The Effect of Differences of Opinion and Valuation Uncertainty on the Pricing of IPOs by Investment Banks

Michael O'Connor Keefe

Victoria University of Wellington

School of Economics and Finance

PO Box 600, Wellington 6140, NZ

michael.keefe@vuw.ac.nz

64 04 463 5708

Current Draft: 29th November, 2010

The Effect of Differences of Opinion and Valuation Uncertainty on the Pricing of IPOs by Investment Banks

Abstract

I model the pricing decision of an investment bank that manages a book-built IPO and faces a stochastic downward sloping demand curve for the firm's IPO shares. The model distinguishes between the effect of differences of opinion and valuation uncertainty on the pricing decision of the investment bank and hence average IPO initial returns. Without relying upon information asymmetries between agents, the model predicts several empirical regularities: average positive initial returns, the ubiquitous use of the over-allotment option, partial adjustment of offer prices to observable information, and extreme initial returns in a bubble. The model provides insight into the how investment banks interpret and price information. For example, news stories about the IPO firm attract potential new investors, many of which are plausibly sentiment investors. My model shows two effects of news stories on the stochastic demand curve. First, the new investors cause an outward shift out in the demand curve. Second, the new investors, many of whom are sentiment investors, increase valuation uncertainty. The investment bank raises the offer price due to the shift in demand caused by news stories, but only partially because of the increase in valuation uncertainty.

Keywords: Initial Public Offering, Investment Bank, Issuing Firm, Underwriter, Offer Price, First Day Returns, Firm Commitment Contract, Demand Uncertainty, Differences of Opinion, Bubble, Market Shock

JEL Classification Codes: D40 - General Market Structure and Pricing, D81-Criteria for Decision-Making under Risk and Uncertainty, G24 - Investment Banking

1 Introduction

Many financial economists explain the average high initial returns (underpricing) of initial public offerings (IPOs) using models of asymmetric information between the agents. For example, Baron (1982) models asymmetry between the issuing firm and the investment bank, Rock (1986) models asymmetry between pre-market investors in the IPO, and Benveniste and Spindt (1989) models asymmetry between the investment bank and the pre-market investors. In all of these models, an agent or agents knows the after-market value of the IPO. High initial returns are an outcome of mechanisms to solve these asymmetry problems. Although these models provide excellent insight into the tensions between agents, they overlook the importance of uncertainty about the future. Knight (1921, pg. 198 Chap. VII) writes,

> If all changes were to take place in accordance with invariable and universally known laws, they could be foreseen for an indefinite period in advance of their occurrence, and would not upset the perfect apportionment of product values among the contributing agencies, and profit (or loss) would not arise.

In this paper, I explore the implications of uncertainty on the initial returns of IPOs.

In a book built firm commitment IPO, an investment bank agrees to sell a set number of shares at an offer price on behalf of an issuing firm. As part of the IPO issuance process, the investment bank builds the book of orders, the majority of which are from institutional investors. After this book-building period, but just prior to the shares trading on an exchange, the investment bank sets the offer price and sells the shares. The investment bank then manages the transition to secondary market trading, also known as the after-market. On average, the clearing price in the secondary market exceeds the offer price, which results in average positive initial returns. However, in the event that demand for the IPO firm's stock is weak, the investment bank supports the price of the issue (Aggarwal, 2000). Therefore, prior to setting the offer price, the investment bank accounts for the potential after-market cost of price supporting the issue. In contrast to the Baron (1982), Benveniste and Spindt (1989), Rock (1986) who model information asymmetries between agents, I model the ex-ante offer price decision of the investment bank, which faces a stochastic downward sloping demand for the IPO firm's shares in the secondary market.

In my model stochastic demand arises from both differences of opinion and valuation uncertainty. As a result, the model distinguishes between the effect of differences of opinion and valuation uncertainty on the investment bank's pricing decision and hence average IPO initial returns. Without relying upon information asymmetries between agents, the model predicts several empirical regularities: average positive initial returns, the ubiquitous use of the over-allotment option, partial adjustment of offer prices to observable information, and extreme initial returns in a bubble. The model provides insight into the how investment banks interpret and price information. For example, news stories about the IPO firm attract potential new investors. My model shows two effects of news stories on the stochastic demand curve. First, the new investors shift out the demand curve. Second, the new investors, many of whom are sentiment investors, increase valuation uncertainty. The investment bank raises the offer price due to the shift in demand caused by news stories, but only partially because of the increase in valuation uncertainty.

In traditional asset pricing models with no market frictions, the demand curve for a stock is perfectly elastic. This implies that if the investment bank prices the IPO at the intrinsic value of the stock, then investors demand as many shares as issued. In contrast to the traditional asset pricing framework, Miller (1977) posits that a downward sloping demand curve arises from differences of opinion. Harris and Raviv (1993) model trading in speculative markets where traders have common information but differ in their interpretation of the information. Harris and Raviv (1993, p. 474) write,

> It seems to us that people often share common information yet disagree as to the meaning of this information, not only in the evaluation of risky assets but also in the evaluation of political candidates, economic policies, and the outcomes of horse races. One example is the variety of opinions among financial analysts and macroeconomists regarding future movements of interest rates, exchange rates, gross national product, and stock prices despite the fact all the analysts have access to the same economic data.

In the difference of opinion setting of Harris and Raviv (1993), investors hold different opinions about the market value of the issuing firm.

Miller (1977) posits that differences of opinion give rise to a downward sloping demand curve for IPOs. He notes that for a downward sloping demand to persist investors with more pessimistic valuations can not easily affect prices. Ofek and Richardson (2003) show that recently issued IPOs are difficult to short sell, which limits pessimistic investors from injecting their opinion into the market price. However, Shleifer and Vishny (1997) show that limits to arbitrage imply that an asset's price may not always reflect it's intrinsic value even in the absence of short sale constraints.

To explain how a downward sloping demand curve is generated using differences of opinion, I provide the following simple example. Suppose there are 15 investors each of whom value the IPO by multiplying the sales of the issuing firm times the average market value to sales ratio of comparable traded firms. All 15 investors have the same information set and 6 possible comparable firms, but each differ in their opinion of which 4 firms out of the 6 should be used as benchmarks. Because the number of possible combinations of 4 taken from 6 is 15, each investor values the IPO based on a different set of comparable firms. As a result, each of the 15 investors plausibly differs in their opinion regarding their mean assessment of the value of the IPO. This generates a downward sloping demand curve. As in Miller (1977), I interpret the slope of the demand curve as representing differences of opinion among investors. Further, because investors value the IPO using a set of comparable firms, investors also face uncertainty about their valuations, which adds a stochastic component to the demand curve. In this framework, the slope of the demand curve represents differences of opinion whereas the standard deviation around the curve represents valuation uncertainty.

In the real world, there are many other choices besides which 4 firms to take from a possible 6. These choices include a far greater list of possible comparable firms and different valuations methods as well as different interpretations of economic data, industry competitiveness, news articles, and management capability etc. These additional choices add to the heterogeneity of valuation opinions and the uncertainty about those opinions. Further, several empirical papers provide evidence in support of a downward sloping demand curve for equities. Shleifer (1986) finds that newly included stocks on the S&P 500 index have a positive abnormal return, which is caused by increased demand for the shares by index funds. Field and Hanka (2001) find an abnormal return of -1.5 percent around lock-up expirations. Huang et al. (2009) investigate Seasoned Equity Offerings (SEOs) in China that list on both the A and B exchange, but which conduct the SEO only on the A exchange. They find a negative abnormal return on the A exchange listed shares but not on the B exchange listed shares. Gao and Ritter (2010) identify demand elasticity as an important determinant of the issuer choosing either fully marketed or accelerated SEO. This research suggests the existence of a downward sloping demand curve for stocks.

Recent empirical evidence supports a cooperative relationship between institutional investors and investment banks. First, Griffin et al. (2007) find institutional investors who are clients of the investment bank tend to price support the issue as net buyers during the first few days of trading. The net buyer position of the investment bank's clients is larger for cold than hot IPOs. Second, Aggarwal et al. (2002) show that institutional investors receive favorable allocations of hot IPOs. This is consistent with both the investment bank rewarding institutional investors for revealing private information or with investment banks expecting institutional investor price support in cold markets. Third, Aggarwal (2003) finds that institutional investors flip hot issues, but price support cold issues. If the institutional investor trades through the investment bank, the investment bank benefits from flipped shares for a hot IPO through increased commissions.

Therefore, even risk neutral institutional investors evaluate the cost of price support. As a result, the institutional investors of the investment bank help underwrite the offer and therefore must account for the possibility that less shares are demanded at the offer price than are offered.

To model the implications of differences of opinion about the value of the IPO, I define after-market demand, q, for IPO shares as stochastic with $q = \tilde{\theta} - bP$ where $\tilde{\theta}$ is distributed $U[\alpha, \beta]$. In this model of demand, the observed demand curve is based on the draw of θ from the uniform distribution. The slope of the demand curve, b, represents differences of opinion about the mean valuation. The standard deviation, θ, represents uncertainty about the mean valuation. For example, if all investors agree about the benchmarks, then each investor has a common expected value, but valuation uncertainty remains due to the different valuation ratios associated with the benchmark stocks. At the extreme, if all investors agree on one benchmark stock then there is neither differences of opinion or valuation uncertainty; hence, a perfectly elastic demand curve. Because the expected value of $\tilde{\theta}$ is $(1/2)(\beta - \alpha)$, the expected clearing price is $P_c^e = \frac{1}{2b}(\beta + \alpha) - \frac{X}{b}$ for a given number of shares X. If the investment bank chooses an offer price less than expected clearing price, $P_0 < P_c^e$, then based on the Law of Large Numbers over multiple issues the average underpricing is positive.

Because the investment bank enters into a firm commitment contract, the payoff to the investment bank is dependent on realized demand, q, and the offer price, P_0. The investment bank commits to purchase X shares. In most cases, the investment bank also has an over-allotment option, which allows the investment bank to sell up to $X(1 + v)$ shares where v is generally 15%. The payoff to the investment bank is dependent on whether $q < X$, $X < q < X(1+v)$, or $q > X(1+v)$. The choice of P_0 affects the probability of these cases. For example, a higher offer price increases the probability that $q < X$. Furthermore, if realized demand, q, is less than the shares the investment bank contractually agrees to sell, X, then the investment bank absorbs a loss equal to $P_0(X - q)$ [1].

In practice, the investment bank rarely absorbs an underwriting loss, but instead supports the price of the issue with the assistance of both its repeat institutional customers and members of the selling syndicate. First, Griffin et al. (2007) show repeat institutional investors of the investment bank are net buyers of cold issues. Second, the lead investment bank uses penalty bids to discourage sales of cold issues in the after-market. If the lead investment bank assesses a penalty bid, a member of the selling syndicate forfeits their selling concession fee. Aggarwal (2000) finds that average first day trading volume is 67.59% for IPO without penalty bids, 56.14% for IPOs in which the lead investment bank elects to not assess the penalty, and 48.59% for IPOs in which the lead investment bank assesses penalty bids. The implication of either

[1] I explore the implications of a different cost function in Section 6.

the threat or use of penalty bids is that after-market sales are restricted, especially in weak demand states. Lastly, in cold markets the investment bank short covers, by purchasing shares in the open market rather than electing to execute the over-allottment option to deliver shares. Ellis et al. (2000) note that investment banks make trading commissions on short covering transactions; however, they forfeit the opportunity to earn fees on the sales of over-allottment shares. In summary, in the case of weak demand for an issue, investment banks purchase shares and assess or threaten to assess penalty bids while repeat institutional clients purchase shares. I don't model these mechanisms, but instead model the cost to the investment bank and repeat institutional investors of providing a price guarentee to the issuing firm as $P_0(X - q)$, which represents the need to purchase shares at the offer price in the event demand is less than the number of shares purchased from the issuing firm.

This paper is related to the IPO literature about ex-ante valuation uncertainty. Beatty and Ritter (1986) extend the ideas of Rock (1986) by noting high ex-ante uncertainty attracts informed investors, which exacerbates the winner's curse problem and investment bank underprices in equilibrium so that uninformed investors are willing to purchase the IPO. Roosenboom (2007) studies the French IPOs where investment banks produce valuation information. Despite producing valuation information, which should minimize the winner's curse problem, investment banks still file below their expected valuation. In this setting, the information production by the investment bank prior to the offering should minimize the informational disadvantage of uninformed investors, yet underpricing persists. As in Chen and Wilhelm (2008), I assume a cooperative relationship between the investment bank and institutional investors, but model secondary market demand using differences of opinion and valuation uncertainty. Unlike Beatty and Ritter (1986), I do not require a winner's curse argument to generate underpricing.

The portion of this paper related to pricing in a bubble is closely related to the theoretical work of Ljungqvist et al. (2006), Derrien (2005), Cornelli et al. (2006). In Ljungqvist et al. (2006) the issuing firm chooses an issuing strategy to maximize the surplus (issued price less intrinsic value) due to wealth constrained sentiment investors. In Derrien (2005) the issuing firm chooses the offer price by estimated demand from sentiment traders as well as extracting private information from informed institutional investors. In Cornelli et al. (2006) the expected value of the shares in the secondary market is a weighted average of information from the sentiment investors in a grey market and informed investors. When sentiment investors have an inflated value of the IPO firm, the issuer and the institutional investors split the surplus. In my model, I provide a general framework that shows underpricing is expected with or without the presence of sentiment investors; however, I show that overly-optimistic sentiment investors diminish the ability of the investment bank to interpret market signals and exacerbate underpricing. Further, because the investment bank can not perfectly interpret market signals, my model explains the partial adjustment of offer prices from comparable market returns, news articles, and other observable variables, independent of the presence of sentiment investors.

