Research highlights

- Extreme value theory suitable to analyze a momentum strategy’s extreme returns
- A momentum strategy has a fat left tail and a thin right tail
- Asymmetry caused by losers' fat right tails and winners’ fat left tails
- The tail asymmetry strongly reduces a momentum strategy’s prospective utility

Asymmetric extreme tails and prospective utility of momentum returns

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Abstract

We use extreme value theory to analyse the tails of a momentum strategy's return distribution. The asymmetry between the fat left tail and thin right tail strongly reduces a momentum strategy's prospective utility levels.

Keywords: Extreme value theory; Asymmetric tails; Prospective utility

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1. Introduction

Menkhoff and Schmeling (2006) show that prospect theory provides a possible explanation for the puzzling high momentum returns. They were the first to compare the prospective utility levels of a momentum strategy and a market portfolio. Their excellent study suggests that the answer to the puzzle lies in the "higher probability of extreme losses,"[1] but they do not focus on those extreme returns, nor distinguish between the winner and loser stocks' tail returns. Our study fills this gap.

Our main contribution is that we use extreme value theory (EVT) to analyse how a momentum strategy's asymmetric extreme tails affect prospective utility. The distinguishing factor of our analysis is that we focus on the extreme tails of the return distribution. We use prospect theory to appraise the impact of extreme tail behaviour of a momentum strategy and market portfolio on utility levels.

2. Extreme value theory

EVT is especially suitable for approximating the likelihood that an extreme return occurs, see e.g. Embrechts et al. (1997) or Straetmans et al. (2008). Moreover, EVT allows for a separate analysis of the left and right tail. Hence, EVT seems an appropriate choice to analyse in detail the extreme tail risks of a momentum strategy.

In our EVT-approach, it is assumed that the tails of the return distribution diminish by a power instead of an exponent, as is the case under (log)normality. The Power Law is used to calculate the probability that return R exceeds tail value Q:

$$p = P[R > Q] = CQ^{-\alpha}, \qquad (1)$$

where parameter C is a positive constant, and parameter Q is a return value deep in the tail of the distribution. The parameter α is known as the tail-index, which

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[1] Menkhoff and Schmeling (2006), p. 366.

appraises the fatness of the tail: a high (low) tail-index implies that the tail is thin (fat). The Hill (1973) method is used to estimate the tail-index $\hat{\alpha}$, where $\hat{\alpha} = 1/\hat{\gamma}$

and: $$\hat{\gamma} = \frac{1}{k}\sum_{j=1}^{k} \ln\left(\frac{x_j}{x_{k+1}}\right), \tag{2}$$

where variable x_j is the *j-th* order statistic; thus $x_j \geq x_{j-1}$ for $j = 2, \ldots, N$, where N equals the number of return observations in the sample. The parameter k in equation (2) represents the number of extreme tail returns used.

Next, we calculate the expected shortfall, which is a quantile estimator with a corresponding confidence level p. The expected shortfall measures the expected return level, given that a certain return level is exceeded. Based on Danielsson et al.,(2006) we estimate the expected shortfall as follows:

$$ES = \left(\frac{\alpha}{\alpha - 1}\right) x_{k+1} \left(\frac{k}{Np}\right)^{\hat{\gamma}}, \tag{3}$$

Based on Straetmans et al. (2008), we conduct 600 block-bootstrapped simulations to generate test statistics on the equality of tail-index estimates as well as expected shortfall estimates, using a block size of $N^{1/3}$.

3. Data and methodology

We use total monthly returns from CRSP on stocks traded on NYSE, AMEX or NASDAQ, from 1926-2009, inclusive. Our momentum portfolio is constructed identical to Jegadeesh and Titman (2001).[2] The market return is calculated as the return on a value weighted portfolio of all stocks in the sample, less the 30-day T-bill yield.

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[2] We use the conventional 6/6 strategy (both the formation and holding periods equal six months). We ran our procedure with the Menkhoff and Schmeling (2006) 12/1/1 strategy and obtained qualitatively similar results. A further robustness test using daily returns also generated qualitatively similar results – both available upon request.

