Default Option and Optimal Capital Structure in Real Estate Investment

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Abstract

This article investigates the determinants of optimal capital structure in real estate investment in a framework where an investor incurs transaction costs when purchasing a property through debt financing. The interaction of these costs and the stochastic evolution of the property value confers on the investor an option value from waiting, which the investor must take into account when deciding the date at which to purchase the property. At the optimal date of purchasing, the investor also chooses a loan-to-value ratio, which balances the tax shield benefit and the transaction costs. After the investment is made, the investor will default once the value of the property falls below the balance of debt. We relate several factors that characterize the evolution of the property value and the mortgage loan to the investment timing, the debt-to-loan value ratio, the default timing and probability, as well as the net value of investment.

Key words: Default, Optimal Capital Structure, Real Estate Investment, Real Options, Transaction Costs

JEL Classification: G13, G31, G32

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I. Introduction

This article investigates the investment and financing decisions of a real estate investor who considers acquiring a self-used commercial or residential property through debt financing. Some articles have considered the optimal financing decision of an investor who intends to purchase an income-generating property. See, for example, Cannaday and Yang (1995, 1996), Gau and Wang (1990), and McDonald (1999).[1] All of these articles allow an investor to have neither the option to default nor the option to delay purchasing the property. This study significantly differs from them by allowing these two options.

This article, which belongs to the burgeoning literature that applies the real options approach to investment (Dixit and Pindyck, 1994), assumes that an investor chooses an optimal timing and a loan-to-value ratio to maximize the net expected present value of property. As Brueggeman and Fisher (2006, chapter 4) suggest, a mortgage loan borrower, who is also the buyer of a property in our framework, incurs sunk costs such as statutory costs and third-party charges. The interaction of these sunk costs and the stochastic property value confers on the investor an option value to delay purchasing the property. Consequently, the investor will not purchase until the value of the property reaches a threshold level. At the optimal date of purchasing, the investor also decides a loan-to-value ratio that involves the tradeoff as follows: the investor enjoys more tax deductible benefits from interest payments and/or capital depreciation, but suffers larger sunk cost when borrowing more debt.[2] After the mortgage contract is signed, however, the investor is also able to default when the value of the property falls short of the balance of the mortgage loan.

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[1] Ever since the seminal paper by Modigliani and Miller (1958), the determinants of corporate borrowing have been a heated topic in the corporate finance literature. See, for example, the survey paper by Harris and Raviv (1991), and Myers (2003). This topic receives little attention, however, in the real estate investment literature. See the discussions in Gau and Wang (1990) and Clauretie and Sirmans (2006, chapter 15).

[2] This tradeoff significantly differs from that addressed in the finance literature, which also allows the tax advantages of borrowing, but considers the costs associated with either financial distress, or the conflict of interest between equity and debt holders. See, for example, Harris and Raviv (1991) and Myers (2003).

Several articles have also allowed an investor to have the option to default when investigating the issue regarding the value of mortgage loan. Some of them even allow the investor to have the option to repay off the loan by further assuming that the interest rate is stochastic over time.[3] However, they focus on neither the choice of investment timing nor the choice of financing decisions.[4] By contrast, we abstract from the prepayment option by assuming that the interest rate is constant over time. This is reasonable for a mortgage loan on a commercial property because substantial penalties on prepayment discourage the investor to pay off the loan before the maturity date (Cannaday and Yang, 1996).

The remaining sections are organized as follows. We first present the basic assumption of the model, and then derive the conditions for the default timing, investment timing and the loan-to-value ratio decided by an investor who intends to hold the property indefinitely. We then analyze the determinants of default timing given that the mortgage loan is already set. The option value to default, in turn, will affect the choices of the investment timing and financing decisions. However, we find that most of our theoretical predictions regarding these two choices are indefinite. Consequently, we employ plausible parameters in order to carry out some numerical comparative-statics testing in the following section. The last section concludes and offers suggestions for future research.

II. The Model

Consider an investor who chooses the timing to purchase a property, as well as the percentage of debt to finance this purchase. The value of the property, $H(s)$, evolves as a geometric Browian motion as

$$\frac{dH(s)}{H(s)} = (\mu - \lambda)ds + \sigma d\Omega(s), \tag{1}$$

3 See, for example, Hilliard, Kau, and Slawson (1998) and Schwartz and Torous (1992).

4 This article also differs from Gau and Wang (1990) and McDonald (1999), as these two studies assume that an investor bears the cost associated with bankruptcy (Stiglitz, 1972) when the investor fails to pay off debt obligations. This article, however, assumes that the loan provider bears this bankruptcy cost.

where $d\Omega(s)$ is the increment of a standard Wiener process, μ is the total expected return from holding the property, λ is the service flow rate, and σ is the instantaneous volatility of the property value. The property in consideration may be a residential property or a self-owned commercial property, and thus we can interpret the service flow rate as the imputed rental rate. Suppose that we start from $t=0$, and denote $H(0)$ as the initial property value. At the initial date, the investor also chooses a loan-to-value ratio, denoted as M. Consequently, the investor carries a mortgage loan with an initial balance $MH(0)$ and injects an initial equity equal to $(1-M)H(0)$. We assume that the mortgage interest rate is equal to a constant r and that the mortgage pays interest only before the maturity date T. We also assume that all agents are risk neutral, and employ the constant interest rate as their discount rate. Thus, at each instant after the purchase, an investor pays interest equal to $rMH(0)$. We also assume that the investor suffers a sunk cost equal to fM^{ε}, where $f>0$, and $\varepsilon>0$. This cost is novel to the literature. As Brueggeman and Fisher (2006, chapter 4) suggest, a mortgage loan borrower, who is also the buyer of a property in our framework, incurs statutory costs and third-party charges. The former includes certain charges for legal requirements pertaining to title transfer, recording of the deed, and other fees required by state and local law. The latter includes charges for services such as legal fees, appraisals, surveys, past inspection, and title insurance. These costs, however, are unrecoverable after the mortgage contract is signed.[5] The total cost paid by the investor at the instant of purchase is thus given by

$$C(H(0),M)=(1-M)H(0)+fM^{\varepsilon}. \tag{2}$$

In contrast to the literature on the optimal debt of real estate investment, we allow the investor to have the option to default. Following Leland (1994), we assume that the investor will choose to default when the property value declines to a value equal to $H_b(s)$ at instant s such that the

[5] Broadly speaking, the transaction sunk costs also include the opportunity cost in the form of time spent in negotiating with both the property seller and the mortgage loan provider.

investor has a value exactly equal to zero.

Denote $V(H(t),t)$ as the investor's value at time t. Applying risk-neutral valuation, and following Leland and Tuft (1996) yields

$$V(H(t),t) = H(t) + V^e(H(t),T), \tag{3}$$

where

$$V^e(H(t),T) = \int_t^T e^{-r(s-t)}[-(1-\tau)rMH(0) + \frac{\tau\delta H(0)}{n}][1-F(s;H(t),H_b(s))]ds \tag{4}$$

$$+e^{-r(T-t)}E_t[1-F(T;H(t),H_b(T))]ATER(T),$$

$F(s;H(t),H_b(s))$ is the cumulative distribution function of the first passage time to default at time s starting from time t, δ is the proportion of the property that is depreciable capital (that is, not the land), n is the length of the depreciation period (39 years for commercial real estate in the U.S.), τ is the income tax rate, E_t is the expectation operator applied at time t, $ATER(T)$ is after-tax equity reversion from selling the property at time T. Note that for a residential mortgage, we need to set δ equal to zero given that no depreciation is allowed for tax purposes. On the right-hand side of Equation (3) the first term is the sum of the after-tax interest payment and the expected gain from deductions of depreciation allowance. The after-tax equity reversion for the investor at time T is given by:[6]

$$ATER(T) = H(T) - MH(0) - \tau[H(T) - H(0) + \frac{\delta T}{n}H(0)], \tag{5}$$

where $H(T)$ is the selling price at date T at which the investor pays off the loan. Consequently, at this date, the investor must also pay off the loan balance, $MH(0)$, and pay taxes on the capital gain of $H(T) - H(0) + \delta TH(0)/n$.

