Graphical Models of Multivariate Volatility

Alethea Rea[1,2], William Rea[1,3], Marco Reale[1], Carl Scarrott[1], and Giuseppe Storti[4]

[1.] Department of Mathematics and Statistics, University of Canterbury, New Zealand

[2.] Department of Statistics, University of Auckland, New Zealand

[3.] Department of Economics and Finance, University of Canterbury, New Zealand

[4.] Dipartimento di Scienze Economiche e Statistiche, Universita Delgi Di Studi Salerno, Italy

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Extended Abstract

Financial markets are always interested in the return and the risk of assets. In statistical terms the return and the risk deal with the mean and the variance (or first and second moments) of the returns respectively.

To study financial time series, assumptions about the distributions of the observations are often made. With stock market indices it is often assumed that daily closing prices all come from a lognormal distribution with mean μ and variance σ^2.

Past studies of the prices of financial assets have shown that there are periods of time where prices movements are large and other periods when the movements are smaller. Thus the variance of such assets is not constant with time. A process with changing volatility or variance over time is called heteroscedastic.

Volatility transmission is a recent area of research that looks at whether the price volatility in one financial market gives useful information when trying to forecast volatility in a second (or subsequent) market. Volatility transmission can also be called spillover effects in the literature. This research looks at volatility transmission, or spillover effects, between the US, UK and Japan stock market indices.

In order to understand volatility transmission between financial assets a multivariate model is essential. This research firstly looks at the standard econometric tools for measuring multivariate volatility or risk.

The price movements between one day and the next are usually not meaningfully predictable because the day to day financial asset returns are usually not correlated. However, it has been empirically observed that the squared returns, used as a proxy for variance, are often correlated. Thus the return series themselves are uncorrelated but not independent.

A univariate model for this type of behaviour is the auto regressive conditional heteroskedasticity or ARCH model. The ARCH model uses a linear combination of p most recent lags to estimate the current volatility. The underlying assumption, however, is that the returns follow a random walk.

The ARCH model was generalised by Bollerslev and multivariate extensions of the generalised ARCH models have been formulated over the years. A multivariate model allows us to study the interaction of several financial assets through volatility channels. That is, it allows the study of the similarity of volatilities observed in different financial markets and the underlying transfer of volatility between the financial assets. We give an application to the Standand and Poor's Composite 500, FTSE 100 and the Nikkei 225; three widely followed stock indices of New York, London and Tokyo stock exchanges.

Using methods and ideas from the branch of mathematics known as graph theory we develop a graphical model for returns and, more importantly, volatilities. The graphical models have been widely applied to non-time series data because they can naturally incorporate an ordering of variables to allow inference to be made about causal links between them. In this case the graphical model finds causal relationships in the time series data by exploiting the time ordering of the variables, rather than using subject-specific knowledge to specify the ordering, and objectively finding the links which are significant. It can easily be extended to allow the study of contemporaneous relationships.

Both the logarithmic returns and the squared logarithmic returns are modelled using graphical modelling. In order to model the squared log returns we a multivariate ARCH. This model is similar to vector autoregressive time series models allowing graphical modelling to be easily adapted. The graphical model for the squared logarithmic returns offers a causal insight into volatility channels.