}{.10 - .04} \qquad (26)$$

So, fully imputed dividends and dividends in the form of share repurchases are irrelevant to firm value up to the point that $E(K_1)$ remains zero and are undesirable beyond that point, i.e., these dividends are irrelevant so long as they are not so high as to induce share issues. In addition, unimputed dividends are undesirable and debt is also undesirable because it no longer generates tax benefits and yet still incurs the debt premium penalty. So, following equation (26), V_0 = $53.3m.

The final case differs from the third only in the absence of imputation credits. Since imputed dividends add nothing to firm value, the absence of the imputation credits does not change the result: debt and unimputed dividends are undesirable, dividends in the form of share repurchases are irrelevant to value so long as they do not induce share issues, and V_0 = $53.3m.

These results are summarised in Table 2. Unimputed dividends are always undesirable. In the tax regime prevailing until the recent reduction in the corporate tax rate and the introduction of the PIE tax regime, the payment of interest and fully imputed dividends generates equivalent and substitutable tax benefits. However, fully imputed dividends are superior, because they do not give rise to the debt premium effect and raise firm value by about 10% in the example considered, debt is therefore not desirable, and dividends in the form of repurchases are neutral so long as they do not induce share issues. In the absence of imputation credits, fully imputed dividends

[20] The change in the tax regime may change the discount rate and will change both the probability distribution of X_1^u and the relationship between IC_1^u and X_1^u. However, we do not attempt to analyse these effects. It is still possible to specify the optimal financial policies for the assumed values for these parameters.

and dividends in the form of share repurchases are neutral so long as they do not induce share issues, debt is now desirable because it does not reduce the level of imputation credits but the optimal leverage level is low and the impact on firm value is trivial Under New Zealand's present tax regime involving a corporate tax rate of 30% and the PIE tax regime, the tax benefits from both debt and fully imputed dividends are now both zero whilst the tax effect of dividends in the form of share repurchases remains zero. Consequently, debt is undesirable and both fully imputed dividends and dividends in the form of share repurchases are irrelevant to value so long as they are not so high as to induce share issues.

5. Conclusions

This paper has analysed optimal dividend and debt policy within a conventional value-maximising multi-period DCF context, and allows for differential personal taxation over types of income, the effect of dividends and interest on the level of share issues (and hence share issue costs), the effect of dividends and interest on the level of internally financed investment (with possibly negative NPV), and the debt premium effect. Furthermore, examples of both non-imputation and imputation regimes have been considered.

In the non-imputation regime considered here, corresponding to the US, debt generates a tax benefit and some level of it is optimal (depending upon the size of the debt premium and the tax benefit) in conjunction with residual dividends paid in the form of share repurchases. Across a range of possible values for the debt premium and the tax benefit, and relative to zero dividends and debt, the valuation gain from optimal debt and dividends is about 9% of firm value, with the great majority of this coming from dividend policy rather than debt policy.

In the imputation regime considered here, corresponding to New Zealand, the payment of interest and fully imputed dividends generates equivalent and substitutable tax effects. Unimputed dividends are not desirable and dividends in the form of share repurchases are neutral so long as they do not induce share issues. Debt is not desirable if imputation credits exist and otherwise exerts only a slight effect upon firm value. The tax benefits from fully imputed dividends were positive until recent tax changes, and raised firm value by about 10% in the example considered, but these tax benefits have now been driven down to zero by recent tax changes.

APPENDIX

This appendix seeks to determine the tax rate on defacto dividends that take the form of share repurchases and are treated by the tax authorities as share repurchases. Suppose a firm with N shares and price per share P_0 unexpectedly generates cash flow of X and immediately disburses it to shareholders via a share repurchase, involving m shares at the ex-repurchase price P_1. So $X = mP_1$. Also, since the unexpected cash flow is immediately disbursed, then the equity value is unchanged, i.e.,