Following the traditional literature such as in Gau and Wang (1990), Cannaday and Yang (1996), and McDonald (1999), we assume that the investor acts in his/her interest when choosing the investment timing and the loan-to-value ratio. Equivalently, this implies that the investor chooses

[6] We assume that T is smaller than n. We must replace T by n if T is larger than n.

an appropriate timing and loan-to-value ratio so as to maximize the expected present value of the investment $V^e(\cdot)$ in Equation (3) net of the investment cost $C(\cdot)$ in Equation (2).

Given that the investor incurs sunk costs in purchasing a property and that the property value is stochastic (Dixit and Pindyck, 1994, p.139), we are unable to find a non-stochastic timing of investment. Instead, the investment rule takes the form where the investor will not purchase the property unless the initial property value reaches a critical level, denoted by H^*. At that instant, the investor will choose a loan-to-value ratio, denoted by M^*. After the investment is made, the investor will default whenever the property value declines to $H_b(s)$ at instant s.

Using risk-neutral valuation and applying Ito's lemma yields the differential equation applying to $V^e(H(t),t)$ as given by:

$$\frac{\sigma^2}{2}H(t)^2\frac{\partial^2 V^e(\cdot)}{\partial H(t)^2}+(r-\lambda)H(t)\frac{\partial V^e(\cdot)}{\partial H(t)}+\frac{\partial V^e(\cdot)}{\partial t}+[-(1-\tau)rM+\frac{\delta\tau}{n}]H(0)=rV^e(\cdot)\,. \tag{6}$$

Equation (6) has an intuitive interpretation. If we treat V^e as an asset value, then the expected capital gain of the investment (the sum of the first three terms on the left-hand side) plus the dividend (the last term on the left-hand side) must be equal to the return required by the investor (the term on the right-hand side). Furthermore, two boundary conditions applied to $V^e(\cdot)$ are given as follows:

$$V(H(T),T)=H(T)-MH(0)-\tau[H(T)-H(0)+\frac{\delta T}{n}H(0)]\,, \tag{7}$$

and

$$V(H_b(s),s)=H(s)+V^e(H_b(s),s)=0\,. \tag{8}$$

Equation (7) indicates that the value of investment should be equal to after-tax equity reversion from selling the property when the mortgage loan is due. Equation (8) indicates that for any instant prior to maturity, i.e., $0<s<T$, the investor will default once the value of investment just falls to zero.

The investor will consider the option value of default decision when deciding a loan-to-value ratio, M. In order to investigate this choice, we define $G=\partial V^e/\partial M$. Differentiating

Equation (6) term by term with respect to M yields

$$\frac{\sigma^2}{2}H(t)^2\frac{\partial^2 G(\cdot)}{\partial H(t)^2}+(r-\lambda)H(t)\frac{\partial G(\cdot)}{\partial H(t)}+\frac{\partial G(\cdot)}{\partial t}-(1-\tau)rH(0)=rG(\cdot)\,. \tag{9}$$

The choice of M is derived by differentiating the net value of investment, $V(H(0),T)-C(H(0),M)$, with respect to M equal to zero. Evaluating the result at $H(0)=H^*$ and $M=M^*$ yields

$$G(H^*,0)+H^*-f\varepsilon M^{*\varepsilon-1}=0\,. \tag{10}$$

Prior to the date at which the investment is made, the investor is not forced to invest in a certain period of time. Consequently, the investor's option value of waiting does not depend on the calendar date, and thus we can denote this option value as $F_1(H(t))$, which is given by

$$\frac{\sigma^2}{2}H(t)^2\frac{\partial^2 F_1(\cdot)}{\partial H(t)^2}+(r-\lambda)H(t)\frac{\partial F_1(\cdot)}{\partial H(t)}=rF_1(\cdot)\,. \tag{11}$$

The choice of the investment timing is characterized by both the value-matching condition

$$F_1(H^*)=V^e(H^*,T)-C(H^*,M^*)\,, \tag{12}$$

and the smooth-pasting condition

$$\frac{\partial F_1(H^*)}{\partial H(0)}=\frac{\partial V^e(H^*,T)}{\partial H(0)}-\frac{\partial C(H^*,M^*)}{\partial H(0)}\,, \tag{13}$$

where we have evaluated Equation (12) and (13) at $H(0)=H^*$ and $M=M^*$.

Appendix A shows the hypothetical case where an investor applies for a mortgage loan that never matures and thus holds the property for an infinite horizon. This seemingly restrictive assumption, however, simplifies our analysis because the investor will then not default unless the property value declines to a constant value, which we denote as H_*. As a result, the default, investment and financing decisions are respectively satisfied the three equations given by:

$$H_*=\frac{[M^*(1-\tau)-(1-e^{-rn})\frac{\delta\tau}{nr}]H^*\beta_2}{(\beta_2-1)}\,, \tag{14}$$

$$X(H^*, M^*) = -(1-\frac{1}{\beta_1})(M^* + (1-e^{-rn})\frac{\delta}{nr})H^*\tau - (\frac{1}{\beta_1} - \frac{1}{\beta_2})(\frac{H^*}{H_*})^{\beta_2} H_* + fM^{*\varepsilon} = 0\,, \tag{15}$$

$$Y(H^*, M^*) = \tau H^* + (1-\tau)H^*(\frac{H^*}{H_*})^{\beta_2} - f\varepsilon M^{*\varepsilon-1} = 0\,, \tag{16}$$

where β_1 and β_2 are respectively given by

$$\beta_1 = \frac{1}{2} - \frac{r-\lambda}{\sigma^2} + \sqrt{(\frac{1}{2} - \frac{r-\lambda}{\sigma^2})^2 + \frac{2r}{\sigma^2}} > 1\,, \tag{17}$$

and

$$\beta_2 = \frac{1}{2} - \frac{r-\lambda}{\sigma^2} - \sqrt{(\frac{1}{2} - \frac{r-\lambda}{\sigma^2})^2 + \frac{2r}{\sigma^2}} < 0\,. \tag{18}$$

Equation (14) is derived based on the condition that at the default point the gross value of investment is equal to zero. Equation (15) is derived based on the condition that an investor balances the immediate benefit from purchasing a property against the benefit from waiting for a more favorable state of nature. Equation (16) is derived based on the condition that an investor trades off the benefit from the tax advantages of debt financing against the sunk cost of debt financing. We can use Equations (14), (15) and (16) simultaneously to derive the solution for the choices of the loan-to-value ratio, M^*, the critical level of the value of housing that triggers investment, H^*, and that for the critical level of the value of property that triggers default, H_*.