I provide insight into Miller (1977), who reasons that divergence of opinion is measured by the slope of the inverse demand curve, $1/b$. Miller (1977) also posits that, for a given issue size, the investment bank will set a progressively higher offer price as the slope of the $1/b$ increases. My results match the intuition of Miller (1977) in that the investment bank chooses a higher offer price as demand becomes more elastic. Miller (1977, pg 1156) writes,

> Incidentally, if underwriters ignore the above arguments and price new issues on the basis of their own best estimates of the prices of comparable seasoned securities, they will typically underprice new issues. The mean of their appraisals will resemble the mean appraisal of the typical investor, and this will be below the appraisal of the most optimistic investors who actually constitute the market for the security.

Despite systematic underpricing, the empirical evidence of Purnanandam and Swaminathan (2004) suggests that investment banks overprice IPOs relative to their fundamentals; therefore, Miller's argument that investment banks set the price based on such fundamentals is not supported by the empirical evidence. In my model, the investment bank prices based on the stochastic demand curve and not based on the intrinsic value of the IPO.

Second, I formally show if an investment bank, through marketing efforts, increases the elasticity of demand then it increases the offer price. These model findings match the empirical findings of Gao and Ritter (2010), who posit that an objective of fully marketed SEOs is to increase the elasticity. Third, my model demonstrates how investment banks only partially adjust for observable variables, when those variables are not a perfect signal of underlying demand. Fourth, I show how the signal is further attenuated in a bubble, which results in even further underpricing.

The paper proceeds as follows. In Section 2, I describe the model. In Section 3, I solve the model for the investment bank's profit maximizing offer price as functions of the standard deviation of demand and the slope parameter. I also show the effect of the over-allotment option on the offer price, the issuing firm's proceeds, and the investment bank's profit. In Section 4, I solve for the optimal offer price as a function of the positive and negative market shocks as well as the subjective probability of the bubble state continuing. In Section 5, I discuss the empirical predictions of the model and studies related to those predictions. In Section 6, I explore the implications to the model of a different cost function. In Section 7, I conclude. To minimize technical details, I provide derivations in Appendix 1 and proofs of propositions in Appendix 2.

2 A Model of Stochastic Demand

Assume that the issuing firm exogenously specifies a quantity of shares, X, that the investment bank commits to place in the IPO. The investment bank sets the offer price, P_0, and then the actual demand for the offer, q, is revealed. There are two distinct outcomes - demand q exceeds the quantity of shares sold X or demand q is less than the quantity of shares that the investment bank committed to underwrite. The profit, π, earned by the investment bank varies by realized demand, q, as follows:

$$\text{if } q > X \Rightarrow \qquad \pi_1 = fP_0X - FC \tag{1}$$

$$\text{if } q < X \Rightarrow \qquad \pi_2 = fP_0X - FC - P_0(X - q) \tag{2}$$

where f represents the percentage of the proceeds received by the investment bank and FC represents the fixed costs of the investment bank in managing the offer.

I assume the investment bank and institutional investors have the same information set and neither can precisely estimate the demand curve for the issue. In addition, because the repeat clients of institutional investor are net buyers in cold markets and therefore share in the responsibility for price support in the event $q < X$. I assume the investment bank and institutional investors are risk neutral. I model demand as stochastic where $q = \tilde{\theta} - bp$ and $\tilde{\theta}$ is distributed $U[\alpha, \beta]$. The expected clearing price, P_c^e is the *ex-ante* expected price where investment bank sells exactly X shares and is defined as

$$\begin{aligned} P_c^e &= \frac{(E(\theta) - X)}{b} \\ &= \frac{1}{2b}(\beta + \alpha) - \frac{X}{b} \end{aligned} \tag{3}$$

since $E[\theta] = (1/2)(\beta - \alpha)$. If the investment bank chooses the offer price equal to the expected clearing price, $P_0 = P_c^e$, then based on the Law of Larger Number over multiple issues average underpricing tends toward zero.

If the investment bank chooses offer price, P_0, then expected profit, $E[\pi]$ to the investment bank is given by

$$E[\pi] = Prob(q \geq X)\pi_1 + Prob(q < X)E[\pi_2|q < X] \tag{4}$$

To this basic model, I sequentially add an over-allotment option, observable market shocks, and an asset pricing bubble. In each setting, I solve equation (4) for the offer price that maximizes expected profits to the investment bank and compare that price to the expected clearing price in equation (3). To depict the economic implications, I parameterize the model and graph the variables under study.

3 The Effect of Demand Uncertainty on the Pricing Decision of the Investment Bank

I make several assumptions in my model. First, I assume that the issuing firm has deep knowledge of its business and industry; however, because an IPO occurs only once for most firms, the issuing firm lacks personnel who understand the institutional details of IPOs. Because keeping specialized in-house resources are cost prohibitive, the issuing firm outsources the required IPO expertise to the investment bank. Second, I assume that the bargaining power of the issuing firm is highest prior to contracting with the investment bank. Chen and Ritter (2000) note that when the issuing firm selects the underwriter during the "beauty contest" phase, the investment bank does not commit to a specific offer price, but does provide a price range. After the issuing firm selects the investment bank, the investment bank gains bargaining power. I assume the investment bank uses its bargaining power to choose the final offer price, provided the offer price is above the reservation price of the issuing firm. If the investment bank chooses an offer price below the reservation price of the issuing firm, the issuing firm withdraws the IPO. I assume the issuing firm's objective is to maximize the proceeds from the IPO, $P_0X(1-f)$, whereas the investment bank's objective is to maximize expected profitability as given in (4).

3.1 The Optimal Offer Price

In Appendix 1, I derive the expected profit to the investment bank.

$$E[\pi] = fP_0X - FC - \frac{\frac{1}{2}P_0(X + bP_0 - \alpha)^2}{\beta - \alpha} \tag{5}$$

The first two terms in (5) represent the profit to the investment bank should $q > X$. The third term in (5) is negative since $P_0 > 0$, $X > 0$, $b > 0$, and $\beta > \alpha > 0$ and represents as the expected reduction in profit due to the probability that $q < X$. When an investment bank raises P_0, expected profit increases from the increased revenue associated with the first term, but decreases due to the possible costs supporting the issue in the after-market in the third term. Therefore, the optimal expected profit maximizing offer price occurs where the marginal revenue associated with the first term equals the marginal costs associated with third term in (5).

In Appendix 2, I solve for the expected profit maximizing offer price, which I find to be:

$$P_0^* = \frac{1}{3b}(\sqrt{6fX(\beta - \alpha) + (X - \alpha)^2} - 2X + 2\alpha) \tag{6}$$

In Appendix 2, I then compare equation (6) with the expected clearing price, equation (3), which leads to the following proposition.

Proposition 1. *Given an exogenous number of shares X and stochastic demand, the investment bank chooses an expected profit maximizing offer price, P_0^*, such that P_0^* is less than the expected clearing price P_c^e.*

To provide economic intuition about Proposition 1, I parameterize the model. I specify the stochastic demand curve as $\alpha = 90$, $\beta = 110$, and $b = 2$ and assume the issuing firm exogenously sets $X = 80$ shares. I follow Chen and Ritter (2000) and set the gross spread, $f = .07$. Lastly, I set $FC = 32$. In this example, the expected clearing price is $P_c^e = \frac{1}{2b}(\beta + \alpha) - \frac{X}{b} = 10$ and the investment bank's expected profit maximizing offer price is $P_0 = 7.96$. If the investment bank has the bargaining power, it will choose $P_0 = 7.96$, which results in expected underpricing of 25.6%. If the issuing firm has bargaining power, then it negotiates the highest offer price possible with the investment bank. As shown in Figure 1, the investment bank makes zero expected profit at an offer price $P_0 = 9.85$. At any price above $P_0 = 9.85$, the investment bank withdraws from the issue. If the investment bank's expected profit maximizing price is below the reservation price of the issuing firm and the investment bank has bargaining power, the investment bank chooses the reservation price, conditional on that price being less than or equal to the zero expected profit price. For example, if the reservation price of the issuing firm were 8.25 the investment bank would be forced to increase P_0 from 7.96 to 8.25. These examples illustrate that underpricing occurs under different competitive market structures and bargaining authority.

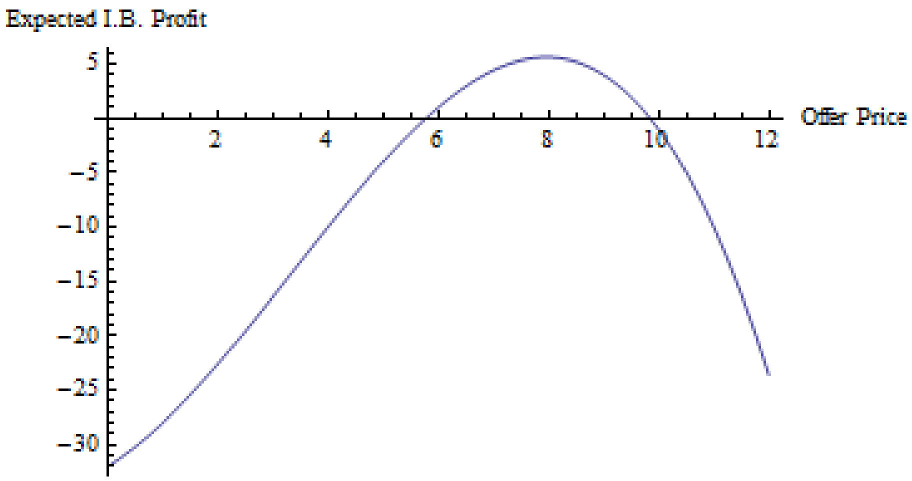

Figure 1: This figure plots the expected investment bank profit against Offer Price, P_0. Demand for IPO shares, q, is defined by $q = \tilde{\theta} - bP$ where θ is distributed $U[\alpha, \beta]$. The model is parameterized as follows: $\alpha = 90$, $\beta = 110$, and $b = 2$. The issuing firm exogenously sets share supply of $X = 80$. The expected profit of the investment bank is $E[\pi] = fP_0X - FC - \frac{P_0}{2(\beta-\alpha)}(X + bP_0 - \alpha)^2$ where $f = .07$ and $FC = 32$.

This model finds underpricing as a profit maximizing behavior for the investment bank when the demand curve is stochastic; implying, underpricing is a natural outcome of the pricing decision if the investment bank has bargaining power at the end of the book-building period. Expected initial returns decrease as the bargaining power of the issuing firm increases; however, underpricing is possible even in the extreme case that the issuing firm forces the investment bank to price at the zero expected profit condition. In summary, the investment bank trades off the increased revenue associated with a higher offer price against the possible costs of supporting the issue in the after-market should $q < X$.

3.2 The Effect of Demand Volatility on the Pricing Decision of the Investment Bank

In this section, I investigate the relationship between demand volatility and the pricing decision of the investment bank. The variance of demand is

$$\begin{aligned} Var[q] &= Var[\theta - bP_0] \\ \sigma_q^2 &= Var[\theta] = \frac{1}{12}(\beta - \alpha)^2 \end{aligned} \tag{7}$$

This implies $\sigma_q = \frac{1}{2\sqrt{3}}(\beta - \alpha)$. Using $\mu = (\alpha + \beta)/2$, I solve for β and α as functions of μ and σ_q and find that $\alpha = \mu - \sqrt{3}\sigma_q$ and $\beta = \mu + \sqrt{3}\sigma_q$. I substitute $\alpha = \mu - \sqrt{3}\sigma_q$ and $\beta = \mu + \sqrt{3}\sigma_q$ into (5) and find

$$E[\pi] = fP_0X - FC - \frac{\frac{1}{2}P_0(X + bP_0 - \mu + \sqrt{3}\sigma_q)^2}{2\sqrt{3}\sigma_q} \tag{8}$$

As in (5), the first two terms in (8) represent the profit to the investment bank should realized demand, q, exceed shares sold, X. The last term in (8) is negative since $P_0 > 0$ and $\sigma_q > 0$, which implies the third term represents the expected reduction in expected profit due to probability $q < X$. When an investment bank raises P_0, expected profit increases from the revenue increase associated with the first term, but decreases due to the expected underwriting costs in the third term of (8). Therefore, the optimal expected profit maximizing offer price occurs where the marginal revenue associated with the first term equals the marginal costs associated with third term in (8).

In Appendix 2, I solve for the profit maximizing offer price to show:

Proposition 2. *Given an exogenous number of shares X and stochastic demand, the investment bank chooses an offer price, P_0^*, that is decreasing with the standard deviation of demand, σ_q.*

To make clear the economic intuition of Proposition 2, I parameterize the model by setting $b = 2$, $X = 80$, $f = .07$, $FC = 32$, and $\mu = E[\theta] = 100$. In this model, an increase in σ_q increases the range of the distribution of θ while keeping the expectation constant. In Figure 2, I plot the relationship between the investment bank's profit maximizing price and the standard deviation of demand σ_q. In a world without demand uncertainty such that $\sigma_q = 0$, the investment bank prices the offering at exactly the market clearing price of 10, which results in exactly zero underpricing. As the standard deviation of demand increases, the investment bank chooses progressively lowers the offer price. By construction the $E[\theta] = 10$ regardless of σ_q, which implies underpricing increases with demand uncertainty.