$$NP_0 = (N - m)P_1$$

Thus

$$N(P_1 - P_0) = mP_1 = X \tag{27}$$

We focus upon an investor i who holds the same proportion of shares sold as not sold under the repurchase, and this proportion is denoted α_i.[21] The investor then receives $\alpha_i X$ and the tax rate on this disbursement of $\alpha_i X$ is the incremental taxes paid as a proportion of $\alpha_i X$. Incremental taxes are paid in respect of both the shares that are not sold to the company in the course of the share repurchase as well as those that are sold. For each of the shares that are not sold, totalling $\alpha_i(N-m)$ shares, the share price rises from P_0 to P_1 as a result of the event, and this capital gain generates a tax payment at the investor's effective capital gains tax rate t_{gi}. For the $\alpha_i m$ shares that are sold to the company, the resulting tax payment is $\alpha_i m(P_1 - H)t_{Si}$ where t_{Si} is the statutory capital gains tax rate and H is the average historic cost of the shares. However, sale of the shares to the company merely accelerates the eventual sale of the shares that would otherwise have occurred in the absence of a repurchase, and this alternative scenario would give rise to a capital gains tax obligation of $\alpha_i m(P_0 - H)t_{gi}$. So, the tax obligation arising from the share repurchase is simply the increment. Thus, investor i's tax rate on the share repurchase (type 1 dividend) is as follows:

$$t_{di}^1 = \frac{\alpha_i (N - m)(P_1 - P_0)t_{gi} + \alpha_i m[(P_1 - H)t_{Si} - (P_0 - H)t_{gi}]}{\alpha_i X}$$

[21] This would literally occur under a pro-rata repurchase offer. However, even in the absence of a pro-rata repurchase offer, the matching proportion scenario characterises the average investor by definition.

$$= \frac{(N-m)(P_1 - P_0)t_{gi}}{X} + \frac{m[(P_1 - H)(t_{gi} + t_{Si} - t_{gi}) - (P_0 - H)t_{gi}]}{X}$$

$$= \frac{(N-m)(P_1 - P_0)t_{gi}}{X} + \frac{m[(P_1 - P_0)t_{gi} + (P_1 - H)(t_{Si} - t_{gi})]}{X}$$

$$= \frac{N(P_1 - P_0)t_{gi}}{X} + \frac{m(P_1 - H)(t_{Si} - t_{gi})}{X}$$

$$= \frac{N(P_1 - P_0)t_{gi}}{N(P_1 - P_0)} + \frac{m(P_1 - H)(t_{Si} - t_{gi})}{mP_1} \qquad \text{using equation (27)}$$

$$= t_{gi} + (t_{Si} - t_{gi})\left[1 - \frac{H}{P_1}\right]$$

Since H is positive and the investor's statutory rate on capital gains (t_{Si}) exceeds their effective rate on capital gains (t_g), then their tax rate on share repurchases must be less than t_{Si} and may even be less than t_{gi}. If share repurchases occur shortly after the investor purchased the shares then H is approximately equal to P_1, and therefore the tax rate on repurchases becomes t_{gi}. As the interval from the average purchase date till the date of repurchase increases, the tax rate on repurchases diverges from t_{gi}, but it could increase or reduce. So, to an acceptable degree of approximation, the investor's tax rate on repurchases is equal to the investor's effective tax rate on capital gains.

Table 1: Summary of Results Under a Classical Tax Regime

	T = .24		T = .17	
	p_c = 4.42	p_c = 3.80	p_c = 4.42	p_c = 3.80
Firm Value with no debt/divs	$49.6m	$49.6m	$49.6m	$49.6m
Optimal debt level	$8.27m	$9.62m	$11.92m	$13.86m
Optimal expected dividends	$3.15m	$3.14m	$3.10m	$3.09m
Firm Value	$53.8m	$53.9m	$54.5m	$54.7m
Value gain	8.5%	8.7%	9.9%	10.3%
Value gain from optimal debt	1.0%	1.2%	2.6%	3.0%
Value gain from optimal divs	7.5%	7.5%	7.3%	7.3%

This table shows the optimal levels for debt and dividends along with the associated firm values in a classical tax system, under four possible combinations of parameter values. The impact of the optimal debt and dividend levels upon firm value (in % terms) is also shown.