Our paper significantly differs from the literature on optimal capital structure in a real estate investment by allowing an investor to have the option to default. We are thus able to calculate the probability of default at any instant s after the investor purchases the property at time $t = 0$. This probability is equal to the first passage time distribution for $H(0)$ to reach $H_b(s)$ before time s, and is thus given by (see, e.g., Harrison, 1985; Leland, 2004)

$$L(H(0), H_b(s), s) = N(h_1(s)) + (\frac{H(0)}{H_b(s)})N(h_2(s))\,, \tag{19}$$

where $N(\cdot)$ denotes cumulative normal distribution functions,

$$h_1(s) = \frac{-\ln\frac{H(0)}{H_b(s)} - vs}{\sigma\sqrt{s}}, \quad h_2(s) = \frac{-\ln\frac{H(0)}{H_b(s)} + vs}{\sigma\sqrt{s}}, \tag{20}$$

and

$$v = r - \lambda - \frac{\sigma^2}{2}.$$

Substituting $H(0) = H^*$, $H_b(s) = H_*$, and let s approaches infinity yields

$$L(H^*, H_*, \infty) = 1, \text{ if } r - \lambda - \frac{\sigma^2}{2} < 0, \tag{21}$$

$$= (\frac{H^*}{H_*})^{-\frac{2(r-\lambda-\frac{\sigma^2}{2})}{\sigma^2}} < 1, \text{ if } r - \lambda - \frac{\sigma^2}{2} > 0.$$

Equation (21) indicates that the value of property will eventually touch the default point if the rate of return on housing investment is expected to be negative ($r - \lambda - \frac{\sigma^2}{2} < 0$). If, instead, the rate of return is expected to be positive, then the value of property has some probability, but not for sure to touch the default trigger over an infinite period of time.

Proposition 1 states how various exogenous forces affect the default point characterized by Equation (14) and the default characterized by Equation (21).

Proposition 1. *An investor is more likely to default* (*both* H_* *and* $L(H^*, H_*, \infty)$ *will increase*) *when* (*i*) *the investor borrows more* (M *increases*)*;* (*ii*) *the investor is allowed to depreciate less capital* (n *increases*)*;* (*iii*) *the property generates more service flow* (λ *increases*)*;* (*iv*) *the property value becomes more volatile* (σ *increases*)*;* (*v*) *the investor is taxed at a higher rate* (τ *increases*)*;* (*vi*) *the property has less capital for deprecation* (δ *decreases*)*; and* (*vii*) *the mortgage rate decreases* (r *decreases*).

Proof: Differentiating H_* and $L(H^*, H_*, \infty)$ with respect to M, n, λ, σ, τ, δ, and r

yields the results.

Proposition 1 thus offers several testable implications that can be investigated for future empirical study. We, however, find only one factor, namely, the sunk cost, yields definite impacts on choices of the investment timing, the loan-to-value ratio, and the default point and probability, as stated below.

Proposition 2. *An investor who incurs more sunk costs when applying for a mortgage loan to purchase a property will delay investment and default earlier, but will change neither the debt level nor the likelihood of default.*

Proof: See Appendix B.

The intuition behind Proposition 2 is explained by using Figure 1. In that Figure, line TT depicts the relationship between H^* and M^* addressed in Equation (15), while line DD depicts the relationship between M^* and H^* addressed in Equation (16). Let us start from an initial equilibrium point A_0, which is the intersection of lines TT and DD, such that an investor will purchase when the property value reaches H_0^*. At that instant, the investor will borrow at a loan-to-value ratio equal to M_0^*. Given the loan-to-value ratio, as the investor incurs a larger sunk cost, then the net investment value will decrease, and thus the investor will wait for a better state to purchase the property. This is shown by a shift of line TT upward to line $T'T'$. On the other hand, given the investment timing, an investor who incurs a larger sunk cost will borrow less because the marginal cost of debt financing will increase. This is shown by a shift of line DD leftward to $D'D'$. The new equilibrium point, A_1, the intersection of line $T'T'$ and $D'D'$, indicates that the investor will delay purchasing the property, but will not change the level of the loan-to-value ratio.

Given that the investor delays the purchase, the default point will also increase (because H^* is positively related to H_* as indicated in Equation (14)), but the probability of default will remain unchanged as the wedge between these two trigger points remains the same.

The comparative-statics results of all the other exogenous forces on the investment and financing decisions are all ambiguous. However, we can use Figure 1 to explain the results in the numerical examples in the next section by employing plausible parameters. We will consider both cases, that is, where the holding period is infinite and where it is finite. Appendix C shows the procedures to find the solutions for the latter case.

III. Numerical Analysis

The benchmark case we choose is as follows: The sunk cost $f = 1$, the income tax rate $\tau = 20\%$; the number of years allowed for depreciation $n = 39$ years; the proportion of depreciable capital $\delta = 0.5$; the mortgage rate $r = 7.5\%$ per year; the service flow rate is equal to 5% per year, i.e., $\lambda = 5\%$; the cost elasticity of debt financing $\varepsilon = 1.5$; the instantaneous volatility of the growth rate of the property value is equal to 12.5% per year, i.e., $\sigma = 12.5\%$ per year; and the holding period is infinite $\bar{t} = \infty$.[7] Given this set of benchmark parameter values, we find that the investor will not purchase a property until the property value reaches 5.6619 ($H^* = 5.6619$). At that instant, the investor will use 65.51% debt to finance this purchase ($M^* = 65.51\%$).

Insert Table 1 here

Table 1 shows the results for f changing in the region (0.5, 1.5), τ in the region (15%, 25%),

[7] According to Goetzmann and Ibbotson (1990), during the period of 1969 to 1989, the annual standard deviation for REITs on commercial property was equal to 15.4%. The volatility of the growth rate of the property value in our benchmark case is just a little smaller.

n in the region (37, 41), δ in the region (0.4, 0.6), r in the region (6.5%, 8.5%), λ in the region (4%, 6%), ε in the region (1.4, 1.6), σ in the region (10%, 15%), and $\bar{t}$ in the region of (10, ∞), holding all the other parameters at their benchmark values. Table 1 indicates the following results. First, an investor will wait for a better state to purchase a property (H^* increases), but will not alter the debt level nor the likelihood to declare default (M^* and $L(H^*, H_*, \infty)$ remain unchanged) if the transaction cost (f) increases. This conforms to the conclusion in Proposition 2. The net investment value also increases with the sunk cost because the investor purchases the property at a better state.

Second, an investor who expects the property value to be more volatile will invest earlier, borrow less and thus receive a lower return, but will be more likely to default. We can use Figure 1 to explain this result. Let point A_0 still characterizes the initial equilibrium. The benefit from borrowing will decrease when the property value becomes more volatile because default will then become more likely. This is captured by a shift from line DD to line $D'D'$. An investor who expects the property value to be more volatile will thus be more hesitant to invest. Consequently, line TT will shift up, but will reach a level below line $T'T'$ such that the new equilibrium will be to the south-west of A_1, indicating that the investor will invest earlier, and borrow less. Given that the investor invests at a state of nature where the property value is lower, the net investment value will thus be lower.

Third, an investor who pays more taxes (τ increases) will invest earlier, default later, and also borrow more. Figure 2 explains the reason behind the above result. Suppose that point A_0 denotes the initial equilibrium point. An investor who needs to pay more taxes will borrow more either because the marginal benefit from borrowing increases or the marginal cost of debt financing decreases. This is shown by a shift from line DD to line $D'D'$. On the other hand, the investor will also purchase the property earlier because waiting will yield less value than investing immediately. This is shown by a shift from line TT to $T'T'$. The new equilibrium point is at

A_1, which indicates that the investor will borrow more and purchase earlier. We see that a higher tax rate also leads to a lower default probability, since the wedge between the investment trigger and default trigger points shirnks even though the investor borrows more. The net investment value increases as a result of a smaller portion injected by the equity investment.

Fourth, an investor will invest earlier, default later, but borrow less if (i) the investor is allowed to depreciate less rapidly (n increases); (ii) the proportion of depreciable capital decreases (δ decreases); (iii) the investor faces a higher mortgage rate (r increases); (iv) the investors receives a lower service flow (λ decreases); and (v) the sunk cost responds less to a change in the debt level (ε decreases).