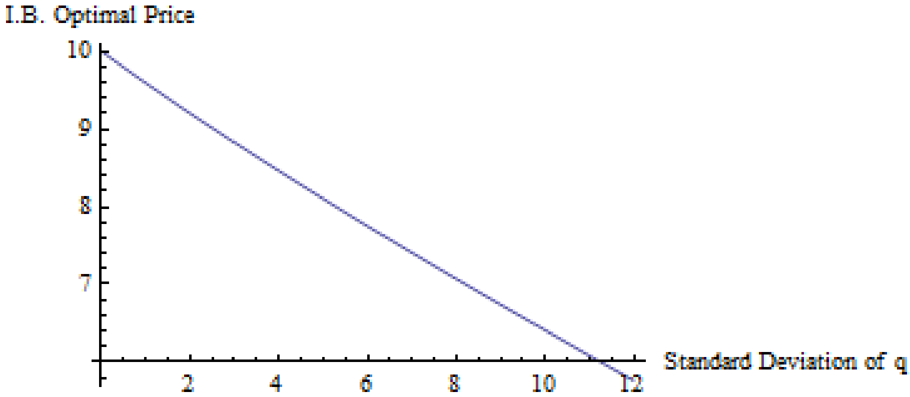

Figure 2: This figure plots the profit maximizing investment bank price against the standard deviation of demand, σ_q. Demand for IPO shares, q, is defined by $q = \tilde{\theta} - bP$ where θ is distributed $U[\alpha, \beta]$. The model is parameterized as follows: $b = 2$, $X = 80$, $f = .07$, $FC = 32$, and $E[\theta] = 100$ where $\alpha = 100 - \sqrt{3}\sigma_q$ and $\beta = 100 + \sqrt{3}\sigma_q$. The expected profit of the investment bank is $E[\pi] = fP_0X - FC - \frac{P_0(X+bP_0-\mu+\sqrt{3}\sigma_q)^2}{4\sqrt{3}\sigma_q}$

3.3 The Effect of an Over-allotment Option on the Pricing Decision of the Investment Bank and Proceeds to the Issuing Firm

In this section, I assume the investment bank exercises an over-allotment option should realized demand, q, exceed the number of shares issued, X, which is exogenously specified by the issuing firm. If there is an over-allotment option, the investment bank has the option to sell up to $X(1 + v)$ shares, where v represents the percentage of shares in the over-allotment option. In this case realized profit, π, depends on the realization of q as follows:

$$\pi = \begin{cases} P_0Xf - FC - P_0(X - q) & \text{if } q \leq X \\ P_0qf - FC & \text{if } X < q < X(1+v) \\ P_0X(1+v)f - FC & \text{if } X(1+v) \leq q \end{cases} \tag{9}$$

As in the previous cases, the investment bank chooses P_0. The choice of P_0 affects the probability of drawing from any of the three regions noted above. For example, a low P_0 decreases the probability that $q \leq X$. In Appendix 1, I solve for the expected profit of the investment to find:

$$E[\pi] = fP_0(\frac{X + vX(\beta - X(1 + \frac{1}{2}v) - bP_0)}{\beta - \alpha}) - FC - \frac{P_0(X + bP_0 - \alpha)^2}{2(\beta - \alpha)} \tag{10}$$

The expected profit equation (10) with the over-allotment option differs from the expected profit equation (5) without the over-allotment because of an additional expression in the first term. Relative to the case without the over-allotment option, the expected profit of the investment bank increases if the additional term $\beta - X(1 + \frac{1}{2}v) - bP_0$ is positive.

In practice, investment banks set the offer price, oversell the issue, and based on observed demand decide how much of the over-allotment option to exercise. In Appendix 2, I take the derivative of the expected profit with respect to P_0, set the derivative to zero, solve for P_0, select the largest root, and simplify to find the profit maximizing offer price with an over-allotment option, which is

$$\begin{aligned} P_0^{ov} = \frac{1}{3b}(&\sqrt{v^2 fX^2(4f - 3) + vfX(2X + 6\beta - 8\alpha) + 6fX(\beta - \alpha) + 2(X - \alpha)^2} \\ &- 2X(1 + fv) + 2\alpha) \end{aligned} \tag{11}$$

If $v = 0$, then (11) reduces to the case without an over-allotment option as defined by (6). To understand the effect of the over-allotment option on the offer price, I take the derivative of P_0^{ov} with respect to v to find:

Proposition 3. *Given an exogenous number of shares X, stochastic demand, and an over-allotment option v, the investment bank chooses an offer price, P_0^{ov}, that is decreasing with the size of the over-allotment option, v.*

As shown in Appendix 2, the offer price is decreasing in the over-allotment percent, v, when the percent of proceeds to the investment bank is less than 75%, the over-allotment option is less than 100%, and the quantity selected is less than the minimum demand at a price of zero. All of these conditions bound the the percent of proceeds, f, the over-allotment option, v, and the quantity chosen is well within any plausible real world standard.

To depict the economic implications of the Proposition 3, I parameterize the model as follows: $\alpha = 90$, $\beta = 110$, $X = 80$, $f = .07$, $FC = 32$. In Figure 3, I plot the investment bank's optimal price against the over-allotment percent as specified by the contract. The graph shows that the optimal offer price is decreasing in the over-allotment percentage and further that the curve is concave. The rate of decline, however, is small.

In Figure 4, I plot expected investment bank profit against the over-allotment percent, v. The graph shows that expected profit first increases and then decreases as the over-allotment percent, v, increases. The expected profit of the investment bank declines with an over-allotment percent above 17%, which is similar to the real world average of 15%.

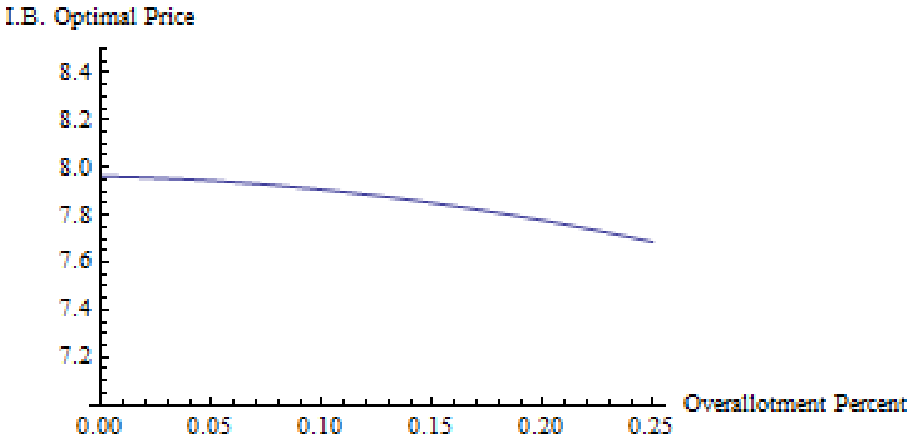

Figure 3: This figure plots the expected investment bank profit maximizing price against the over-allotment percent v. The investment bank can choose to exercise the over-allotment option by issuing up to $X(1+v)$ shares. Secondary market demand is specified by $q = \tilde{\theta} - bP$ where θ is distributed $U[\alpha, \beta]$. The model is parameterized as follows: $\alpha = 90$, $\beta = 110$, $X = 80$, $f = .07$, $FC = 32$.

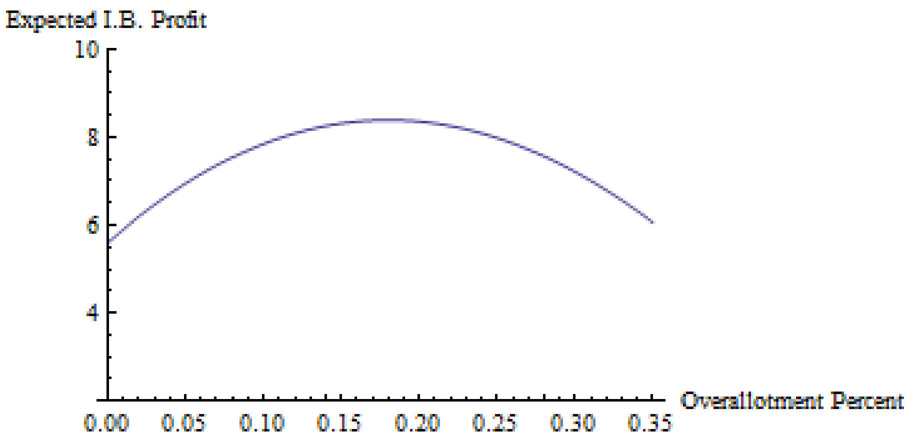

Figure 4: This figure plots the investment bank's expected profit against the over-allotment percent v. The investment bank can choose to exercise the over-allotment option by issuing up to $X(1+v)$ shares. Secondary market demand is specified by $q = \tilde{\theta} - bP$ where θ is distributed $U[\alpha, \beta]$. The model is parameterized as follows: $\alpha = 90$, $\beta = 110$, $X = 80$, $f = .07$, $FC = 32$.

In Figure 5, I plot expected issuing firm expected proceeds against the over-allotment percent, v. The graph shows that expected proceeds first increases and then decreases as the over-allotment percent, v, increases. If the objective of the issuing firm is to maximize proceeds from the IPO, the results in Figure 5 support that an over-allotment option of 15% is optimal. Furthermore, an over-allotment of 15% increases both the expected profitability of the investment bank and the expected proceeds to the issuing firm. This implies the use of an over-allotment option benefits both issuing firm and investment bank.

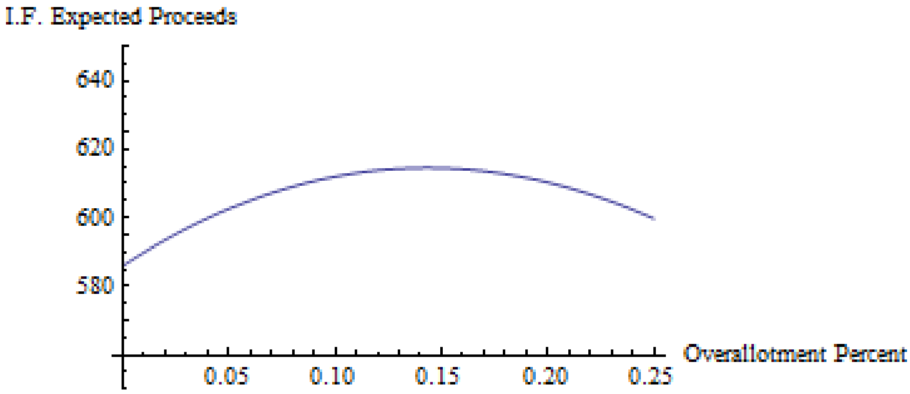

Figure 5: This figure plots the issuing firm's expected proceeds against the over-allotment option percent v. The investment bank can choose to exercise the over-allotment option by issuing up to $X(1+v)$ shares. Secondary market demand is specified by$q = \tilde{\theta} - bP$ where θ is distributed $U[\alpha, \beta]$. The model is parameterized as follows: $\alpha = 90$, $\beta = 110$, $X = 80$, $f = .07$, $FC = 32$.

3.4 The Effect of Price Elasticity of Demand on the Pricing Decision

In this section, I investigate the relationship between underpricing and the elasticity of demand. The elasticity of demand is

$$\begin{aligned} E[\varepsilon_{q,p}] &= \frac{E[dq/dP]}{q/P} \\ &= \frac{-bP_0^*}{E[\theta] - bP_0^*} \end{aligned} \tag{12}$$

Equation (12) shows expected elasticity depends on both the slope coefficient of the demand curve, b, and the offer price chosen by the investment bank. Also, equation (3) shows the expected market clearing price depends on b and equation (6) shows the optimal offer price depends on b. As a result, for a given issue size, the expected clearing price, optimal offer price, and elasticity all depend on b. In Appendix 2, I show that for any given b, the expected underpricing and elasticity are constants. Therefore, underpricing is a result of demand uncertainty and is not influenced by either the slope coefficient b or demand curve elasticity. As the demand curve becomes more elastic (i.e. demand curve shifts out due to b decreasing), the investment bank adjusts P_0^* so that they expect the same underpricing and elasticity.

Proposition 4. *Given an exogenous number of shares X and stochastic demand, the investment bank chooses an offer price, P_0^*, with the same expected elasticity and underpricing for any demand slope coefficient b. Further, the investment bank's optimal offer price decreases with the slope coefficient of demand b.*

To depict Proposition 4, I parameterize the model as follows: $\alpha = 90$, $\beta = 110$, $X = 80$, $f = .07$, and $FC = 32$. Demand is specified by $q = \tilde{\theta} - bP$. If $b = 2$ and $P = 10$, then $\varepsilon_{q,p} = -.25$; implying inelastic demand (ie. a 100% increase in price results in a 25% decrease in demand). I solve for the price that maximizes the expected profit of the investment bank as a function of b. I also solve for the expected clearing prices as a function of b. In Figure 6, I plot the relationship between the investment bank's expected profit maximizing offer price and the coefficient associated with price, b. This formalizes the argument of Miller (1977)[2], who reasoned that, for a given issue size, the investment bank will set a progressively higher offer price as the slope, $1/b$ of the inverse demand curve increases. In this example, to clear $X = 80$ shares, the investment bank sets an increasingly higher price as b decreases.