To assess the tails of the various return distributions, we first determine how many order-statistics are required to best estimate the tail-index. Following Straetmans et al. (2008), we generate Hill-plots which depict the relationship between the tail-index estimate, α, and the number of order statistics, *k*. From a visual inspection of the Hill-plots we observe that the tail-index estimate is relatively stable when the number of order statistics is around 25 for the momentum strategy and around 15 for the market portfolio. Moving further out into the tail, the tail-index estimate becomes biased. Using fewer observations renders the estimate statistically less reliable.

4. Asymmetric tail risks

Researchers have long known that stock return distributions have fat tails. However, whether momentum investors' winner and loser portfolios similarly have fat tails has received less attention. We find they do, and the tails are asymmetric. We further find that it is precisely this tail asymmetry - fat winner's left tail and (shorted) fat loser's right tail - which causes the momentum portfolio to have proportionally fat left and thin right tails.

Table 1. Monthly return characteristics

		Winners	Losers	Winners - Losers	Market
Returns	Minimum	-32.60%	-39.30%	-53.17%	-29.04%
	Maximum	43.54%	78.20%	32.91%	38.27%
	Mean	1.50%	0.61%	0.89%	0.61%
	St. dev.	7.32%	9.25%	5.84%	5.47%
	Skewness	-0.33	1.32	-2.53	0.17
	Kurtosis	3.72	11.41	21.50	7.56
	Left α	2.72	2.85	1.87	3.24
	Right α	2.95	1.91	3.58	2.45
	(Left-Right) α	-0.23^{*}	0.95^{*}	-1.71^{*}	0.79^{*}

Table 1 reports monthly momentum return and market return characteristics. The Left (Right) α denotes the estimated index of the return distribution's left (right) tail. The superscript * indicates significance at the one percent level of the bootstrapped differences between the left and right tail-index.

Table 1 shows that the winner and loser portfolios exhibit very different risk-return characteristics. Although like others we find that the momentum portfolio yields a positive mean, the momentum portfolio has the highest kurtosis and the most negative skewness of all four portfolios in Table 1.

From Table 1 we observe that the loser's right tail-index is significantly lower then its left tail-index. By contrast, the winners' tail-index estimate shows an opposite effect. The losers have relatively fat right tails and thin left tails, and the winners have relatively thin right tails and fat left tails – a momentum investor is exposed to the short losers' fat right tail with no compensating left tail. This effect is corroborated by the tail-indices for the momentum portfolio; the left-tail-index is lower and the right tail-index is higher than any of the other tail-index estimates.

The expected shortfall estimates in Figure 1 are further evidence of the momentum strategy's highly asymmetric tail risks. For all confidence levels the momentum's left tail expected shortfall significantly exceeds that of the right tail[3].

< PLACE FIGURE 1 AROUND HERE >

The economic importance of the difference between the left and right tail expected shortfall becomes especially pronounced for the most extreme momentum returns. For the 5% confidence level (once every 20 months) the sum of the momentum returns' left and right tail expected shortfalls equals -4.73%. For the 0.5% confidence level (once every 200 months) this sum reaches a much more negative level of -34.92%. By contrast, the market's expected shortfall estimates are markedly more symmetric; the right tail expected shortfall levels are quite similar to their left tail counterparts. As a result, for the market portfolio the sum of two tails' expected shortfalls hovers around zero. Hence, the momentum strategy has, compared to the

[3] Details of this bootstrapped significance test are available upon request.

market, highly undesirable tail return characteristics. A block-bootstrapped t-test shows that the difference between the two portfolios' expected shortfalls is statistically significant for most confidence levels.

5. Extreme tail returns' impact on prospective utility

Menkhoff and Schmeling (2006) introduce prospect theory to momentum strategy analysis. In this section we follow their lead and analyse how prospective utility levels are affected by extreme tail returns. In prospect theory, utility is determined by an S-shaped value function. Due to loss aversion, the *dis*utility of a loss is around twice as great as the utility of an equal-sized gain. We refer to Benartzi and Thaler (1995) for details on prospect theory, its parameter estimates and calculation approach.