We can use Figure 3 to explain the results of the above five scenarios. Similar to Figure 2, starting from the initial equilibrium A0, line TT will shift downward to line $T'T'$, while line DD will shift rightward to line $D'D'$. The investor will still accelerate the purchase, but will borrow less, as indicated by the new equilibrium point A1. We also see that default probability will decrease since the invetor borrows less. The net investor value decreases as a result of a larger portion injected by the equity investment.

Finally, we find that as the mortgage loan lasts longer, an investor will borrow more debt, delay investment, and receive a higher return for the investment. This result is reasonable because the investor can enjoy deductions on both interest payments each year and depreciation allowance consecutively for thirty nine years. The investor will thus gradually increase the loan-to-value as the mortgage loan lasts longer, and also enjoy a higher return for investment.

Figure 4 shows the default points for the mortgage loans with maturity from 15 years to infinite years. It shows that for finite maturity loans, the default point moves upward rapidly as the maturity date approaches. Figure 5 shows the default probability over time for these different mortgage loans. It shows that a shorter maturity mortgage loan always has a higher default probability than a longer one when these two loans are signed at the same date. It also shows that for finite maturity loans, the default probability increases abruptly when the maturity date approaches.

IV. Conclusion

This article investigates the determinants of optimal capital structure in real estate investment in a framework where an investor incurs transaction costs when purchasing a property through debt financing. The interaction of these costs and the stochastic evolution of the property value confers on the investor an option value from waiting, which the investor must take into account when deciding the date at which to purchase the property. At the optimal date of purchasing, the investor also chooses a loan-to-value ratio, which balances the tax shield benefit and the transaction costs. After the investment is made, the investor will default once the value of the property falls below the balance of debt. We relate several factors that characterize the evolution of the property value and the mortgage loan to the investment timing, the debt-to-loan value ratio, the default timing and probability, as well as the net value of investment.

This article employs a simplified model, and thus can be extended in the following respect. First, we can assume that future interest rates are also stochastic over time, thus allow for prepayment option as addressed in Hilliard et al. (1998) and Sshwartz and Torous (1992). Second, we can also endogenize the discount point as addressed in Cannaday and Young (1995). Finally, we can also assume that the mortgage rate is adjustable as addressed in Kau et al. (1993).

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Appendix A: The case for $T = \infty$

When $T = \infty$, then $\partial V^e(\cdot)/\partial t = 0$, and $E_t ATER(T) e^{-r(T-t)} = 0$. For this case, suppose that $F_2(H(t))$ denotes the value of $V^e(H(t), T)$ in the region where $t \geq 0$ and $H(t) > H_*$ where H_* is the critical level of the property value that triggers default at $t = 0$. As a result, we can rewrite Equation (6) as:

$$\frac{\sigma^2}{2} H(t)^2 \frac{\partial^2 F_2(H(t))}{\partial H(t)^2} + (r - \lambda) H(t) \frac{\partial F_2(H(t))}{\partial H(t)} - (1-\tau) rMH(0) + \frac{\tau\delta}{n} H(0) = rF_2(H(t)) . \tag{A1}$$

Substituting $F_2(H(t)) = H(t)^\beta$ into the homogenous part of Equation (A1) yields the quadratic equation for solving β:

$$\phi(\beta) = -\frac{\sigma^2}{2}\beta(\beta - 1) - (r - \lambda)\beta + r = 0 . \tag{A2}$$

The solution to the homogenous part of Equation (A1) is thus given by

$$F_2(H(t)) = B_1 H(t)^{\beta_1} + B_2 H(t)^{\beta_2} , \tag{A3}$$

where β_1 and β_2 are, respectively, the larger and smaller roots of β in Equation (A2), and B_1 and B_2 are constants to be determined. The solution to the non-homogeneous part of Equation (A1) is given by

$$F_2(H(t)) = -(1-\tau) rMH(0) + \frac{\tau\delta}{nr} H(0)(1 - e^{-rn}) . \tag{A4}$$

The solution for $F_2(H(t))$ in Equation (A2) is thus given by:

$$F_2(H(t)) = B_1 H(t)^{\beta_1} + B_2 H(t)^{\beta_2} - (1-\tau) rMH(0) + \frac{\tau\delta}{nr} H(0)(1 - e^{-rn}) . \tag{A5}$$

The terms B_1, B_2, and H_*, are solved simultaneously from the boundary conditions as follows:

$$\lim_{H(t)\to\infty} B_1 H(t)^{\beta_1} + B_2 H(t)^{\beta_2} = 0 , \tag{A6}$$

$$V(H_*) = H_* + F_2(H_*) = 0 , \tag{A7}$$

and

$$\frac{\partial V(H_*)}{\partial H(0)} = 1 + \frac{\partial F_2(H_*)}{\partial H(0)} = 0 . \tag{A8}$$

Equation (A6) is the limit condition, which states that the investor's option value from defaulting the mortgage loan is worthless as the housing price approaches infinity. This condition requires that $B_1 = 0$. Equation (A7) is the value-matching condition, which states that, at the optimal timing of default, the investor has a value equal to zero, as suggested by Leland (1994). Equation (A8) is the smooth-pasting condition, which guarantees that the investor will not derive any arbitrage profits by deviating the optimal default strategy. Solving equations (A6)-(A8) simultaneously yields

$$H_* = \frac{[M(1-\tau) - (1-e^{-rn})\frac{\tau\delta}{nr}]H(0)\beta_2}{(\beta_2 - 1)}, \tag{A9}$$

and

$$B_2 = -\frac{1}{\beta_2} H_*^{1-\beta_2}. \tag{A10}$$

Denote $F_1(H(t))$ as the option value of waiting at the region where investment has not made and $H(t) < H^*$. Then we can rewrite Equation (11) as:

$$\frac{\sigma^2}{2} H(t)^2 \frac{\partial^2 F_1(H(t))}{\partial H(t)^2} + (r-\lambda)H(t)\frac{\partial F_1(H(t))}{\partial H(t)} = rF_1(H(t)). \tag{A11}$$

The solution for $F_1(H(t))$ is given by

$$F_1(H(t)) = A_1 H(t)^{\beta_1} + A_2 H(t)^{\beta_2}. \tag{A12}$$

The terms A_1, A_2, and H^* are solved from the boundary conditions as follows:

$$\lim_{H(t)\to 0} F_1(H(t)) = 0, \tag{A13}$$

$$F_1(H^*) = H^* + F_2(H^*) - C(H^*, M^*), \tag{A14}$$

and

$$\frac{\partial F_1(H^*)}{\partial H(0)} = 1 + \frac{\partial F_2(H^*)}{\partial H(0)} - \frac{\partial C(H^*, M^*)}{\partial H(0)}. \tag{A15}$$

Equation (A13) is the limit condition, which indicates that the option from waiting is worthless if the housing price approaches its minimum permissible of zero. This requires that $A_2 = 0$. Equation

(A14) is the value-matching condition, which states that at the optimal timing of investment, the investor should be indifferent between investing and not investing. Equation (A15) is the smooth-pasting condition, which prevents the investor from deriving any arbitrage profits by deviating the optimal investment strategy.