Table 2: Summary of Results Under a Dividend Imputation Regime

	$T < T_c$		$T = T_c$	
	$IC = 0$	$IC > 0$	$IC = 0$	$IC > 0$
Firm value with no divs/debt	$53.3m	$53.3m	$53.3m	$53.3m
Optimal imputed dividends	n/a	$4.06m	n/a	irrelevant
Optimal unimputed dividends	0	0	0	0
Optimal share repurchases	irrelevant	irrelevant	irrelevant	irrelevant
Optimal Debt	$3.91m	0	0	0
Firm Value	$53.4m	$58.7m	$53.3m	$53.3m
Value gain	0.2%	10%	0	0
Value gain from optimal debt	0.2%	0	0	0
Value gain from optimal divs	0	10%	0	0

This table shows the optimal levels for debt and three classes of dividends along with the associated firm values in an imputation tax system, under four possible combinations of parameter values. The impact of the optimal debt and dividend levels upon firm value (in % terms) is also shown.

REFERENCES

Berk, J. and DeMarzo, P. 2007, *Corporate Finance,* Pearson Addison Wesley.

Boyle, G. 1996, 'Corporate Investment and Dividend Tax Imputation', *The Financial Review* 31, 465-482.

Brennan, M. 1970, 'Taxes, Market Valuation and Corporate Financial Policy', *National Tax Journal*, vol.23, pp.417-27.

Cliffe, C. & Marsden, A. 1992, 'The Effect of Dividend Imputation on Company Financing Decisions and the Cost of Capital in New Zealand', *Pacific Accounting Review*, vol. 4, pp. 1-30.

DeAngelo, H. and Masulis, R. 1980, 'Leverage and Dividend Irrelevancy under Corporate and Personal Taxation', *The Journal of Finance*, vol. 35, pp. 452-467.

Fung, W. and Theobald, M. 1984, 'Dividends and Debt Under Alternative Tax Systems', *Journal of Financial and Quantitative Analysis*, vol. 19 (1), pp. 59-72.

Goldman Sachs JBWere. 2006, *New Zealand Company Beta Book*.

Graham, J. 2000, 'How Big are the Tax Benefits of Debt', *Journal of Finance,* vol. 55, pp. 1901-1941.

Green, R. and Hollifield, B. 2003, 'The Personal-Tax Advantages of Equity', *Journal of Financial Economics*, vol. 67, pp. 175-216.

Lally, M. 1992, 'The CAPM Under Dividend Imputation', *Pacific Accounting Review*, vol. 4, pp. 31-44.

________ 2000, 'Valuation of Companies and Projects under Differential Personal Taxation', *Pacific-Basin Finance Journal*, vol. 8, pp. 115-133.

________ and Marsden, A. 2004, 'Tax-Adjusted Market Risk Premiums in New Zealand: 1931-2002', *Pacific-Basin Finance Journal*, vol. 12 (3), pp 291-310.

Masulis, R. and Trueman, B. 1988, 'Corporate Investment and Dividend Decisions under Differential Personal Taxation', *Journal of Financial and Quantitative Analysis*, vol. 23, pp. 369-385.

Miles, J. and Ezzell, J. 1985, 'Reformulating Tax Shield Valuation: A Note', *Journal of Finance,* vol.40, pp.1485-92.

Miller, M. 1977, 'Debt and Taxes', *Journal of Finance*, vol.32, pp.261-76.

Modigliani, F., and M. Miller, 1958, The Cost of Capital, Corporation Finance and the Theory of Investment, *American Economic Review* 48, 261-297.

_________________________ 1961, 'Dividend Policy, Growth and the Valuation of Shares', *The Journal of Business*, vol. 34, pp. 411-433.

_________________________ 1963, Corporate Income Taxes and the Cost of Capital: A Correction, *American Economic Review* 53, 433-443.

Protopapadakis, A. 1983, 'Some Indirect Evidence on Effective Capital Gains Taxes', *Journal of Business*, vol.56, pp.127-38.

Reilly, F. and Brown, K. 2006, *Investment Analysis and Portfolio Management*, 8th edition, Thomson South-Western.

Skinner, D. 2008, 'The Evolving Relation between Earnings, Dividends and Stock Repurchases', *Journal of Financial Economics*, vol. 87, pp. 582-609.