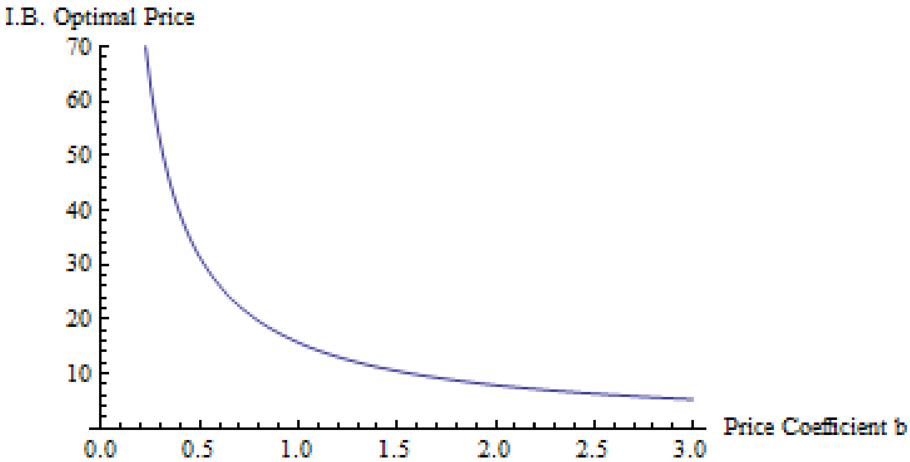

Figure 6: This figure plots the profit maximizing investment bank price against the coefficient b from the demand equation, $q = \tilde{\theta} - bP$ where θ is distributed $U[\alpha, \beta]$. The model is parameterized as follows: $\alpha = 90$, $\beta = 110$, $X = 80$, $f = .07$, $FC = 32$.

At $b = 1$ the investment bank's expected profit maximizing offer price is $P_0 = 15.93$, the expected elasticity of demand is $E[\varepsilon_{q,p}] = -.189$, the expected market clearing price is 20 and expected underpricing is 27.6%. At $b = 3$ the investment bank's expected profit maximizing offer price is $P_0 = 5.30$ and the expected clearing price is 6.66; however, both the elasticity of demand and expected underpricing remain at the same levels of $-.189$ and 27.6% respectively. In this model, for b such that $0 < b < \theta - X$, the investment bank chooses an optimal offer price such that both the expected elasticity and underpricing are constants, ceterus paribus. These

[2]Miller exposues his arguments using inverse demand where price is a function of quantity.

results imply it is the uncertainty of demand and not the steepness of the demand curve that causes underpricing.

4 The Effect of Market Shocks and Bubbles on the Pricing Decision of the Investment Bank

To model the effect of changes to IPO pricing from market variables over the filing period, I add an exogenous demand shock $\epsilon = \frac{\delta}{2}(\beta - \alpha)$ where δ is a scaling factor. There are two possible demand states. In a high demand state, $S = H$, both the lower and and upper bounds of the distribution of $\tilde{\theta}$ shift up by $\epsilon = \frac{\delta}{2}(\beta - \alpha)$. Likewise in a low demand state, $S = L$, both the lower and upper bounds of the distribution of $\tilde{\theta}$ shift down by $\epsilon = \frac{l\delta}{2}(\beta - \alpha)$. If the high demand state occurs, $S = H$, then the distribution of $\tilde{\theta}$ changes from $U[\alpha, \beta]$ to $U[\alpha + \delta(\frac{\beta-\alpha}{2}), \beta + \delta(\frac{\beta-\alpha}{2})]$. If the low demand state occurs, $S = L$, then the distribution of $\tilde{\theta}$ changes from $U[\alpha, \beta]$ to $U[\alpha - l\delta(\frac{\beta-\alpha}{2}), \beta - l\delta(\frac{\beta-\alpha}{2})]$. I use $l\delta$ for the negative shock to provide asymmetry in the outcomes.

To estimate expected profit when facing observable market returns, the investment bank needs to define a subjective probability that the market returns are both applicable to the IPO and reflect the underlying economic value of the IPO firm. The investment bank estimates the subjective probability of a high state, $Prob(S = H) = \lambda$ and the subjective probability of a low state $Prob(S = L) = (1 - \lambda)$. For example, if the investment bank observes high market returns during the book-building period that are clearly applicable to the IPO and which is based on improving fundamentals, then the investment bank assigns the subjective probability, $Prob(S = H) = \lambda = 1$. Likewise, if the investment banker observes high market returns and inflated valuation ratios, then the investment bank assigns a subjective probability $Prob(S = H) = \lambda < 1$.

I define a bubble as a shift upward in the demand curve for reasons that are not based on a change in the intrinsic value of the IPO. Because the investment bank has information about the intrinsic value of the IPO, the investment bank is aware of the existence of the bubble and therefore faces a conundrum when pricing the IPO in a bubble market. If the investment bank fully incorporates the bubble price into the offer price and the bubble pops, then the probability the investment bank incurs a loss of $P_0(X - q)$ increases. Assuming the investment bank rationally evaluates the intrinsic value of the IPO, the investment bank recognizes the existence of the bubble. To price the offer, the investment bank evaluates both the subjective probability the bubble continues and the demand shocks from the continuation or implosion of the bubble. Institutional investors will also factor the bubble into their pricing evaluation.

In a bubble environment, the investment bank expects their repeat institutional clients to help stabilize the issue in the after-market. These repeat institutional investors can not flip their allocations and expect future allocations of hot IPOs.

In this two state setting there are 4 possible outcomes. The investment banks choice of P_0 affects the probability that $q < X$ in both the high and low states. As a result, the investment bank chooses P_0 to maximize expected profit as follows:

$$\begin{aligned} E[\pi] = \lambda \{ Prob(q > X|S = H)\pi_1 + Prob(q < X|S = H)E[\pi_2|(q < X) \cap (S = H)] \} \\ + (1-\lambda) \{ (Prob(q > X|S = L)\pi_1 + Prob(q < X|S = L)E[\pi_2|(q < X) \cap (S = L)] \} \end{aligned} \quad (13)$$

In Appendix 1, I derive the expected profit as function of the parameters to find:

$$\begin{aligned} E[\pi] = fP_0X - FC - \frac{(1-\lambda)P_0(X + bP_0 - \alpha + \frac{l\delta}{2}(\beta-\alpha))^2}{2(\beta-\alpha)} \\ - \frac{\lambda P_0(X + bP_0 - \alpha + \frac{\delta}{2}(\beta-\alpha))^2}{2(\beta-\alpha)} \end{aligned} \quad (14)$$

I solve for the investment bank optimal choice of P_0 in (14). Because the solution is very long and does not provide economic insight, I report the closed-form solution in Appendix 1. To explore the effect of market returns on the investment bankers offer price decision, I explore the implications where the the market returns are fully observable and applicable to the IPO. In the case of a positive observable market shock, the investment banker assigns subjective probability $Prob(S = H) = \lambda = 1$. In the case of a negative observable market shock, the investment banker assigns subjective probability $Prob(S = H) = \lambda = 1$.

To show the economic implications of positive observable market returns, I parameterize the model as follows: $\alpha = 90$, $\beta = 110$, $b = 2$, $X = 80$, $f = .07$, and $FC = 20$. In Figure 7 , I plot both the expected clearing price, P_c^e and the optimal offer price, P_0^*, as a function of the observable positive market shock, δ. The figure shows that the offer price (dashed line) increases at the same rate as the expected clearing price (solid line). The implication is that if observable market variables are a perfect signal of changes in the IPO valuation, then the investment bank fully adjusts of the offer price.

To show the economic implications of negative observable market shocks, I parameterize the model as follows: $\alpha = 90$, $\beta = 110$, $b = 2$, $X = 80$, $f = .07$, and $FC = 20$. In addition, I assume the issuing firm has a reservation price of 7 and that this reservation price is common knowledge. In Figure 8, I plot both the expected clearing price, P_c^e and the offer price, P_0^*, as a function of the observable negative market shock, $l\delta$. Figure 8 shows that the offer price (dashed line) decreases at the same rate as the expected clearing price (solid line) until the reservation price, 7, of the issuing firm is reached. For negative market shocks, $\delta > .25$, the

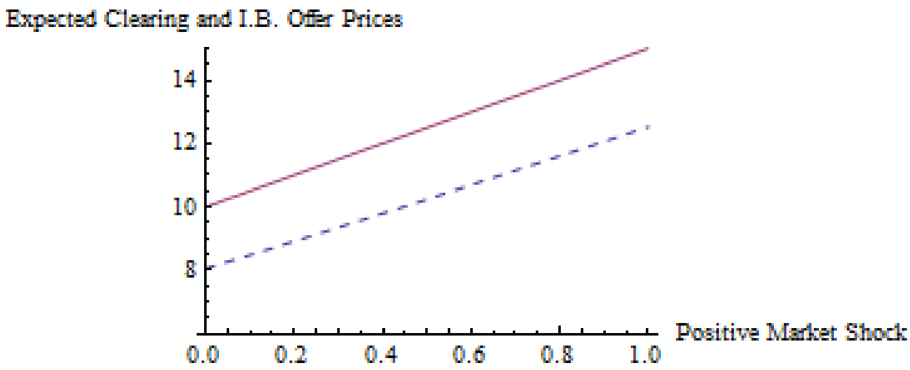

Figure 7: This figure plots both the expected clearing price (solid line) and the optimal offer price (dashed line) as a function of the observable positive market shock, δ. The model is parameterized as follows. Demand is given by $q = \tilde{\theta} - bP$ with $\tilde{\theta}$ distributed $U[90, 110]$ and $b = 2$. The issuing firm exogenously specifies the number of shares to sell, $X = 80$. The investment bank earns $f = .07$ of the proceeds, incurs $FC = 20$, and commits to placing $X = 80$ shares.

investment bank optimally would price the issue at less than 7, but can not as the issuing firm would withdraw the IPO. Therefore, the investment bank prices the offer at 7 until it reaches a zero profit condition at a negative shock of $\delta = .45$.

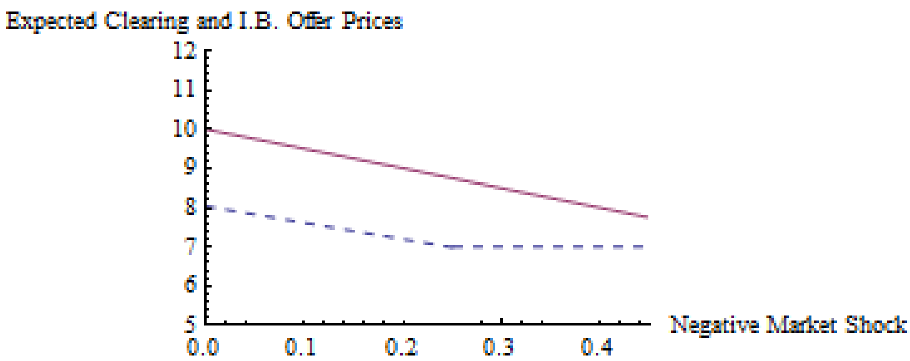

Figure 8: This figure plots both the expected clearing price (solid line) and the offer price (dashed line) as a function of the observable positive market shock, $l\delta$. The model is parameterized as follows. Demand is given by $q = \tilde{\theta} - bP$ with $\tilde{\theta}$ distributed $U[90, 110]$ and $b = 2$. The issuing firm exogenously specifies the number of shares to sell, $X = 80$. The investment bank earns $f = .07$ of the proceeds, incurs $FC = 20$, and commits to placing $X = 80$ shares. The issuing firm has a reservation price of 7.

The above examples illustrate that positive observable market variables have a limited effect on the magnitude of initial IPO returns when the investment bank's subjective probability of either a positive or negative market shock is one. Intuitively, when the investment bank is no longer certain about the applicability of the observable market variables, the range of possible outcomes increases. In fact, the highest variance occurs when the investment banker has a subjective probability of 50% for either the the high or low state occuring. This insight yields the next proposition, which I demonstrate using a simulation as shown in Table 1.

Proposition 5. *Given an exogenous number of shares, X, and stochastic demand, the investment bank chooses an offer price, P_0, such that underpricing decreases as the prior subjective probability, λ, either increases or decreases from a starting point of .5. In addition, underpricing increases with the size of the negative bubble shock, l.*

To make the proposition clearer, I parameterize the model as follows. Demand is given by $q = \tilde{\theta} - bP$ with $\tilde{\theta}$ distributed $U[90, 110]$ and $b = 2$. The issuing firm exogenously specifies the number of shares to sell, $X = 80$. The investment bank earns $f = .07$ of the proceeds, incurs $FC = 20$, and commits to placing $X = 80$ shares. I plot expected underpricing, conditional on the realization of a bubble state, against the investment bank's prior subjective probability, λ, that a bubble state occurs. Figure 9 shows that underpricing, conditional on the realization of a bubble, decreases as the subjective probability that the bubble state continues increases. At the extreme $\lambda = 1$, the investment bank is certain there is not a bubble, and the case reverts to the positive shock case. When the investment bank has a subjective probability that the bubble will pop, the investment bank hedges by further underpricing the offering, which results in apparent partial adjustment.

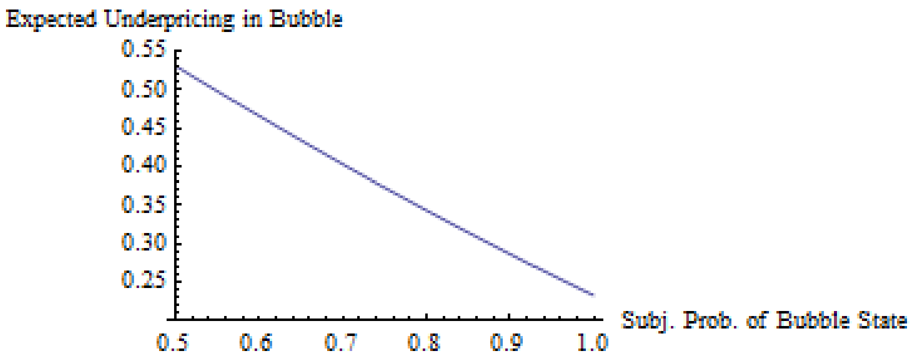

Figure 9: This figure plots expected underpricing, conditional on the realization of a bubble state, against the investment bank's prior subjective probability, λ, that a bubble state continues. The model is parameterized as follows. Demand is given by $q = \tilde{\theta} - bP$ with $\tilde{\theta}$ distributed $U[90, 110]$ and $b = 2$. The issuing firm exogenously specifies the number of shares to sell, $X = 80$. The investment bank earns $f = .07$ of the proceeds, incurs $FC = 20$, and commits to placing $X = 80$ shares. The positive bubble demand shock is specified by $\delta = .25$ with a negative bubble shock $l = 2$ times δ. I solve for the investment bank's profit maximizing offer price and then compute underpricing, conditional on the realization of a bubble, as a function of the prior subjective probability, λ.