Solving Equations (A13)-(A15) simultaneously yields

$$X(H^*,M^*) = -(1-\frac{1}{\beta_1})[M^* + \frac{\delta}{nr}(1-e^{-rn})]H^*\tau - (\frac{1}{\beta_1}-\frac{1}{\beta_2})(\frac{H^*}{H_*})^{\beta_2} H_* + fM^{*\varepsilon} = 0 . \quad \text{(A16)}$$

For H^* to be an interior solution, it is required that the second-order condition holds. This requires that the derivative of the left-hand side of Equation (A16) with respect to H^* is negative, and thus

$$\Delta_{11} = \partial X(H^*,M^*)/\partial H^* < 0 \quad \text{(A17)}$$

Furthermore, the choice of M is found by setting the derivative of $F_1(H^*)$ in Equation (A14), or equivalently, $H^* + F_2(H^*) - C(H^*,M^*)$, with respect M equal to zero. This yields

$$Y(H^*,M^*) = \tau H^* + (1-\tau)H^*(\frac{H^*}{H_*})^{\beta_2} - f\varepsilon M^{*\varepsilon-1} = 0 . \quad \text{(A18)}$$

It is required that the following second order condition holds:

$$\Delta_{22} = \partial Y(H^*,M^*)/\partial M^* < 0 . \quad \text{(A19)}$$

Appendix B: Proof of Proposition 2

Totally differentiating $X(H^*,M^*)=0$ in Equation (15) and $Y(H^*,M^*)=0$ in Equation (16) with respect to f yields

$$\Delta_{11}\frac{\partial H^*}{\partial f} + \Delta_{12}\frac{\partial M^*}{\partial f} + \Delta_{13} = 0 , \quad \text{(B1)}$$

$$\Delta_{21}\frac{\partial H^*}{\partial f} + \Delta_{22}\frac{\partial M^*}{\partial f} + \Delta_{23} = 0 , \quad \text{(B2)}$$

where $\Delta_{11} = \partial X / \partial H^*$, $\Delta_{12} = \partial X / \partial M^*$, $\Delta_{13} = \partial X / \partial f$, $\Delta_{21} = \partial Y / \partial H^*$, $\Delta_{22} = \partial Y / \partial M^*$, $\Delta_{23} = \partial Y / \partial f$.

As a result,

$$\frac{\partial H^*}{\partial f} = (-\Delta_{13}\Delta_{22} + \Delta_{12}\Delta_{23}) / \Delta > 0 , \tag{B3}$$

$$\frac{\partial M^*}{\partial f} = (-\Delta_{11}\Delta_{23} + \Delta_{21}\Delta_{13}) / \Delta = 0 . \tag{B4}$$

where $\Delta = \Delta_{11}\Delta_{22} - \Delta_{21}\Delta_{21} > 0$, which also indicates that the slope of DD is steeper than that of TT .

Appendix C: The case for finite $\bar{t}$

We can follow Brennan and Schwartz (1978) and Hull and White (1990) to find H^* and M^*. Let $y = \ln H$ such that $y^* = \ln H^*$, $U(y) = F_1(H)$, and $Z^e(y,t) = V^e(H,t)$. As a result, Equation (11) can be rewritten as:

$$\frac{\sigma^2}{2}\frac{dU^2(y)}{dy^2} + (r - \lambda - \frac{\sigma^2}{2})\frac{dU(y)}{dy} - rU(y) = 0 \text{, if } y(0) < y^* . \tag{C1}$$

Similarly, Equation (6) can be rewritten as:

$$\frac{\sigma^2}{2}\frac{\partial^2 Z^e(y,t)}{\partial y^2} + (r - \lambda - \frac{\sigma^2}{2})\frac{\partial Z^e(y,t)}{\partial y} + \frac{\partial Z^e(y,t)}{\partial t} + [-(1-\tau)rM + \frac{\delta\tau}{n}]e^{y^*} = rZ^e(y,t) , \tag{C2}$$

if $t \geq 0$, and $y(0) \geq y^*$.

Furthermore, Equation (7) can be rewritten as:

$$Z^e(y(T),T) = e^{y(T)}(1-\tau) - e^{y^*}(M + \frac{\tau\delta T}{n} - \tau) . \tag{C3}$$

Finally, Equation (2) can be rewritten as:

$$C(e^{y^*}, M) = (1-M)e^{y^*} + fM^{\varepsilon} . \tag{C4}$$

Let

$$G(H,t) = \partial V^e(H,t) / \partial M \ . \qquad \text{(C5)}$$

Differentiating Equation (6) term by term with respect to M yields

$$\frac{\sigma^2}{2} H^2 \frac{\partial^2 G(H,t)}{\partial H^2} + (r-\lambda) H \frac{\partial G(H,t)}{\partial H} + \frac{\partial G(H,t)}{\partial t} - (1-\tau) r H^* = rG(H,t) \ . \qquad \text{(C6)}$$

The choice of M is then derived by setting the derivative of $W = H + V^e(\cdot) - C(\cdot)$ with respect to M equal to zero. That is:

$$\frac{\partial W}{\partial M} = H^* + G(H^*, 0) - f \varepsilon M^{*\varepsilon - 1} = 0 \ , \qquad \text{(C7)}$$

where M^* is the optimal value of M. Let $g(y,t) = G(H,t)$. As a result, Equation (C7) can be written as

$$g(y^*, 0) + e^{y^*} - f \varepsilon M^{*\varepsilon - 1} = 0 \ , \qquad \text{(C8)}$$

Furthermore, Equation (C6) can be transformed into:

$$\frac{\sigma^2}{2} \frac{\partial^2 g(y,t)}{\partial y^2} + (r - \lambda - \frac{\sigma^2}{2}) \frac{\partial g(y,t)}{\partial y} + \frac{\partial g(y,t)}{\partial t} - rg(y,t) - (1-\tau) r e^{y^*} = 0 \ . \qquad \text{(C9)}$$

We can implement the explicit finite difference method (Hull and White, 1990) to solve for M^* and H^*. We begin by choosing a small time interval, Δt, and a small change in y, Δy. A grid is then constructed for considering values of g when y is equal to

$$y_0 \ , \ \ y_0 + \Delta y \ , \ \ \ldots , \ \ y_{\max} \ ,$$

and time is equal to

$$0 \ , \ \ \Delta t \ , \ \ \ldots , \ \ T \ .$$

The parameters y_0 and $y_{\max}$ are the smallest and the largest values of y, and $t = 0$ is the current time. We will denote $y_0 + i\Delta y$ by y_i ($i = 1, \cdots, n$), $j\Delta t$ by t_j ($j = 1, \cdots, m$), and the value of the derivative security at the (i, j) point on the grid by $g_{i,j}$. The partial derivatives of

g with respect to y at node (i, j) are approximately as follows:

$$\frac{\partial g}{\partial y} = \frac{g_{i+1,j} - g_{i-1,j}}{2\Delta y}, \tag{C10}$$

$$\frac{\partial^2 g}{\partial y^2} = \frac{g_{i+1,j} + g_{i-1,j} - 2g_{i,j}}{(\Delta y)^2}, \tag{C11}$$

and the time derivative is approximately

$$\frac{\partial g}{\partial t} = \frac{g_{i,j} - g_{i,j-1}}{\Delta t}. \tag{C12}$$

Equation (C8) indicates that $e^{y^*} = [f\varepsilon M^{*\varepsilon-1} - g(y^*, 0)]$. Substituting this and Equations (C10) – (C12) into Equation (C9) yields:

$$g_{i,j-1} = ag_{i-1,j} + bg_{i,j} + cg_{i+1,j} - \frac{\Delta t}{(1+r\Delta t)} r(1-\tau)[f\varepsilon M^{*\varepsilon-1} - g_{i^*,0}], \tag{C13}$$

where

$$a = \frac{1}{(1+r\Delta t)}[\frac{\sigma^2 \Delta t}{2(\Delta y)^2} - (r - \lambda - \frac{\sigma^2}{2})\frac{\Delta t}{2\Delta y}], \tag{C14}$$

$$b = \frac{1}{(1+r\Delta t)}[1 - \frac{\sigma^2 \Delta t}{(\Delta y)^2}], \tag{C15}$$

$$c = \frac{1}{(1+r\Delta t)}[\frac{\sigma^2 \Delta t}{2(\Delta y)^2} + (r - \lambda - \frac{\sigma^2}{2})\frac{\Delta t}{2\Delta y}], \tag{C16}$$

and i^* is the value of i that makes $g_{i,0}$ reach the lowest level. The choice of M, M^* is the one that makes the left-hand side of Equation (C13) equal to $g_{i^*,0}$.

