BETA, LEVERAGE AND DEBT POLICY

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Abstract

This paper develops a beta gearing model for situations in which firms may adjust their debt level with lag in accordance with a particular leverage ratio, and therefore embraces both the Hamada and Miles-Ezzell models as special cases. Implementation of this model shows that the Miles-Ezzell rather than the Hamada model is a much better approximation for debt resetting frequencies less than ten years, and also for most of the interval between successive debt resets even when the debt resetting frequency exceeds ten years. This suggests that the Miles-Ezzell model is significantly superior to the Hamada model.

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1. Introduction

The relationship between the beta of a firm's equity and its leverage has been extensively examined in the literature, starting with the work of Hamada (1972). Hamada assumes that debt is fixed in dollar terms, that the firm's debt is free of systematic risk, that a classical tax regime prevails[1], and that the firm's financing comprises only ordinary shares and straight debt. Subsequently Conine (1980) has extended the model to take account of systematic risk on a firm's bonds, Miles and Ezzell (1985) have extended the model to deal with situations in which the firm's leverage ratio rather than the level of debt is fixed, Taggart (1991) has extended the model to deal with non-classical tax regimes, and Ehrhardt and Shrieves (1995) have extended the model to deal with the presence of warrants and convertible debt. Despite these later developments, Hamada's model appears to be favoured (see Copeland et al, 2005, p. 576; Ross et al, 2002, section 17.7; Damodaran, 1997, Ch. 11; Bodie et al, 2009, p. 614). The preference for Hamada's model over that of Conine, and that of Ehrhardt and Shrieves, could be attributed to the difficulties in estimating additional parameters within these models coupled with a belief that the results from using these more complex models would not be materially different from that of the Hamada model. However, since the only additional parameter in Miles and Ezzell's model (the risk free rate) is readily observable, the preference for Hamada's model over that of Miles and Ezzell cannot be justified in this way; the justification must instead be that firms do not adjust their debt levels in line with changes in equity values or that they do so with sufficiently significant delays that the Hamada model is a better approximation than that of the Miles and Ezzell model. The first of these possibilities is clearly not true; equity values have grown significantly over time (Bernstein and Arnott, 2003, Figure 4) and debt levels have broadly kept pace with this (Fama and French, 1999, Figure 1). Accordingly, the use of the Hamada rather than the Miles and Ezzell model can only be justified if firms adjust their debt levels with sufficiently significant delays that the Hamada model is a better approximation to that from the Miles and Ezzell model.

[1] A classical tax regime is one in which interest generates a corporate tax deduction whilst debt and equity returns are equally taxed at the personal level. So, the overall tax benefit from debt is the corporate tax savings with no mitigation arising from higher personal taxation on interest than on equity returns.

This paper seeks to examine this issue. To do so we construct a model of the relationship between a firm's equity beta and its leverage level in the situation in which a firm adjusts its debt level to maintain a target leverage ratio, but may do so with some lag. All of the other assumptions of the Hamada model are retained, i.e., the firm's debt is free of systematic risk, a classical tax world prevails, and the firm's financing comprises only ordinary shares and straight debt. So, the resulting model embraces the models of Hamada and Miles and Ezzell as special cases. This allows us to examine the question of which of the Hamada model and the Miles and Ezzell model is a better approximation.

2. Analysis

Suppose a firm resets its debt level every *N* years, to achieve a leverage ratio at each such point of *L*, and the debt level was last reset at time 0, i.e., $B_0 = LV_0$ where V_0 is the levered value at time 0. Letting V_t denote the value at time *t* ($t = 0, 1, 2 \ldots N$) of the firm at time *t*, *k* denote the appropriate discount rate on V_N, X_j^u denote the unlevered cash flow in year *j*, k_u denote the appropriate discount rate on the unlevered cash flows, R_f denote the risk free rate and T_c denote the corporate tax rate, then this value V_t can be expressed as the present value of the unlevered cash flows until the next debt reset point plus the present value of the interest tax shields over the same period plus the present value of the firm's value at the next debt reset point, as follows:

$$V_t = \sum_{j=t+1}^{N} \frac{E(X_j^u)}{(1+k_u)^{j-t}} + \sum_{j=t+1}^{N} \frac{B_0 R_f T_c}{(1+R_f)^{j-t}} + \frac{E(V_N)}{(1+k)^{N-t}} \tag{1}$$