In Figure 10, I plot the expected underpricing, conditional on the realization of a bubble state, against δ and l. The parameter δ specifies the magnitude of an upward shift in the

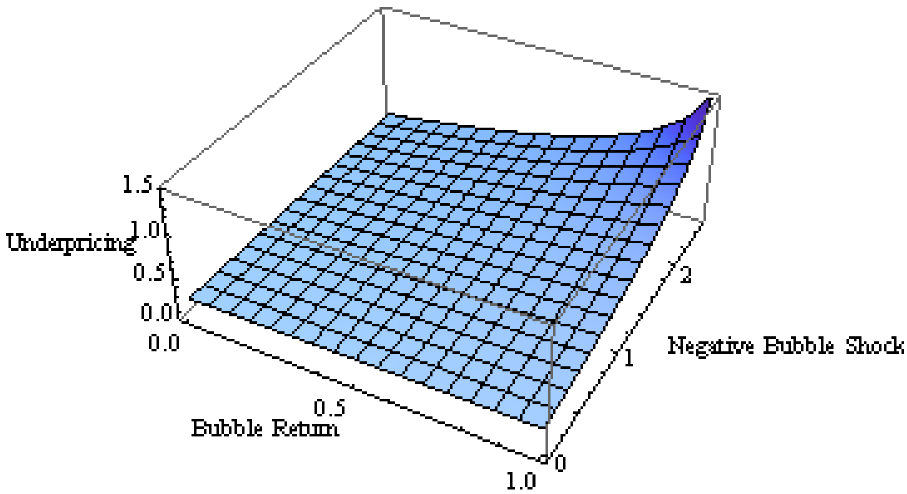

Figure 10: This figure plots the expected underpricing, conditional on the realization of a bubble state, against δ and l. The parameter δ specifies the magnitude of an upward shift in the demand curve in the bubble state whereas the parameter l specifies the magnitude that the downward shift in the demand curve exceeds the upward shift if the bubble pops. The model is parameterized as follows. Demand is given by $q = \tilde{\theta} - bP$ with $\tilde{\theta}$ distributed $U[90, 110]$ and $b = 2$. The issuing firm exogenously specifies the number of shares to sell, $X = 80$. The investment bank earns $f = .07$ of the proceeds, incurs $FC = 20$, and commits to placing $X = 80$ shares. The prior subjective probability of the investment bank that the bubble continues is $\lambda = .80$. I solve for the investment bank's profit maximizing offer price and then compute underpricing, conditional on the realization of a bubble, as a function of both δ and l.

demand curve in the bubble state whereas the parameter l specifies the magnitude that the downward shift in the demand curve exceeds the upward shift if the bubble pops. I set $\lambda = .80$. Figure 10 shows that underpricing, conditional on the realization of a bubble, increases in both the bubble return and the size of the negative bubble shock. With $\lambda = .80$, the investment bank believes there is only a 20% probability the bubble will pop. Even with this modest probability, the investment bank hedges by setting the offer price lower as both the bubble size and shock increases.

5 Empirical Predictions

The model distinguishes between differences of opinion and valuation uncertainty. If during the book-building period the investment bank reduces differences of opinion, b, among investors then the investment bank increases the offer price. For example, if the investment bank convinces each investor to use the same valuation method and matches, then the investment bank fully eliminates differences of opinion between investors; nonetheless, if the common valuation ratios have the same volatility as the prior ratios then each investors valuation uncertainty remains the same. In this example, the offer price increases but the expected underpricing remains the same.

Prediction 1. Underpricing is a profit maximizing strategy for an investment bank given demand uncertainty and a firm commitment contract.

Ritter and Welch (2002) find that from 1980-2001 the average first trading day return of initial public offerings is 18.8% and that underpricing is an empircial regularity in all countries. At the end of the book-building period, investment banks are in a strong bargaining position relative to the issuing firm and, as a result, strongly influence the choice of the offer price. Under this assumption, underpricing is consistent with a profit maximizing strategy of the investment bank.

The ability of the investment banks to choose the offer price is plausible, but not required, for underpricing. Under my model, investment banks may underprice in expectation, when they operate in a competitive market (i.e. they earn zero economic profit in expectation). I illustrate this scenario in Figure 1. The expected clearing price is 10. With fixed costs set to 32, the investment bank breaks even in expectation at a offer price of $P_0 = 9.85$, which is below the expected clearing price. Muscarella and Vetsuypens (1989) find that self-underwritten IPOs have positive average initial returns. Because firms often set internal transfer prices to produce zero economic profit, this study also shows that even if the case of zero expected economic profit the investment bank may price below the expected clearing price.

Prediction 2. IPO initial return increases as valuation uncertainty increases.

Lowry et al. (2010) explore the relationship between initial return and the volatility of initial returns. In their sample of IPOs from 1965 - 2005 , they find an 87.7% correlation between average monthly IPO initial return and the monthly standard deviation of initial returns. They define several variables related to valuation uncertainty and show a strong relationship between the variables associated with valuation uncertainty and the time series of both initial return and the volatililty of initial returns. They interpret book-building as a mechanism to reduce uncertainty and the consequential average high initial returns associated with uncertainty. My model also supports the idea that if an investment bank reduces valuation uncertainty during the bookbuilding period, then the investment bank increases the offer price and initial return decreases in expectation. As a result, the expected proceeds to the issuing firm and the profit to the investment bank

Prediction 3. The expected profit maximizing offer price of the investment bank decreases with the size of over-allotment option, v.

Kutsuna et al. (2009) explore the price formation process in using a sample 487 book-built IPOs in Japan from 1997-2003. Prior to 2002 the over-allotment option was not allowed in Japan. They create two dummy variables to test the impact of the over-allotment option on two pricing revisions and initial return. First, they set a dummy to one for all IPOs within the period where the over-allotment option is allowed. Second, they set a dummy to one if the specific IPO uses an over-allotment option. Both dummies are negatively related to each of the two separate offer revisions, but only the coefficient associated with the period dummy is statistically significant in explaining the revision from the first estimated price to the midpoint of the initial filing range. Overall, the empirical evidence suggests the use of the over-allotment option has a small negative effect on the offer price. These results are consistent with my model results as depicted in Figure 3, which shows a small negative relationship between price and the over-allotment percent, v. In my parameterized example, the offer price declines from approximately 7.95 to 7.85 as the over-allotment percent changes from zero to 15%.

Prediction 4. Because the expected issuing firm proceeds and investment bank profit are concave in the percent over-allotment option, v; there is an optimal size of the over-allotment for both the issuing firm and investment bank. Because the use of an over-allotment option is in the joint interest of the issuing firm and investment bank, the over-allotment option should be widely implemented in practice.

Using the sample 6,351 IPOs between 1981-2007[3], I find that only 75 don't use an over-allotment option. Further, the 75 IPOs that don't use the over-allotment option tend to be clustered early in the sample period with 55 IPOs not using the option in the 1980's. The near ubiquitous use of the option supports the perspective that the option benefits both the issuing firm and investment bank.

The empirical literature on the use of over-allotment options tends to focus on the benefits to the investment bank. Aggarwal (2003) using a sample of 114 IPOs details the use of the over-allotment option as a mechanism for after-market price support. First, the investment bank oversells the issue. If the demand is not high, the investment bank closes out its short position by purchasing shares. If demand is high, then the investment bank executes the over-allotment option to deliver the over-sold shares. Using a sample of 306 NASDAQ IPOs, Ellis et al. (2000) show how the use of the over-allotment option as a mechanism for price support generates trading fees. Because investment banks operate as a market makers on the NASDAQ, they gain trading fees when they short cover for low demand IPOs. These empirical studies clearly show that the over-allotment option benefits investment banks. To my knowledge there is not an empirical study that shows how the option benefits issuing firms; yet, it is hard to posit

[3]See Section **??** for a discussion of the sample.

an industry structure where the over-allotment option is ubiquitously used but only benefits investment banks.

Prediction 5. Investment banks increase the offer price if the absolute value of the slope coefficient of the demand curve b decreases (i.e. demand becomes more elastic).

Kandel et al. (1999) analyze 27 Israeli IPOs conducted using a uniform price auction. The offer price is set based on the clearing price from the auction. In this auction setting, investors ex-ante don't have access to demand elasticity information; however, investors ex-post can infer elasticity from information released by the investment bank after the auction. Kandel et al. (1999) find a positive correlation between initial return and elasticity. If the investors observe ex-post that demand for the offer was elastic they bid up the price. In contrast to the auction results, where investors recognize demand elasticity ex-post, in book-built IPOs, investment banks can estimate demand elasticity ex-ante and price the issue accordingly. In my model, the investment bank increases the offer price if their estimation of the stochastic demand curve becomes more elastic (i.e. b decreases). Because the investment bank pro-actively adjusts the offer price, the expected initial return does not change.

Prediction 6. Investment banks increase the offer price if the demand curve shifts out.

The empirical evidence is consistent with the idea investment bankers influence valuations. Purnanandam and Swaminathan (2004) find that IPOs are overvalued relative to industry ratios. Cogliati et al. (2008) analyze European IPOs in which the investment bank includes valuation estimates in the prospectus. They reverse engineer the valuation models to uncover the implied cash flow growth rates. They compare the implied cash flow growth rates to actual realizations and find the implied cash flow growth rates exceed actual realizations. However, these studies do not distinguish between whether the investment bank increases prices in reaction to high demand states or strategically influences prices.

Kim and Ritter (1999) write,

> In practice, investment bankers who suspect that they have a hot deal on their hands may be tempted to choose comparables with high multiples to justify a high price. Furthermore, they will generally pick comparables that will not make the IPO look overpriced.

To my knowledge there is not an empirical study that shows investment bankers strategically influence comparables, but my model indicates they have an incentive to engage in this activity.

Prediction 7. The investment bank fully adjusts the offer price using observable information, conditional on that information representing a perfect signal of the underlying value of the IPO issuing firm's stock.

Cook et al. (2006) posit that news stories about the IPO firm recruit sentiment investors and find that news stories during the six months prior to the offering helps explain IPO initial returns. Under my model, there are two effects. First, the recruitment of new investors through news stories shifts upward the demand curve. Second, the new sentiment investors increase valuation uncertainty as these investors value the IPO differently than the institutional investors. Under these conditions, my model predicts that the investment banks adjust upward the offer price due to the shift in demand, but adjust downward by the increase in volatility of demand. Keefe (2010) finds that the natural log of news articles in the six months prior to the IPO is positively associated with investment bank offer price revisions and IPO initial returns.

There are several studies that investigate the relationship between observable information and IPO initial returns. My model predicts investment banks fully adjust offer prices using observable information only if the information is a perfect signal of the underlying value. Bradley and Jordan (2002) find that public information predicts 35% to 50% of the variation in IPO intitial returns; however, Lowry and Schwert (2004) conclude investment banks almost fully adjust offer prices using observable public information. Keefe (2010) shows that investment banks only partially adjust offer prices upward based on industry returns during the filing period. In the context of my model, if comparable market returns are not a perfect signal of the underlying change in value (ie. $\lambda \neq 1$) then investment banks partially adjust for comparable industry returns. The empirical evidence supports this interpretation.

Prediction 8. Investment banks increase underpricing in a bubble based upon their subjective probability the bubble pops and the magnitude of the expected negative shock. The subjective probability can be interpreted as the investment bankers assessment about the quality of observable market variables.

Shiller (2006, pg. 129) writes,

> Another article, in Barron's, "Burning Up" by Jack Willoughby, included a ranking of Internet companies who were losing money by the number of months until they had burned through all their cash. Willoughby's idea of ranking Internet companies this way made these companies' problems suddenly vivid and clear, and was eminently quotable. His article was a bombshell that led to the kind of skeptical talk that can in turn lead to the undoing of stocks.

When articles like this are published, the investment bank faces a difficult choice. If the investment bank prices the issue at the intrinsic value of the issuing firm and the bubble doesn't pop, the investment bank can claim they were right a few years hence, but in the interim they lose market share. In this model, as the information about the likelihood of the bubble bursting is observable (e.g. burn rate article) the investment bank lowers their subjective probability that the bubble continues.

A bubble may occur when sentiment investors value the IPO above its intrinsic rational valuation. Kaustia and Knupfer (2008) analyze the allocations to individual investors for 57 IPOs in Finland between 1995 and 2000. They categorize each investors' initial IPO experience as hot or cold. An investor with an initial hot IPO experience is more than twice as likely as an investor with an initial cold IPO experience to purchase a subsequent IPO. Kaustia and Knupfer (2008) interpret these findings as supporting reinforcement learning. This behavioral bias explains how sentiment investors, who experience positive returns in the first IPO purchase, may value the IPO differently than institutional investors. Cornelli et al. (2006) analyze the effect of grey-market prices on the subsequent trading behavior of institutions and long run IPO firm returns. In European grey-markets, investors speculate on the future IPO price. When the grey-market price exceeds the midpoint of the filing range, Cornelli et al. (2006) find (1) institutional investors spin their shares and (2) that the long run IPOs returns are lower than benchmark returns. Consistent with my model, Cornelli et al. (2006) also find that that only a portion of the grey-market price is incorporated into the issue price.