Similarly, Equation (C2) can be rewritten as:

$$Z^e{}_{i,j-1} = aZ^e{}_{i-1,j} + bZ^e{}_{i,j} + cZ^e{}_{i+1,j} + \frac{\Delta t}{(1+r\Delta t)}[rM(-1+\tau) + \frac{\delta\tau}{n}]e^{y^*}, \tag{C17}$$

where $Z^e{}_{i^*,0}$ is found at $i = i^*$ such that the left-hand side of Equation (C17), $Z^e{}_{i,j-1}$, is equal to $Z^e{}_{i^*,0}$. For any i, the limited liability of equity requires that

$$e^{y_i} + Z^e_{i,j-1} \geq 0 \quad \text{(C18)}$$

Furthermore, at the maturity date T, Equation (7) suggests that $Z^e_{i,m}$ must satisfy

$$e^{y_i} + Z^e_{i,m} = e^{y_i}(1-\tau) - e^{y_{i^*}}[M^* + \frac{\tau\delta T}{n} - \tau]. \quad \text{(C19)}$$

We also need to impose the optimal condition for the investment timing. The solution to $U(y)$ in Equation (C1) is given by

$$U(y) = A_1 e^{\beta_1 y} + A_2 e^{\beta_2 y}, \quad \text{(C20)}$$

where A_1 and A_2 are constants to be determined, and β_1 and β_2 are defined in Appendix A. The optimal timing is determined by the following boundary conditions:

$$\lim_{y \to 0} U(y) = 0, \quad \text{(C21)}$$

$$U(y^*) = e^{y^*} + Z^e(y^*, 0) - (1-M)e^{y^*} - fM^{*\varepsilon}, \quad \text{(C22)}$$

$$\frac{dU(y^*)}{dy} = Me^{y^*} + \frac{\partial Z(y^*, 0)}{\partial y}. \quad \text{(C23)}$$

Solving Equations (C21)-(C23) simultaneously yields

$$A_2 = 0, \quad \text{(C24)}$$

$$A_1 = (Me^{y^*} + Z^e(y^*, 0) - fM^{*\varepsilon}) / e^{\beta_1 y^*}, \quad \text{(C25)}$$

and

$$A_1 \beta_1 e^{\beta_1 y^*} = Me^{y^*} + \frac{\partial Z^e(y^*, 0)}{\partial y}. \quad \text{(C26)}$$

We need to calculate $\partial Z^e(y^*, 0) / \partial y$ in Equation (C26). Let $R(y,t) = \partial Z^e(y^*, t) / \partial y$.

Differentiating Equation (C2) with respect to y yields [8]

$$\frac{1}{2}\sigma^2 \frac{\partial^2 R(y^*, 0)}{\partial y^2} + (r - \lambda - \frac{\sigma^2}{2}) \frac{\partial R(y^*, 0)}{\partial y} + \frac{\partial R(y^*, 0)}{\partial y} + [rM(-1+\tau) + \frac{\delta\tau}{n}] = rR(y^*, 0). \quad \text{(C27)}$$

Differentiating Equation (C3) with respect to y yields

---

[8] We need to impose $y^* = y$ here because we consider the point at which the investment is just made.

$$R(y(T),T) = -e^{y^*}(M + \frac{\tau\delta T}{n} - \tau). \quad \text{(C28)}$$

As we can transform Equation (C2) into Equation (C17), we can also transform Equation (C27) into the following Equation

$$R_{i,j-1} = aR_{i-1,j} + bR_{i,j} + cR_{i+1,j} + \frac{\Delta t}{(1+r\Delta t)}[rM(-1+\tau) + \frac{\delta\tau}{n}]e^{y^*}. \quad \text{(C29)}$$

The law of motion for $Z^e(y,t)$ shown in Equation (C2) and that for $g^e(y,t)$ shown in Equation (C9) are subject to two optimal conditions shown in Equations (C8) and (C26), and three boundary conditions shown in Equations (C18), (C19), and (C28). Solving these conditions simultaneously yields the solutions for M^*, $g_{i^*,0}$, and $Z_{i^*,0}$, where $Z_{i^*,0}$ is the value of the property in equilibrium. We can further use the relation $H^* = e^{y_{i^*}}$ to find the critical level of the value of property that triggers investment.

Equation (19) in the main text is only applied to the situation where $H_b(s)$ is constant for all s. Given that $H_b(s)$ varies with s when the term of the loan is finite, we must modify this formula. In the discrete approximation, $H_b(s)$ is a sequence of $H_b(t_0)$, $H_b(t_1)$, ... Suppose that the cumulative default probability is denoted as $Def(s)$, then

$$Def(t_0) = L(H_b(t_0))) = 0, \text{ and } Def(t_1) = L(H_b(t_1))). \quad \text{(C30)}$$

The cumulative default probability is given by the first passage time distribution for $H(t_0)$ to reach $H_b(t_2)$ at time $s = t_2$, net of the probability that $H(t_0)$ reaches between $H_b(t_1)$ and $H_b(t_2)$ at time $s = t_1$. That is,

$$Def(t_2) = L(H(0), H_b(t_2), t_2) - (L(H(0), H_b(t_2), t_1) - L(H(0), H_b(t_1), t_1)). \quad \text{(C31)}$$

Table 1

Optimal Operating and Financial Policies for Different Economic and Financial Variables

This table reports the levels of the net investment values ($V^e-(1-M^*)H^*-fM^{*\varepsilon}$), as well as the default probability ($L(H^*,H_*,\infty)$), when the investment trigger (H^*) and the loan-to-value ratio (M^*), and the default trigger (H_*) are all chosen to maximize the net value of investment. Panel A reports the results for the base case. Panel B reports the results when the sunk cost of investment (f) is equal to 0.5 and 1.5. Panel C reports the results when the corporate tax rate (τ) is 15% and 25%. Panel D reports the results when years allowed for depreciation are equal to 37 and 41. Panel E reports the results when the depreciable capital (δ) accounts for 40% and 60%. Panel F reports the results when the mortgage rate of interest (r) is 6.5% and 8.5%. Panel G reports the results when the service flow rate (λ) is equal to 4% and 6%. Panel H reports results when the cost elasticity of debt financing (ε) is equal to 1.4 and 1.6. Panel I reports the results when the volatility of the expected growth of the housing price (σ) is 10% and 15%.. All Panels are reported by holding all other parameters at their benchmark values.