The levered value at time *N* (V_N) can be expressed as the sum of the unlevered value at that point (V_N^U) plus the present value at that point of the subsequent debt tax shields. These debt tax shields are certain within each cycle of *N* years, and therefore warrant discounting at the risk free rate back to the beginning of the cycle followed by discounting at the rate *k* for all earlier years, as follows:

$$V_N = V_N^U + V_N \sum_{t=1}^{N} \frac{LR_f T_c}{(1+R_f)^t} + \frac{E(V_{2N})}{(1+k)^N} \sum_{t=1}^{N} \frac{LR_f T_c}{(1+R_f)^t} + \ldots \tag{2}$$

Letting g denote the annualised expected growth rate in V over a cycle of N years, the last equation can be written as follows.

$$V_N = V_N^U + V_N \sum_{t=1}^{N} \frac{LR_f T_c}{(1+R_f)^t} + \frac{V_N (1+g)^N}{(1+k)^N} \sum_{t=1}^{N} \frac{LR_f T_c}{(1+R_f)^t} + ...$$

$$= V_N^U + V_N \sum_{t=1}^{N} \frac{LR_f T_c}{(1+R_f)^t} \left[\frac{1}{1 - \frac{(1+g)^N}{(1+k)^N}} \right]$$

Solving the last equation for V_N yields the following.

$$V_N = V_N^U \left[1 - \frac{1 - \frac{(1+g)^N}{(1+k)^N}}{\sum_{t=1}^{N} \frac{LR_f T_c}{(1+R_f)^t}} \right] = V_N^U \theta \qquad (3)$$

where θ = [ ]. So V_N is proportional to V_N^U and therefore the appropriate rate for discounting V_N back to time t must be the unlevered discount rate k_u.[2] Substitution of equation (3) into equation (1) yields the following.

$$V_t = \sum_{j=t+1}^{N} \frac{E(X_j^u)}{(1+k_u)^{j-t}} + \sum_{j=t+1}^{N} \frac{B_0 R_f T_c}{(1+R_f)^{j-t}} + \frac{E(V_N^U \theta)}{(1+k_u)^{N-t}} \qquad (4)$$

The firm's debt level does not change over the period from t = 0 until t = N. Accordingly, $B_0 = B_t$ and substitution of this into equation (4) yields the following.

$$V_t = \sum_{j=t+1}^{N} \frac{E(X_j^u)}{(1+k_u)^{j-t}} + \sum_{j=t+1}^{N} \frac{B_t R_f T_c}{(1+R_f)^{j-t}} + \frac{E(V_N^U \theta)}{(1+k_u)^{N-t}}$$

[2] Naturally, the discount rate applied to equity is larger than k_u in recognition of the effect of leverage in raising the risk of equity.

$$= \sum_{j=t+1}^{N} \frac{E(X_j^u)}{(1+k_u)^{j-t}} + B_t T_c \left[\sum_{j=1}^{N-t} \frac{R_f}{(1+R_f)^j} \right] + \frac{E(V_N^U \theta)}{(1+k_u)^{N-t}}$$

$$= \sum_{j=t+1}^{N} \frac{E(X_j^u)}{(1+k_u)^{j-t}} + B_t T_c \delta_t + \frac{E(V_N^U \theta)}{(1+k_u)^{N-t}}$$

where δ_t = [ ]. So the value of the firm can be decomposed into three components. Accordingly, the beta of the entire firm is a value-weighted average of the betas associated with these three components, with the first and last warranting the unlevered beta β_U and the second component warranting a beta of zero. Consequently[3]

$$\beta_{Vt} = \beta_U \left[\frac{V_t - B_t T_c \delta_t}{V_t} \right]$$

It is also true, by definition, that firm value at any time is the sum of the equity value (*S*) and the debt value (*B*) at that time. Accordingly, the beta of the entire firm is a value-weighted average of the betas associated with these two components, and the second component (debt) is assumed to be free of systematic risk, i.e.,

$$\beta_{Vt} = \beta_{et} \left[\frac{S_t}{V_t} \right]$$

The last two equations imply that

$$\beta_{et} \left[\frac{S_t}{V_t} \right] = \beta_U \left[\frac{V_t - B_t T_c \delta_t}{V_t} \right]$$

and hence

$$\beta_{et} = \beta_U \left[\frac{V_t - B_t T_c \delta_t}{S_t} \right]$$

[3] The unlevered beta does not change over the cycle between successive resets of the debt level, and therefore is not subscripted with *t*. However the ratio [ ] does change over the cycle and therefore the beta for the entire firm must change over the cycle; accordingly, it is subscripted with *t*.