Whereas Kaustia and Knupfer (2008) show how individual investors form biased valuations of IPO firms due to a reinforced learning bias, Cornelli et al. (2006) show how investment banks strategically price based upon the direction of this bias. Ofek and Richardson (2003) document the institutional features that allow a bubble to continue. Specifically, they argue that short sales constraints, low institutional ownership and a low float prevented the opinions of pessimistic investors from affecting prices. As the bubble matured, lock-up agreements expired insiders sold their shares. Amazingly, by August 2000 over 80 billion dollars worth of IPO, SEO, and insider sales for 327 firms had been sold to the public.

The research reviewed above supports the idea of investor sentiment influencing IPO issuance; however, Derrien and Kecskes (2009) find that investor sentiment matters little relative to industry fundamentals in explaining issuance activity. They analyze 631 firms in the Canadian petroleum industry and find that fundamentals such as the number of rigs, wells drilled, etc. explain 40% of issuance activity versus only 10% for sentiment. They also take a sub-sample of the firms that sell to individual investors and find that fundamentals continues to explain issuance activity. These results are consistent with Keefe (2010) who finds omitted variables, such as investor sentiment, have the largest economic effect in the bubble on IPO intial return. Based on these studies it appears the affect of sentiment on equity issuance and initial returns is large only during periods where valuations uncertainty is substantial.

6 Robustness

In this section, I evaluate the effect of a different cost function on the model predictions. Specifically, in this section I assume that if realized demand, q, is less than the shares issued, X, the investment bank immediately sells the shares at the clearing price. Under this assumption, the cost function is $(P_0 - P_{q<X})(X - q)$. Ex-post the clearing price, $P_{q<X}$, is based on the realized demand. Ex-ante the expected clearing price, conditional on $q < X$, is $P^e_{q<X} = \frac{1}{2b}(\alpha - X) + \frac{P_0}{2}$ and the expected profit to the investment bank is

$$E[\pi] = fP_0X - FC - \frac{\frac{1}{2}(P_0 - P^e_{q<X})(X + bP_0 - \alpha)^2}{\beta - \alpha} \tag{15}$$

I substitute $P^e_{q<X} = \frac{1}{2b}(\alpha - X) + \frac{P_0}{2}$ into equation (15) to find

$$E[\pi] = fP_0X - FC - \frac{\frac{1}{4}(P_0 - \frac{\alpha - X}{b})(X + bP_0 - \alpha)^2}{\beta - \alpha} \tag{16}$$

The investment bank chooses the offer price, P_0 to maximize expected profit as defined in equation (15). I solve for the expected profit maximizing offer price to find

$$P_0^* = \frac{1}{3b}(\sqrt{12fX(\beta - \alpha)} - 3X + 3\alpha) \tag{17}$$

Because the investment bank resells the shares in the case that $q < X$ both the expected profit equation (16) and the optimal offer price equation (17) differ slightly from equations (5) and (6), respectively. Specifically, in the numerator of the third term of the expected profit equations an expression changes from P_0 in equation (5) to $P_0 - \frac{\alpha - X}{b}$ in (16). The $\frac{\alpha - X}{b}$ represents the reduction in loss relative to the original model the investment bank achieves by selling the shares on the open market. Interestingly, this reduction is inversely related to the slope coefficient b, where a relatively large b (i.e. more elastic demand) allows the investment bank to sell the shares at a price relatively closer to the offer price.

I find that all of the predictions hold with two exceptions. First, under some conditions it is optimal for the investment bank to over-price. These conditions occur when there is a relatively high share quantity, low uncertainty, and high investment banking fees. Second, at low levels of uncertainty the optimal offer price is an increasing function of demand uncertainty; however, this case is plausibly bounded by the region where it is optimal to overprice the issue. Therefore, outside of a special situation of an issue with high share quantity, high investment banking fees, and low uncertainty, all the predictions of my model hold. I review below the effect on the two model predictions that are affected by the change in the cost function.

Prediction 1. Underpricing is a profit maximizing strategy for an investment bank given demand uncertainty and a firm commitment contract.

Using the new cost function, underpricing is a profit maximizing strategy when the expected offer price is less than the expected clearing price. This holds when shares issued, $X < \frac{3}{16}(\frac{\beta-\alpha}{f})$. This implies the investment bank has the incentive to over-price when $X > \frac{3}{16}(\frac{\beta-\alpha}{f})$, which represents a upper bound on the number of shares to issue such that it is in the investment bank's interest to underprice. Two variables effect the bound. First, if $\beta - \alpha$ increases the bound increases. Second, as the fee, f, increases the lower bound decreases. The formula shows that in cases with high fees, f, and low uncertainty, $\beta - \alpha$, the investment bank is motivated to over-price the issue. In my model I assumed the institutional investors and institutions share the same information set so by individual rationality of the institutional investors, the investment bank could not charge above the expected clearing price.

Prediction 2. IPO initial return increases as valuation uncertainty increases.

The optimal offer price, P_0 is a decreasing function of demand uncertainty, σ_q, when $\sigma_q > \frac{6}{\sqrt{3}fX}$. At low levels of demand uncertainty when $\sigma_q < \frac{6}{\sqrt{3}fX}$, the investment bank increases the offer price as uncertainty increases. Above, I show that at low levels of uncertainty, the investment bank has the incentive to over-price. The region where P_0 is an increasing function of σ_q overlaps with the region that the investment bank over-prices and is smaller whenever $fX > \frac{3}{2}$. This region is very small and does not materially affect the results of the main model.

7 Conclusion

I model the pricing decision of an investment bank that manages a book-built IPO and faces a stochastic downward sloping demand curve that arises from both differences of opinion and valuation uncertainty. I extend the model to analyze the effect on the pricing decision of the investment bank due to effort during book-building, the use of an over-allotment option, observable market returns, and asset pricing bubbles. To my knowledge this is the first paper to theoretically model both valuation uncertainty and differences of opinion to explain the IPO pricing decision.

As in Miller (1977), I interpret the slope coefficient of the demand curve as representing differences of opinion. I find that the investment bank increases the price as demand becomes more elastic, but that underpricing is not affected by changes in differences of opinion. Effectively, if the investment bank has knowledge of a deterministic demand curve, even if that demand curve is steep, the investment bank would price at the expected clearing price. Furthermore, for any slope coefficient, the investment bank sets the offer price with the same expected demand elasticity and expected underpricing. This insight leads to a re-interpretation of book-building where the investment bank works to simultaneously resolve uncertainty, reduce differences of opinion to increase demand curve elasticity, market the IPO to recruit new investors, and interpret observable market signals.

My model predicts several empirical regularities: average positive initial returns, the ubiquitous use of the over-allotment option, partial adjustment to observable variables, and large positive initial returns in a bubble. The model provides a means to interpret the actions of investment banks during the filing process. For example, news stories about the IPO firm attract potential new investors, many of which are plausibly sentiment investors. My model shows two effects of news stories on the stochastic demand curve. First, the new investors shift out the demand curve. Second, the new investors, many of whom are sentiment investors, increase valuation uncertainty. The investment bank raises the offer price due to the shift in demand caused by news stories, but only partially because of the increase in valuation uncertainty. Likewise, observable positive industry market returns also shifts out the stochastic demand curve; yet, the investment bank partially adjusts if the shift is not a perfect signal of the underlying value of the IPO firm. Lastly, in an asset pricing bubble the investment banks must account for the possibility of price supporting the IPO in the after-market for upwards of 30 days. During this period there is the risk of the asset prices deflating, which creates additional valuation uncertainty. Therefore, the investment bank again only partially adjust upward the offer price for sentiment demand. Using a simple set of assumptions, my model provides predictions that are consistent with empirical regularities and help interpret the actions of the investment banks.

Because my model distinguishes between shifts in demand, uncertainty, and differences of opinion, it provides normative insights about IPO mechanisms. For example, whereas an auctioned IPO is best at extracting private valuations, and thereby lowering initial return, a book-built IPO may be the best at marketing the issue. If through marketing efforts the investment bank attracts additional investors, the demand curve will simultaneously shift outward and valuation uncertainty increase. Hence, the investment bank will adjust upward the offer price due to the shift in demand, but only partially due to the increased valuation uncertainty. Therefore, the efficacy of book-built versus auction IPOs, at least from the perspective of the issuing firm, can not easily be judged by comparing initial returns as argued by Derrien and Womack (2003), Degeorge et al. (2010), Lowry et al. (2010).

Appendices

A Derivations

Derivation 1. *Derivation of the expected profit of the investment bank from Section 3.1*

I substitute (1) and (2) into (4) to obtain:

$$\begin{aligned} E[\pi] = &\, Prob(q \geq X)(fP_0X - FC) \\ &+ Prob(q < X)(fP_0X - FC - P_0(X - E[q|q < x])) \end{aligned} \tag{18}$$

where

$$\begin{aligned} Prob(q \geq X) &= Prob(\tilde{\theta} - bP_0 \geq X) \\ &= Prob(\tilde{\theta} \geq X + bP_0) \\ &= \frac{\beta - X - bP_0}{\beta - \alpha} \end{aligned} \tag{19}$$

and

$$\begin{aligned} Prob(q < X) &= Prob(\tilde{\theta} - bP_0 < X) \\ &= Prob(\tilde{\theta} < X + bP_0) \\ &= \frac{X + bP_0 - \alpha}{\beta - \alpha} \end{aligned} \tag{20}$$

and

$$\begin{aligned} E[q|q < X] &= E[\theta - bP_0|\tilde{\theta} - bP_0 < X] \\ &= E[\theta|\tilde{\theta} < X + bP_0] - E[bP_0|\tilde{\theta} < X + bP_0] \\ &= \frac{1}{2}(\alpha + X + bP_0) - bP_0 \\ &= \frac{1}{2}(\alpha + X - bP_0) \end{aligned} \tag{21}$$

From(18), I factor out fP_0X and FC and substitute (20) and (21) to find:

$$\begin{aligned} E[\pi] &= fP_0X - FC - Prob(q < X)P_0(X - E[q|q < X]) \\ &= fP_0X - FC - \frac{P_0(X + bP_0 - \alpha)(X - \frac{1}{2}(X - bP_0 + \alpha))}{\beta - \alpha} \\ &= fP_0X - FC - \frac{\frac{1}{2}P_0(X + bP_0 - \alpha)^2}{\beta - \alpha} \end{aligned} \tag{22}$$

Derivation 2. *Derivation of the expected profit of the investment bank with an over-allotment option from Section 3.3*

The investment bank's chooses P_0 to maximize the expected profitability as denoted by:

$$\begin{aligned} E[\pi] = & Prob[q \leq X]P_0Xf - FC - P_0(X - E[q| \leq X]) \\ & + Prob[X < q < X(1+v)]P_0E[q|X < q < X(1+v)]f - FC \\ & + Prob[q > X(1+v)]P_0X(1+v)f - FC \end{aligned} \tag{23}$$

I re-write the expected profit equation (23) in a form which illustrates the expected revenue and costs as follows:

$$\begin{aligned} E[\pi] = & P_0f(X + Prob[X < q < X(1+v)](E[q|X < q < X(1+v)] - X) + Prob[q > X(1+v)]Xv) \\ & - FC - Prob[q \leq X]P_0(X - E[q| \leq X]) \end{aligned} \tag{24}$$

where

$$\begin{aligned} Prob(q \leq X) &= Prob(\tilde{\theta} - bP_0 \leq X) \\ &= \frac{X + bP_0 - \alpha}{\beta - \alpha} \end{aligned} \tag{25}$$

and

$$\begin{aligned} Prob[X < q < X(1+v)] &= Prob(X < \tilde{\theta} - bP_0 < X(1+v)) \\ &= Prob(X + bP_0 < \tilde{\theta} < X(1+v) + bP_0) \\ &= \frac{Xv}{\beta - \alpha} \end{aligned} \tag{26}$$

and

$$\begin{aligned} Prob[q \geq X(1+v)] &= Prob(\tilde{\theta} - bP_0 \geq X(1+v)) \\ &= Prob(\tilde{\theta} \geq X(1+v) + bP_0) \\ &= \frac{\beta - X(1+v) - bP_0}{\beta - \alpha} \end{aligned} \tag{27}$$

and

$$\begin{aligned} E[q|q < X] &= E[\theta - bP_0|\tilde{\theta} - bP_0 < X] \\ &= E[\theta|\tilde{\theta} < X + bP_0] - E[bP_0|\tilde{\theta} < X + bP_0] \\ &= \frac{1}{2}(\alpha + X - bP_0) \end{aligned} \tag{28}$$

and

$$\begin{aligned} E[q|X < q < X(1+v)] &= E[\theta - bP_0|X < \tilde{\theta} - bP_0 < X(1+v)] \\ &= E[\theta|X + bP_0 < \tilde{\theta} < X(1+v) + bP_0] - bP_0 \\ &= \frac{1}{2}(X(1+v) + X) \\ &= X + \frac{1}{2}v \end{aligned} \tag{29}$$

I substitute (25), (26), (27), (28), and (29) into (23) and simplify to find:

$$E[\pi] = fP_0(\frac{X + vX(\beta - X(1+\frac{1}{2}v) - bP_0)}{\beta - \alpha}) - FC - \frac{P_0(X + bP_0 - \alpha)^2}{2(\beta - \alpha)} \tag{30}$$

Derivation 3. *Derivation of the expected profit of the investment bank with observable market shocks and an asset pricing bubble from Section 4*

The investment bank's chooses P_0 to maximize the expected profitability as denoted by:

$$\begin{aligned} E[\pi] = &\lambda \{Prob(q > X|S = H)\pi_1 + Prob(q < X|S = H)E[\pi_2|(q < X) \cap (S = H)]\} \\ &+ (1-\lambda)\{(Prob(q > X|S = L)\pi_1 + Prob(q < X|S = L)E[\pi_2|(q < X) \cap (S = L)]\} \end{aligned} \tag{31}$$