	Loan-to-Value Ratio, M^* (%)	Investment trigger, H^*	Default trigger, H_*	Net investment value, $V^e-(1-M^*)H^*-fM^{*\varepsilon}$	Default Probability, $L(H^*,H_*,\infty)$ (%)
A. Benchmark Case					
$f=1$, $\tau=20\%$, $\delta=50\%$, $r=7.5\%$ $\lambda=5\%$, $\varepsilon=1.5$, $\sigma=12.5\%$	65.51	5.6619	2.2674	0.4041	13.35
B. Sunk Cost of Investment:					
$f=0.5$	65.51	2.8310	1.1337	0.2021	13.35
$f=1.5$	65.51	8.4929	3.4011	0.6062	13.35

Table 1

Optimal Operating and Financial Policies for Different Economic and Financial Variables

	Loan-to-Value Ratio, M^* (%)	Investment trigger, H^*	Default trigger, H_*	Net investment value, $V^e - (1 - M^*)H^* - fM^{*\varepsilon}$	Default Probability, $L(H^*, H_*, \infty)$ (%)
C. Corporate Tax Rate:					
τ =15%	62.82	7.0790	2.9388	0.3551	14.46
τ =25%	67.43	4.7263	1.7909	0.4399	11.83
D. Years Allowed for Depreciation:					
n =37	67.64	5.6969	2.3536	0.4180	14.30
n =41	63.44	5.6202	2.1813	0.3900	12.46
E. Depreciable Capital:					
δ =40%	54.71	5.3696	1.8008	0.3253	9.04
δ =60%	74.37	5.7554	2.6067	0.4565	17.51
F. Contract Rate of Interest:					
r =6.5%	83.96	5.0553	2.4786	0.3297	51.91
r =8.5%	42.59	4.8810	1.2859	0.2799	0.96

Table 1

Optimal Operating and Financial Policies for Different Economic and Financial Variables

	Loan-to-Value Ratio, M^* (%)	Investment trigger, H^*	Default trigger, H_*	Net investment value, $V^e-(1-M^*)H^*-fM^{*\varepsilon}$	Default Probability, $L(H^*,H_*,\infty)$ (%)
G. Service Flow Rate:					
λ =4%	33.27	4.3225	0.8499	0.2356	0.35
λ =6%	88.70	5.0159	2.6573	0.2922	55.74
H. Cost Elasticity of Debt Financing:					
ε =1.4	50.07	5.2024	1.5598	0.3114	7.07
ε =1.6	84.42	5.8563	3.0667	0.4565	24.09
I. Volatility of the Expected Growth of the Housing Price:					
σ =10%	85.09	6.2866	3.5221	0.5023	9.85
σ =15%	50.53	5.0005	1.4237	0.3152	21.54
J. Maturity:					
T =10	48.00	5.2099		0.2899	
T =15	50.18	5.2597		0.2928	
T =20	50.96	5.2945		0.2992	
T =25	52.91	5.3622		0.3146	
T =30	55.09	5.3838		0.3305	

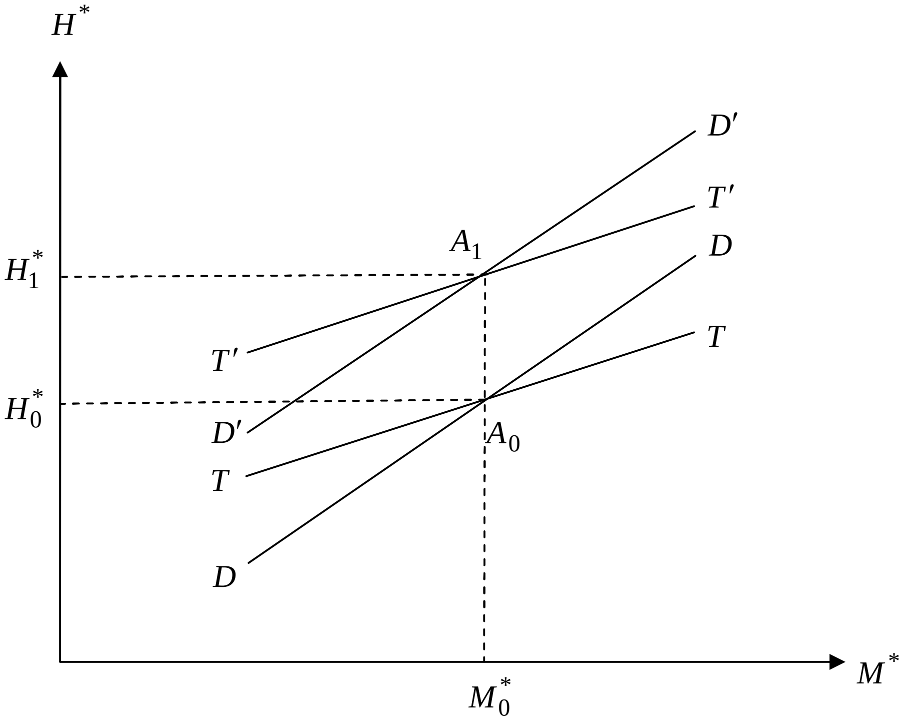

Figure 1: An increase in the sunk cost of investment will delay purchase, but will not affect the loan-to-value ratio.

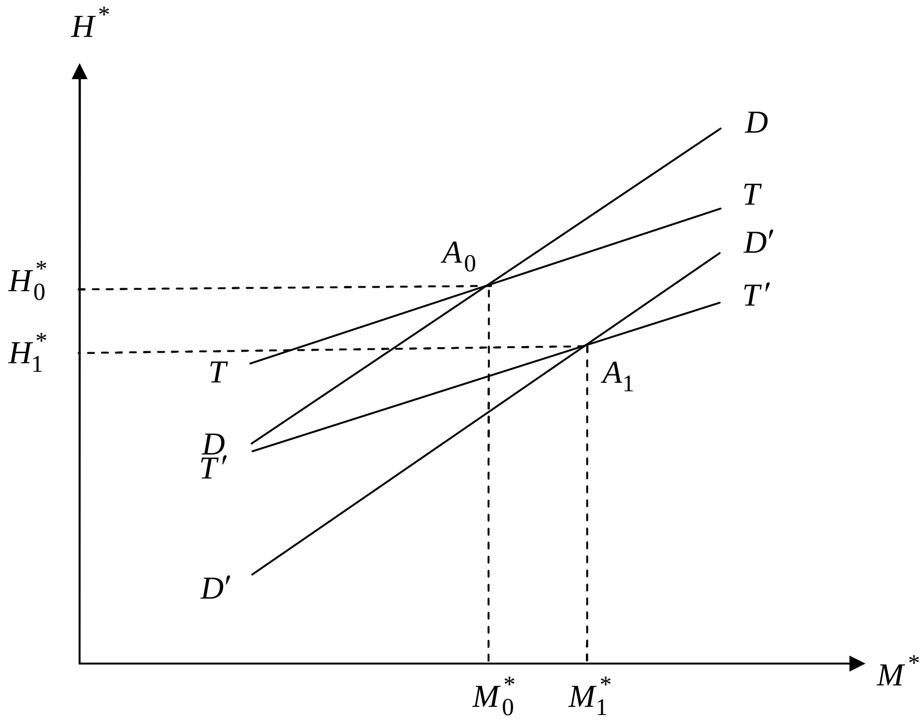

Figure 2: An increase in the tax rate will accelerate purchasing and increase the loan-to-value ratio.

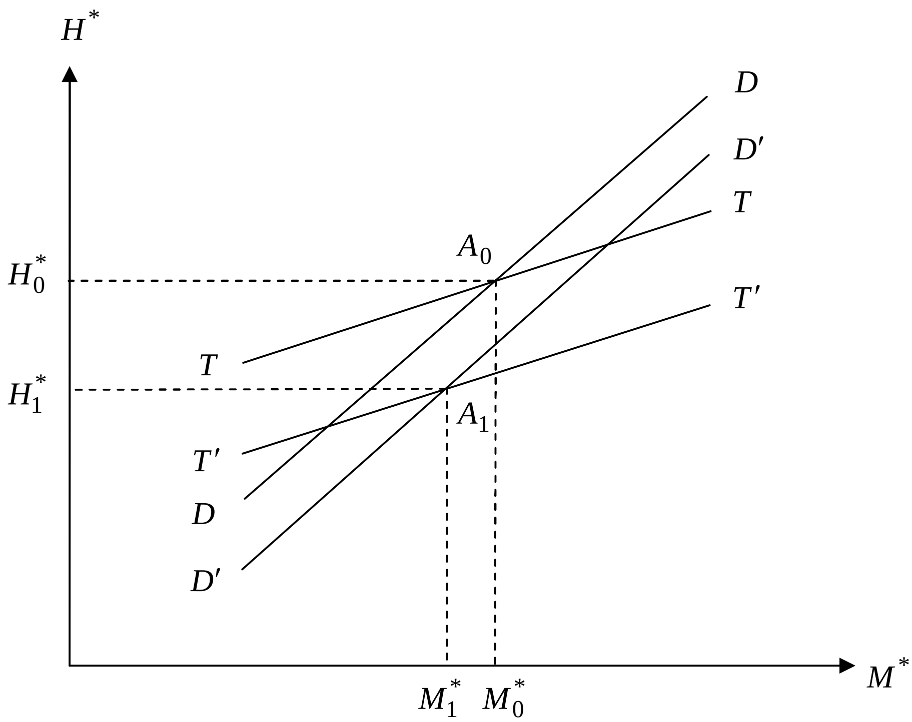

Figure 3: A decrease in either the portion of depreciable capital, the service flow rate, or the elasticity of the sunk cost with respect to debt financing, or an increase in either years allowed for depreciation or the mortgage rate will accelerate purchasing and reduce the loan-to-value ratio.