< PLACE FIGURE 2 AROUND HERE >

Figure 2 demonstrates that the most extreme tail returns are disproportionally important in determining a strategy's total utility[4]. This is most dramatic in evaluation horizons of around a year, and especially when comparing the momentum versus market strategies. We introduce the tail utility contribution which is calculated as the sum of the left and right tails' utility levels, divided by the overall utility. The tail utility contribution thus measures the relative importance of the tails in a portfolio's total utility level. As the evaluation horizon increases, the tail utility contributions strongly increase for the momentum portfolio. By contrast, for the market strategy the tail utility contributions remain around the same level because its tail risks are more symmetric compared to the momentum portfolio. Finally, from Figure 2 we note that

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[4] We have compared utility levels from the 6/6 strategy for evaluation horizons from 1 to 36 months with an annual transaction cost from 1% to 5% to those of the market strategy. Our results are consistent with Menkhoff and Schmeling (2006) which show that the momentum strategy does not clearly outperform the market strategy.

as we use more of the tail observations, i.e. as the confidence level p increases from 0.5% to 5.0%, the tail utility contributions increase for both the momentum and the market portfolio. Such increases are more pronounced for the momentum portfolio due to its more asymmetric tail risks.

6. Conclusion

Using EVT we find that the winner and loser stocks in a momentum portfolio exhibit very different tail characteristics which both contribute to the momentum portfolio's fat left tail and thin right tail. This asymmetry becomes especially evident in the most extreme momentum returns. By contrast, the tail returns of a market portfolio are significantly less asymmetric. We find that it is the asymmetry of the extreme tail risks that strongly reduces a momentum strategy's overall utility levels.

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Figure 1

Figure 1. Asymmetry in Expected Shortfalls

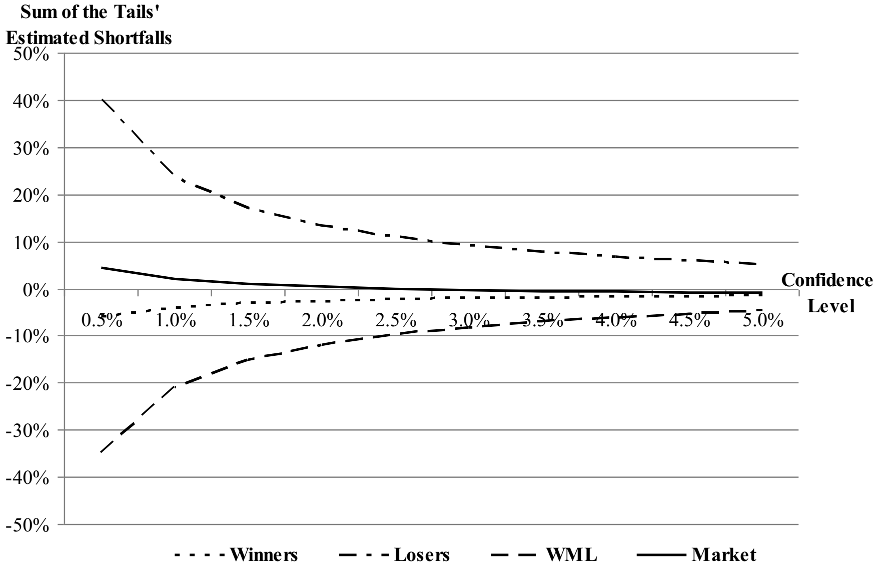

Figure 1 shows the sum of left and right tails' expected shortfalls for each of four portfolios: winners, losers, momentum (WML) and the market. The confidence level *p*, from equation (3), ranges from 0.5% to 5.0%.