$$= \beta_U \left[\frac{S_t + B_t - B_t T_c \delta_t}{S_t} \right]$$

$$= \beta_U \left[1 + \frac{B_t}{S_t}(1 - T_c \delta_t) \right] \tag{5}$$

where

$$\delta_t = \frac{R_f}{1+R_f} + ... + \frac{R_f}{(1+R_f)^{N-t}} \tag{6}$$

The analysis here embraces both the Hamada case, and the Miles and Ezzell case, because they arise as special cases with *N*-*t* = ∞ and *N*-*t* = 1 respectively, i.e., substitution of these extreme values for *N* into equation (6) yields

$$\delta_t = \frac{R_f}{1+R_f} + \frac{R_f}{(1+R_f)^2} + ... = 1$$

and

$$\delta_t = \frac{R_f}{1+R_f}$$

respectively, and substitution of these values for δ_t into equation (5) yields the Hamada model, and the Miles and Ezzell model, respectively.

Examination of equations (5) and (6) reveals a remarkable feature of the model: that the relationship between the levered and unlevered betas depends upon *N*-*t* rather than *N*, i.e., it depends upon the time remaining until the next debt reset rather than the frequency of resetting. Thus, if one firm resets it debt level every ten years and another does so annually, the beta formulas for the two firms will be identical when the first firm has one year to go before its next debt reset.

3. Numerical Results

We now consider some numerical results from the model presented in equations (5) and (6). We start by fixing *t* = 0 and varying *N*, i.e., we determine how the beta gearing formula for the first year within the debt resetting cycle varies with the cycle length. The only effect of varying this cycle length is to change the value for δ_0, in accordance with equation (6). For a range of values for R_f, the results are shown in Table 1. They reveal that, even for $R_f = .07$, δ_0 is less than 0.5 for all values of N less than ten years, i.e., the Miles and Ezzell model is a better approximation for all values of N less than 10 years. Furthermore, at the more likely value of $R_f = .05$, this midpoint value rises to 14 years and rises further to 23 years if $R_f = .03$. Thus, the use of the Hamada formula rather than the Miles and Ezzell formula implies a belief that firms reset their debt levels less frequently than every 10-20 yearly. These figures seem implausible, and therefore the use of the Hamada formula even as an approximation to a world in which debt levels are periodically reset seems clearly unjustified.

We now turn to fixing N and varying t from $t = 0$ to $t = N$-1, i.e., determining how the beta gearing formula varies as we shift forwards in time through the debt resetting cycle. The mathematics of this is identical to reducing N with t set to 0, as discussed in the previous paragraph, because equations (5) and (6) depend upon N-t. For example, suppose that $N = 5$ and $R_f = .05$. In this case, the value for δ_t declines from 0.22 to 0.05 over the course of the cycle, as shown in the first three rows of the penultimate column in Table 1. Thus, the Miles and Ezzell model might be considered to be an acceptable approximation across the entire cycle. However, if $N = 20$ and $R_f = .05$, the value for δ_t declines from 0.62 to 0.05 over the course of the cycle, as shown in the first six rows of the penultimate column of Table 1. Accordingly, the Hamada model would be the better approximation at the beginning of the cycle. However, the Miles and Ezzell model would still be the better approximation over most of the cycle. Thus, even if one elected to use the Hamada model in cases in which N was sufficiently large that this model was the better approximation at the beginning of the cycle, the Miles and Ezzell model will be a better approximation over most of the cycle.

4. Conclusions

This paper develops a beta gearing model to deal with situations in which firms may engage in lagged adjustment of their debt level in accordance with a particular leverage ratio, and therefore embraces both the Hamada and Miles/Ezzell models as special cases. The numerical analysis of this model shows that the Miles/Ezzell model is a much better approximation for debt resetting frequencies less than ten years, and that the Miles/Ezzell model is also a much better approximation for most of the interval between successive debt resets even when the debt resetting frequency is greater than ten years. All of this suggests that the Miles/Ezzell model is significantly superior to the Hamada model and ought to be used in substitution for it.

Table 1: The Relationship Between δ_t and the Debt Resetting Frequency

N-t	δ_t		
	$R_f = .03$	$R_f = .05$	$R_f = .07$
1	.03	.05	.07
3	.08	.14	.18
5	.14	.22	.29
10	.26	.39	.49
15	.36	.52	.64
20	.45	.62	.74
30	.59	.77	.87

This table shows the relationship between δ_t and the term remaining before the next debt resetting point (*N-t*), for a range of values for the risk free rate R_f.

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