By substituting (1) and (2) into (31) and factoring out FC and eP_0X, I find

$$\begin{aligned} E[\pi] = &P_0X(f - e) - FC - \lambda Prob(q < X|S = H)P_0(X - E[q|(q < X) \cap (S = H)) \\ &- (1-\lambda)Prob(q < X|S = L)P_0(X - E[q|(q < X) \cap (S = L)) \end{aligned} \tag{32}$$

where

$$\begin{aligned} Prob(q > X|S = H) &= Prob(\tilde{\theta} - bP_0 \geq X|S = H) \\ &= Prob(\tilde{\theta} \geq X + bP_0|S = H) \\ &= \frac{\beta - X - bP_0 + \delta(\frac{\beta-\alpha}{2})}{(\beta - \alpha)} \end{aligned} \tag{33}$$

$$\begin{aligned} Prob(q < X|S = H) &= Prob(\tilde{\theta} - bP_0 < X|S = H) \\ &= Prob(\tilde{\theta} < X + bP_0|S = H) \\ &= \frac{X + bP_0 - \alpha - \delta(\frac{\beta-\alpha}{2})}{(\beta - \alpha)} \end{aligned} \tag{34}$$

$$
\begin{aligned}
Prob(q > X|S = L) &= Prob(\tilde{\theta} - bP_0 \geq X|S = L) \\
&= Prob(\tilde{\theta} \geq X + bP_0|S = L) \\
&= \frac{\beta - X - bP_0 - \delta l(\frac{\beta-\alpha}{2})}{(\beta - \alpha)}
\end{aligned} \tag{35}
$$

$$
\begin{aligned}
Prob(q < X|S = L) &= Prob(\tilde{\theta} - bP_0 < X|S = L) \\
&= Prob(\tilde{\theta} < X + bP_0|S = L) \\
&= \frac{X + bP_0 - \alpha + \delta l(\frac{\beta-\alpha}{2})}{(\beta - \alpha)}
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
E[q|(q < X) \cap (S = H)] &= E[\theta - bP_0|(\tilde{\theta} - bP_0 < X) \cap (S = H)] \\
&= E[\theta|(\tilde{\theta} < X + bP_0) \cap (S = H)] - bP_0 \\
&= \frac{1}{2}(\alpha + \delta(\frac{\beta - \alpha}{2}) + X + bP_0) - bP_0 \\
&= \frac{1}{2}(\alpha + X - bP_0) + \delta(\frac{\beta - \alpha}{4})
\end{aligned} \tag{37}
$$

$$
\begin{aligned}
E[q|(q < X) \cap (S = L)] &= E[\theta - bP_0|(\tilde{\theta} - bP_0 < X) \cap (S = L)] \\
&= E[(\theta|\tilde{\theta} < X + bP_0) \cap (S = P)] - bP_0 \\
&= \frac{1}{2}(\alpha - \delta l(\frac{\beta - \alpha}{2}) + X + bP_0) - bP_0 \\
&= \frac{1}{2}(\alpha + X - bP_0) - \delta l(\frac{\beta - \alpha}{4})
\end{aligned} \tag{38}
$$

$$
\begin{aligned}
E[\theta|S = H] &= \frac{1}{2}(\alpha + \delta(\frac{\beta - \alpha}{2}) + \beta + \delta(\frac{\beta - \alpha}{2})) \\
&= \frac{1}{2}((\alpha + \beta) + \delta(\beta - \alpha))
\end{aligned} \tag{39}
$$

which implies the expected clearing price in a bubble is $P^e_{bubble} = \frac{1}{2b}[(\alpha + \beta) + \delta(\beta - \alpha) - 2X]$

$$
\begin{aligned}
E[\theta|S = L] &= \frac{1}{2}(\alpha - \delta l(\frac{\beta - \alpha}{2}) + \beta - \delta l(\frac{\beta - \alpha}{2})) \\
&= \frac{1}{2}((\alpha + \beta) - \delta l(\beta - \alpha))
\end{aligned} \tag{40}
$$

which implies the expected clearing price if the bubble pops is $P^e_{popped} = \frac{1}{2b}[(\alpha+\beta) - \delta l(\beta - \alpha) - 2X]$

I substitute (34), (36), (37), and (38) into (32) and simplify to find expected profit as:

$$
\begin{aligned}
E[\pi] = fP_0X - FC - \frac{(1-\lambda)P_0(X + bP_0 - \alpha + \frac{l\delta}{2}(\beta-\alpha))^2}{2(\beta-\alpha)} \\
- \frac{\lambda P_0(X + bP_0 - \alpha + \frac{\delta}{2}(\beta-\alpha))^2}{2(\beta-\alpha)}
\end{aligned} \tag{41}
$$

To find the profit maximizing P_0^*, I take the derivative of equation (41) with respect to P_0, set to zero, select the largest root, and simplify to find:

$$
\begin{aligned}
P_0^* = \\
&\frac{1}{24b^2}(-8bl\alpha\delta\lambda + 8bl\alpha\delta + 8bl\beta\delta\lambda - 8bl\beta\delta - 16bX - 8b\alpha\delta\lambda + 16b\alpha + 8b\beta\delta\lambda + \\
&((8bl\alpha\delta\lambda - 8bl\alpha\delta - 8bl\beta\delta\lambda + 8bl\beta\delta + 16bX + 8b\alpha\delta\lambda - 16b\alpha - 8b\beta\delta\lambda)^2 - \\
&48b^2(8fX\alpha - 8fX\beta - l^2\alpha^2\delta^2\lambda + l^2\alpha^2\delta^2 + 2l^2\alpha\beta\delta^2\lambda - 2l^2\alpha\beta\delta^2 - l^2\beta^2\delta^2\lambda + \\
&l^2\beta^2\delta^2 + 4lX\alpha\delta\lambda - 4lX\alpha\delta - 4lX\beta\delta\lambda + 4lX\beta\delta - 4l\alpha^2\delta\lambda + 4l\alpha^2\delta + 4l\alpha\beta\delta\lambda - \\
&4l\alpha\beta\delta + 4X^2 + 4X\alpha\delta\lambda - 8X\alpha - 4X\beta\delta\lambda + \alpha^2\delta^2\lambda - \\
&4\alpha^2\delta\lambda + 4\alpha^2 - 2\alpha\beta\delta^2\lambda + 4\alpha\beta\delta\lambda + \beta^2\delta^2\lambda))^{.5}
\end{aligned} \tag{42}
$$

B Proofs of Propositions

Proposition 1. *Given an exogenous number of shares X and stochastic demand, the investment bank chooses an expected profit maximizing offer price, P_0^*, such that P_0^* is less than the expected clearing price P_c^e.*

Proof. The first trading day demand, q, for IPO shares is stochastic with $q = \tilde{\theta} - bP$ where $\tilde{\theta}$ is distributed $U[\alpha, \beta]$ and where $\beta > \alpha > 0$, and $b > 0$. The expected profit to the investment bank is given by

$$E[\pi] = fP_0X - FC - \frac{\frac{1}{2}P_0(X + bP_0 - \alpha)^2}{\beta - \alpha}$$

with feasible realizations of the parameters such that $0 \leq X \leq \beta$ and $P > \frac{\beta}{b}$. Since the expected value of $\tilde{\theta}$ is $(1/2)(\beta - \alpha)$, the expected clearing price on the first day of trading is $P_c^e = \frac{1}{2b}(\beta + \alpha) - \frac{X}{b}$. I take the derivative of the expected profit with respect to P_0 and find

$$\frac{\partial E[\pi]}{\partial P} = -\frac{0.5(bP + X - \alpha)^2}{\beta - \alpha} - \frac{bP(bP + X - \alpha)}{\beta - \alpha} + fX \tag{43}$$

and then set (43) equal to zero, solve for P_0^*, select the largest root, and simplify to find:

$$P_0^* = \frac{1}{3b}(\sqrt{6fX(\beta - \alpha) + (X - \alpha)^2} - 2X + 2\alpha) \tag{44}$$

I check that (44) is a profit maximizing offer price, by taking the second derivative of the expected profit equation and simplify to find:

$$\frac{\partial^2 E[\pi]}{\partial^2 P} = \frac{b(3bP + 2X - 2\alpha)}{\alpha - \beta} \tag{45}$$

Since there are two roots, the expected profit function has regions where the function is either convex or concave. If the expected profit function is strictly concave, then the RHS of (45) is negative, which implies the numerator of the RHS of (45) most be positive. This implies that $3bP + 2X - 2\alpha > 0$ or that $P > \frac{2(\alpha - X)}{3b}$. This condition holds when $\alpha > X$, where α represents the demand at $P_0 = 0$ when the realization of $\theta = \alpha$. I substitute (44) into the above concavity condition to find

$$\frac{1}{3b}(\sqrt{6fX(\beta - \alpha) + (X - \alpha)^2} - 2X + 2\alpha) > \frac{2(\alpha - X)}{3b}$$
$$\sqrt{6fX(\beta - \alpha) + (X - \alpha)^2} > 0 \tag{46}$$

Since the LHS of (46) is clearly positive, the expected profit function is strictly concave at the optimal offer price, conditional on the issuing firm choosing X less than α.

I proceed using a proof by contradiction. Suppose $P_0^* > P_c^e$, which implies

$$\begin{aligned}
\frac{1}{3b}(\sqrt{6fX(\beta-\alpha)+(X-\alpha)^2}-2X+2\alpha) &> \frac{1}{2b}(\beta+\alpha)-\frac{X}{b} \\
-6fX\alpha+6fX\beta+X^2-2X\alpha+\alpha^2 &> (1.5\beta-0.5\alpha-X)^2 \\
X(-6f\alpha+6f\beta-3\alpha+3\beta) &> -0.75\alpha^2-1.5\alpha\beta+2.25\beta^2 \\
X &< \frac{0.75\alpha^2+1.5\alpha\beta-2.25\beta^2}{6f(\beta-\alpha)+3(\beta-\alpha)} \qquad (47)
\end{aligned}$$

Since $\beta > \alpha$ the denominator of the RHS of (47) is positive. If the numerator of the RHS of (47) is negative then

$$\begin{aligned}
\alpha^2+2\alpha\beta-3\beta^2 &< 0 \\
\alpha(\alpha+2\beta)-\beta(3\beta) &< 0 \qquad (48)
\end{aligned}$$

Since $\beta > \alpha$ and $3\beta > \alpha+2\beta$, (48) holds. Since $X > 0$ by definition and RHS of (47) is negative since the numerator is negative and the denominator is positive, by contradiction $P_0^* < P_c^e$.

□

Proposition 2. *Given an exogenous number of shares X and stochastic demand, the investment bank chooses an offer price, P_0^*, that is decreasing with the standard deviation of demand, σ_q.*

Proof. I substitute $\alpha = \mu - \sqrt{3}\sigma_q$ and $\beta = \mu + \sqrt{3}\sigma_q$ into (44) and simplify to find

$$P_0^* = \frac{1}{3b}\left\{\sqrt{X^2-12\sqrt{3}fX\sigma-2X\mu+2\sqrt{3}X\sigma+(\mu-\sqrt{3}\sigma)^2}+2(\mu-\sqrt{3}\sigma-X)\right\}$$

I take the derivative with respect to σ and simplify to find

$$\frac{\partial P_0}{\partial\sigma} = \frac{1}{6b}\left\{\frac{\left(12\sqrt{3}fX+2\sqrt{3}X-2\sqrt{3}\left(\mu-\sqrt{3}\sigma\right)\right)}{\sqrt{12\sqrt{3}fX\sigma+X^2-2X(\mu-\sqrt{3}\sigma)+(\mu-\sqrt{3}\sigma)^2}}-4\sqrt{3}\right\} \qquad (49)$$

Equation (49) is clearly negative if

$$\begin{aligned}
12\sqrt{3}fX+2\sqrt{3}X-2\sqrt{3}(\mu-\sqrt{3}\sigma) &< 0 \\
6fX+X-(\mu-\sqrt{3}\sigma) &< 0 \\
X(1+6f) &< \mu-\sqrt{3}\sigma \qquad (50)
\end{aligned}$$

This implies that $\frac{\partial P_0}{\partial\sigma} < 0$ when $X < \frac{\mu-\sqrt{3}\sigma}{1+6f} < \mu-\sqrt{3}\sigma$. From proposition 1, P_0 is concave when $X < \alpha = \mu-\sqrt{3}\sigma$. It remains to be shown that $\frac{\partial P_0}{\partial\sigma} < 0$ for X within $\frac{\mu-\sqrt{3}\sigma}{1+6f} < X < \mu-\sqrt{3}\sigma$.