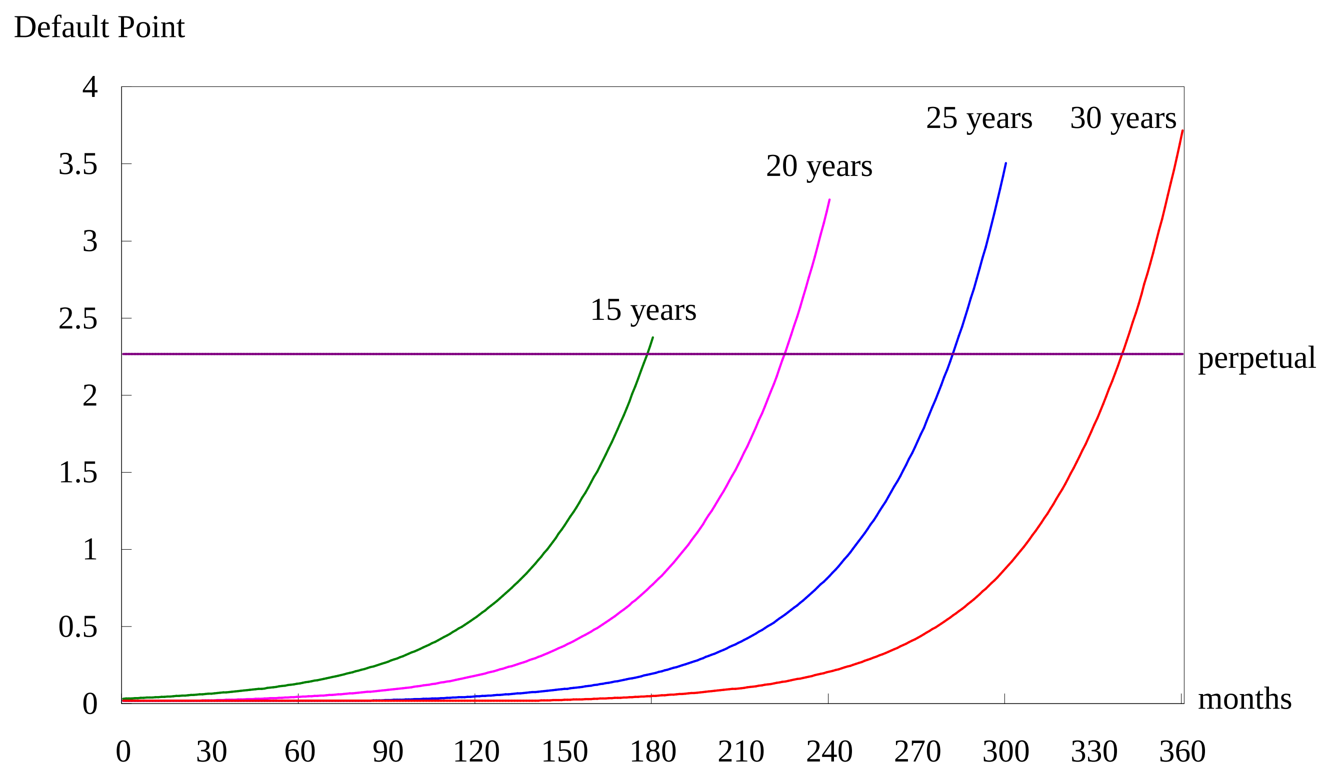

Figure 4: Default Point for different terms of loans over the period of each loan.

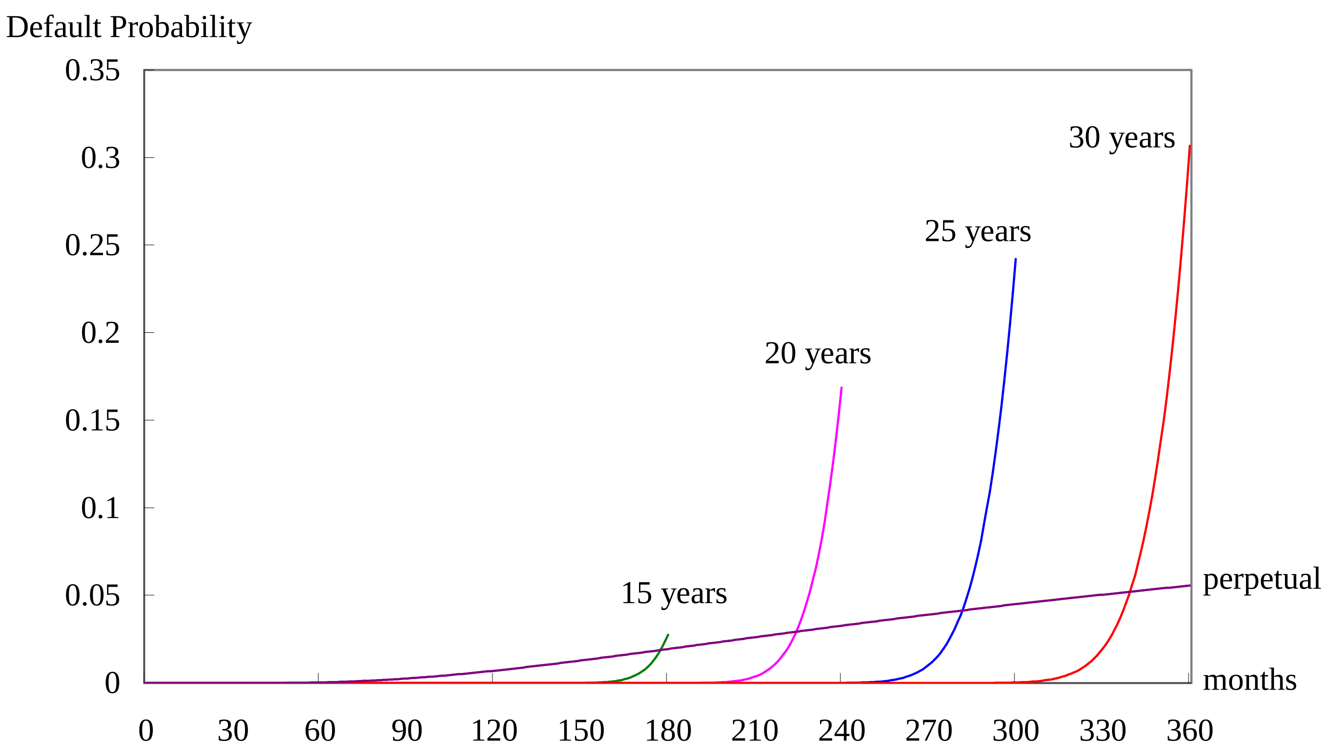

Figure 5: Default Probability for different terms of loans over the period of each loan.