Figure 2

Figure 2. Prospective Utility Tail Contributions

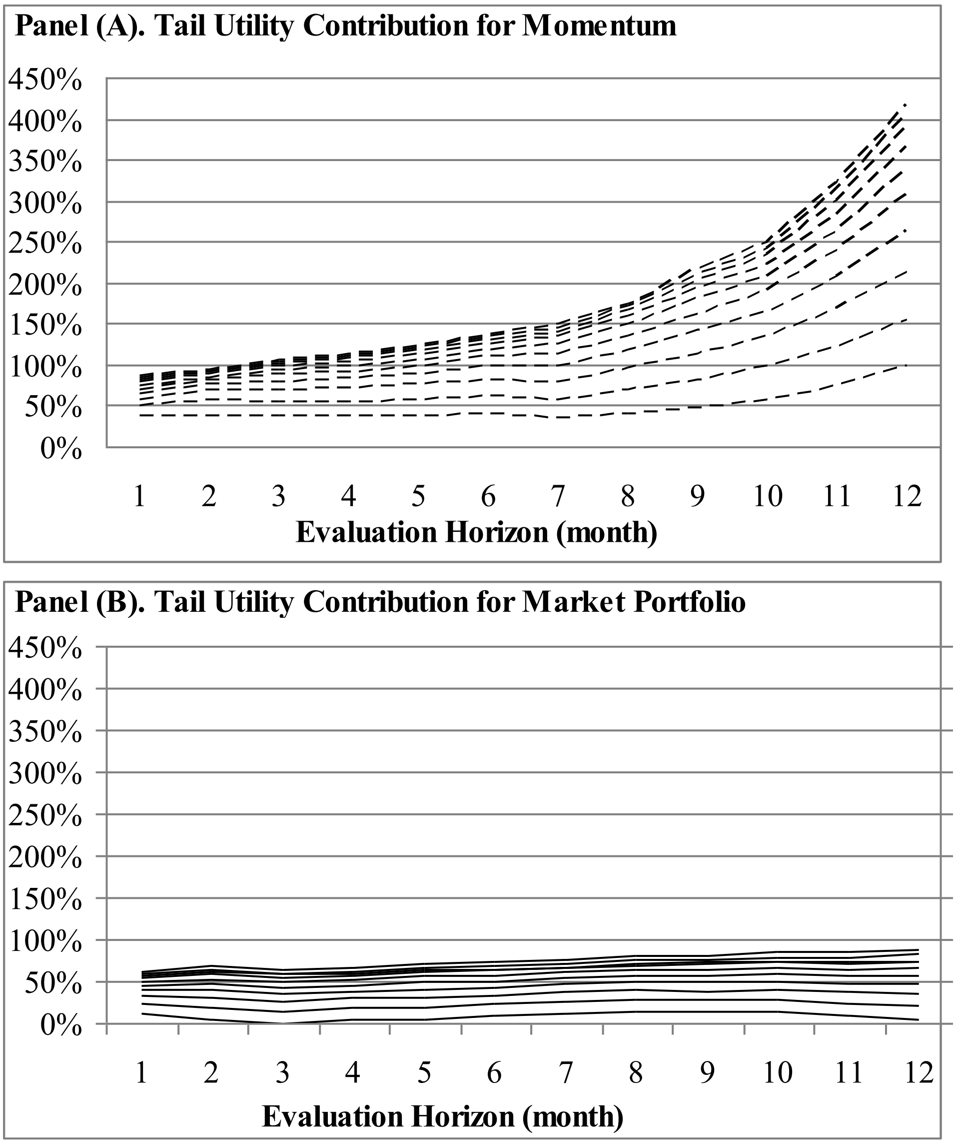

Figure 2 reports the contributions of tail observations in determining prospective utility levels for the 6/6 momentum strategy (in Panel (A)) and the market strategy (in Panel (B)) across various evaluation horizons. Based on Menkhoff and Schmeling (2006), we randomly draw 100,000 consecutive n-month returns from the sample, and then rank them to calculate 200 interval means as inputs into the prospective utility function. The tail utility contribution reports the sum of the utility on the left and right tails divided by total utility. The lines report tail utility contributions for confidence intervals ranging from 0.5% (bottom line in each panel) to 5.0% (highest line in each panel).