If $\frac{\partial P_0}{\partial\sigma}$ is negative, then

$$(6f+1)X+\sqrt{3}\sigma-\mu < 2\sqrt{X((12f+2)\sqrt{3}\sigma-2\mu)+3\sigma^2-2\sqrt{3}\mu\sigma+X^2+\mu^2} \qquad (51)$$

I set $X = \mu - \sqrt{3}\sigma$, which is the highest possible value of X that can be chosen by the issuing firm, and substitute into (51) and simplify to find

$$3\sigma^2(4f^2 - 9) + 4f^2\mu(\mu - 8\sqrt{3}\sigma) < 0 \tag{52}$$

Since $0 < f < 1$ by definition $4f^2 - 9 < 0$, the first term in (52) is negative. The second term is negative when $\mu < 8\sqrt{3}\sigma$. This condition holds because because realized demand can not be negative so $\alpha \geq 0$, which implies $\mu \geq \sqrt{3}\sigma$. □

Proposition 3. *Given an exogenous number of shares X, stochastic demand, and an over-allotment option v, the investment bank chooses an offer price, P_0^{ov}, that is decreasing with the size of the over-allotment option, v.*

Proof. The expected profit to the investment bank with an over-allotment option is given by

$$E[\pi] = fP_0\left(\frac{X + vX(\beta - X(1 + \frac{1}{2}v) - bP_0)}{\beta - \alpha}\right) - FC - \frac{P_0(X + bP_0 - \alpha)^2}{2(\beta - \alpha)}$$

I take the derivative of the expected profit with respect to P_0, set the derivative to zero, solve for P_0, select the largest root, and simplify to find the profit maximizing offer price with an over-allotment option, which is

$$\begin{aligned} P_0^{ov} = \frac{1}{3b}\Big(&\sqrt{v^2fX^2(4f-3) + vfX(2X + 6\beta - 8\alpha) + 6fX(\beta - \alpha) + 2(X - \alpha)^2} \\ &- 2X(1 + fv) + 2\alpha\Big) \end{aligned} \tag{53}$$

I take the partial derivative of P_0^{ov} with respect to v to find

$$\frac{\partial P_0^{ov}}{\partial v} = \frac{1}{3b}\left\{[(2vfX^2(4f - 3) + fX(2X + 6\beta - 8\alpha)]A - 2fX\right\} \tag{54}$$

where $A = \sqrt{v^2fX^2(4f-3) + vfX(2X + 6\beta - 8\alpha) + 6fX(\beta - \alpha) + 2(X - \alpha)^2}$. The partial derivative of P_0^{ov} with respect to v is negative if $4f - 3 < 0$ and $2X + 6\beta - 8\alpha < 0$. The first condition implies that $f < 3/4$, which is plausible given the typical IPO gross spread is 7 percent. The second condition implies that $4\alpha - 3\beta < X$. Using the constraint from Proposition 1 that $X < \alpha$, implies that $4\alpha - 3\beta < X < \alpha$ or that $\alpha < \beta$. Lastly, for $A \in \Re$,

$$6fX(\beta - \alpha) + 2(X - \alpha)^2 > v^2fX^2(3 - 4f) + vfX(8\alpha - 6\beta - 2X) \tag{55}$$

which is true if both $6fX(\beta - \alpha) > vfX(8\alpha - 6\beta - 2X)$ and $2(X - \alpha)^2 > v^2fX^2(3 - 4f)$ hold. In the case of the first inequality,

$$\begin{aligned} 6fX(\beta - \alpha) &> vfX(8\alpha - 6\beta - 2X) \\ 3(\beta - \alpha) &> v(4\alpha - 3\beta - X) \end{aligned} \tag{56}$$

Since $\beta - \alpha > 0$ by assumption, the first inequality (56) holds if the $RHS < 0$ or $4\alpha - 3\beta < X$. Using $X < \alpha$ or $-\alpha < X$ from Proposition 1 implies that if $\alpha < \beta$ then (56) holds. In the case of the second inequality,

$$\begin{aligned}
2(X-\alpha)^2 &> v^2 f X^2(3-4f) \\
2X^2 - 4X\alpha + \alpha^2 &> 3fv^2X^2 - 4f^2v^2X^2 \\
X^2(2 - 3fv^2 + 4f^2v^2) - 4X\alpha + \alpha^2 &> 0 \\
X^2(3fv^2 - 4f^2v^2 - 2) + 4X\alpha - \alpha^2 &< 0 \qquad (57)
\end{aligned}$$

which is true if both $3fv^2 - 4f^2v^2 - 2 < 0$ and $4X\alpha - \alpha^2 < 0$, which hold if $v < 1$, $f < 3/4$, and $X < \alpha$. □

Proposition 4. *Given an exogenous number of shares X and stochastic demand, the investment bank chooses an offer price, P_0^*, with the same expected elasticity and underpricing for any demand slope coefficient b. Further, the investment bank's optimal offer price decreases with the slope coefficient of demand b.*

Proof. The partial derivative of the profit maximizing offer price with respect to b is

$$\frac{\partial P_0^*}{\partial b} = \frac{-A}{3b^2} \qquad (58)$$

where $A = \sqrt{6fX(\beta-\alpha) + (X-\alpha)^2} + 2(\alpha - X)$. Since $X < \alpha$ and $\beta > \alpha$, A>0. This implies the offer price decreases with b. The expected elasticity of demand at P_0^* is

$$\begin{aligned}
E[\varepsilon_{q,p}] &= \frac{E[dq/dP]}{q/P} \\
&= \frac{-bP_0^*}{E[\theta] - bP_0^*} \\
&= \frac{(-b)(\frac{1}{3b})A}{\frac{1}{2}(\beta+\alpha) - (b)(\frac{1}{3b})A} \\
&= \frac{-A}{\frac{3}{2}(\beta+\alpha) - A} \qquad (59)
\end{aligned}$$

Because (59) does not depend on b, the expected elasticity of demand at the optimal offer price does not depend on the slope coefficient b. Lastly, expected underpricing is defined as

$$\begin{aligned}
E[UP] &= \frac{P_c^e - P_0^*}{P_0^*} \\
&= \frac{\frac{1}{2b}(\beta+\alpha) - \frac{X}{b} - \frac{1}{3b}A}{\frac{1}{3b}A} \\
&= \frac{\frac{3}{2}(\beta+\alpha) - 3X - A}{A} \qquad (60)
\end{aligned}$$

Because (60) does not depend on b, expected underpricing is independent of the slope coefficient b. □

Proposition 5. *Given an exogenous number of shares, X, and stochastic demand, the investment bank chooses an offer price, P_0^*, such that underpricing decreases as the prior subjective probability, λ, either increases or decreases from a starting point of* .5*. In addition, underpricing increases with the size of the negative bubble shock, l.*

Proof. For any positive probability λ, the draw on the intercept of the demand curve becomes a mixture of uniform distributions. The expected draw is easily derive; however the variance of the expected draw does not have a closed form solution. In Table 1, I report the mean and standard deviation of the demand curve intercept for 1,000 repetitions of 500 draws for each unique combination of λ, δ, and l. I show that the variance of demand is highest when $\lambda = .50$ and then decreases as λ approaches either 0 or 1. In addition, the variance increases as l or δ increases. Therefore, by Proposition 2, underpricing increases as either l or δ increases and underpricing decreases when λ either increases or decreases from a starting point of .50. $\square$

References

Aggarwal, R. (2000). Stabilization activities by underwriters after initial public offerings. *The Journal of Finance 55*(3), 1075–1103.

Aggarwal, R. (2003). Allocation of initial public offerings and flipping activity. *Journal of Financial Economics 68*(1), 111–135.

Aggarwal, R., N. R. Prabhala, and M. Puri (2002). Institutional allocation in initial public offerings: Empirical evidence. *The Journal of Finance 57*(3), 1421–1442.

Baron, D. P. (1982, Sep.). A model of the demand for investment banking advising and distribution services for new issues. *The Journal of Finance 37*(4), 955–976.

Beatty, R. P. and J. R. Ritter (1986). Investment banking, reputation, and the underpricing of initial public offerings. *Journal of Financial Economics 15*, 213–232.

Benveniste, L. M. and P. A. Spindt (1989). How investment bankers determine the offer price and allocation of new issues. *Journal of Financial Economics 24*(2), 343–361.

Bradley, D. J. and B. D. Jordan (2002). Partial adjustment to public information and IPO underpricing. *The Journal of Financial and Quantitative Analysis 37*(4), 595–616.

Chen, H.-C. and J. R. Ritter (2000). The seven percent solution. *The Journal of Finance 55*(3), 1105–1131.

Chen, Z. and W. J. Wilhelm (2008). A theory of the transition to secondary market trading of IPOs. *Journal of Financial Economics 90*(3), 219–236.

Cogliati, G. M., S. Paleari, and S. Vismara (2008). IPO pricing: Growth rates implied in offer prices. *SSRN: http://ssrn.com/abstract=965450*.

Cook, D. O., R. Kieschnick, and R. A. Van Ness (2006). On the marketing of IPOs. *Journal of Financial Economics 82*, 35–61.

Cornelli, F., D. Goldreich, and A. Ljungqvist (2006). Investor sentiment and pre-ipo markets. *The Journal of Finance 61*(3), 1187–1216.

Degeorge, F., F. Derrien, and K. L. Womack (2010, November). Auctioned IPOs: The u.s. evidence. *Journal of Financial Economics 98*(2).

Derrien, F. (2005). IPO pricing in "hot" market conditions: Who leaves money on the table? *The Journal of Finance 60*(1), 487–520.

Derrien, F. and A. Kecskes (2009). How much does investor sentiment really matter for equity issuance activity? *European Financial Management 15*(4), 787–813.

Derrien, F. and K. L. Womack (2003, Spring). Auctions vs. bookbuilding and the control of underpricing in hot IPO markets. *The Review of Financial Studies 16*(1), 31–61.

Ellis, K., R. Michaely, and M. O'Hara (2000). When the underwriter is the market maker: An examination of trading in the IPO aftermarket. *The Journal of Finance 55*(3), 1039–1074.

Field, L. C. and G. Hanka (2001). The expiration of IPO share lockups. *The Journal of Finance 56*(2), 471–500.

Gao, X. and J. R. Ritter (2010). The marketing of seasoned equity offerings. *Journal of Financial Economics 97*(1), 33–52.

Griffin, J. M., J. H. Harris, and S. Topaloglu (2007). Why are IPO investors net buyers through lead underwriters? *Journal of Financial Economics 85*(2), 518–551.

Harris, M. and A. Raviv (1993). Differences of opinion make a horse rate. *The Review of Financial Studies 6*(3), 473–506.

Huang, Z., L. Zhao, and X. Mao (2009). Is the demand curve for stocks downward-sloping? new evidence from seasoned equity offerings. *SSRN: http://ssrn.com/abstract=1361610*.

Kandel, S., O. Sarig, and A. Wohl (1999). The demand for stocks: an analysis of IPO auctions. *Review of Financial Studies 12*(2), 227–247.

Kaustia, M. and S. Knupfer (2008). Do investors overweight personal experience? evidence from IPO subscriptions. *Journal of Finance 63*(6), 2679–2702.

Keefe, M. O. (2010). Is the IPO offer price revision really a proxy for private information? *SSRN: http://ssrn.com/abstract=1324182*.

Kim, M. and J. R. Ritter (1999). Valuing IPOs. *Journal of Financial Economics 53*(3), 409–437.

Knight, F. H. (1921). *Risk, Uncertainty, and Profit.* Boston: Houghton Mifflin Company, The Riverside Press Cambridge.

Kutsuna, K., J. K. Smith, and R. L. Smith (2009). Public information, IPO price formation, and long-run returns: Japanese evidence. *The Journal of Finance 64*(1), 505–546.

Ljungqvist, A., V. Nanda, and R. Singh (2006). Hot markets, investor sentiment, and IPO pricing. *Journal of Business 79*(4), 1667–1702.

Lowry, M., M. S. Officer, and W. G. Schwert (2010). The variability of IPO initial returns. *The Journal of Finance 65*(2), 425–465.

Lowry, M. and G. W. Schwert (2004). Is the IPO pricing process efficient? *Journal of Financial Economics 71*(1), 3–26.

Miller, E. M. (1977). Risk, uncertainty, and divergence of opinion. *The Journal of Finance 32*(4), 1151–1168.

Muscarella, C. J. and M. R. Vetsuypens (1989). A simple test of baron's model of IPO underpricing. *Journal of Financial Economics 24*(1), 125–136.

Ofek, E. and M. Richardson (2003). Dotcom mania: The rise and fall of internet stock prices. *The Journal of Finance 58*(3), 1113–1137.

Purnanandam, A. K. and B. Swaminathan (2004). Are IPOs really underpriced? *Review of Financial Studies 17*(3), 811–848.

Ritter, J. R. and I. Welch (2002). A review of IPO activity, pricing, and allocations. *The Journal of Finance 57*(4), 1795–1828.

Rock, K. (1986). Why new issues are underpriced. *Journal of Financial Economics 15*, 187–212.

Roosenboom, P. (2007). How do underwriters value initial public offerings? an empirical analysis of the french IPO market. *Contemporary Accounting Research 24*(4), 1217–1243.

Shiller, R. J. (2006). *Irrational Exuberance*. Doubleday Business.

Shleifer, A. (1986). Do demand curves for stocks slope down? *The Journal of Finance 41*(3), 579–590.

Shleifer, A. and R. W. Vishny (1997). The limits of arbitrage. *The Journal of Finance 52*(1), 35–55.

Table 1: Simulation Results

This table shows simulation results for the intercept of the demand curve, $\tilde{\theta}$, where $\tilde{\theta}$ is distributed $U[90, 110]$ and demand is given by $q = \tilde{\theta} - bP$. The bubble demand shock is specified by δ, the negative bubble shock by l, and the investment bank's prior subjective probability the bubble will continue, λ. The table reports the mean and standard deviation of the demand curve intercept for 1,000 repetitions of 500 draws for each unique combination of λ, δ, and l.

	$\delta = 1$ and l=1		$\delta = 1$ and l=1.5		$\delta = 1.5$ and l=1	
λ	$E[\theta]$	σ_θ	$E[\theta]$	σ_θ	$E[\theta]$	σ_θ
1	109.993	5.775	110.018	5.773	114.988	5.777
0.9	107.981	8.331	107.496	9.457	112.009	10.525
0.5	99.986	11.53	97.549	13.765	99.952	16.073
0.1	92.006	8.322	87.502	9.467	88.017	10.695
0	89.993	5.769	85.021	5.769	84.987	5.772