Fixed come hell or high water? Selection and prepayment of fixed rate mortgages outside the US

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Abstract

We examine the decision to prepay a fixed rate mortgage in the UK, Canada, Ireland, Australia and New Zealand. These countries are characterised by having substantial fees which are associated with breaking a fixed rate mortgage. We develop a model which allows for fluctuations both in banks' wholesale rates and also credit spreads. We find that households can achieve economically significant benefits from both following an optimal prepayment strategy contingent on the break fee used by their bank, and also by selection of fixed interest rate term and (where available) break fee structure.

Refinancing of mortgages is a well understood phenomenon in the US, both by households and academics. Theoretical work (such as Kau, Keenan, Muller, and Epperson (1987, 1992, 1993) and Stanton (1995)) has been supplemented and supported by empirical work (such as Deng, Quigley, and van Order (2000)) suggesting that households do indeed manage their mortgages in an optimal fashion.

In contrast, however, refinancing outside the US is relatively unexplored territory. Countries such as the UK, Canada, Australia, New Zealand and Ireland, contrary to the US, do not have widespread securitisation of mortgages (see Murphy (1996)). As a result, individual banks are left bearing any risk concerned with their lending. To mitigate this risk, contracts offered in these countries tend to be simpler than their US counterparts, rendering them easier instruments to hedge.

Firstly, mortgages are recourse loans. If a borrower fails to make payments on his loan, the bank can first seize the house securing the loan, and can then hold the borrower responsible for any remaining liability. This largely eliminates the first option available to many US households with respect to their debt: the default option.[1]

As well as differing from a default perspective, most loans only allow borrowers to fix their interest rates for fairly short periods of time (rarely exceeding five years) relative to the mortgage's overall life. Hence these fixed rate loans are more similar to hybrid adjustable rate mortgages (ARMs) in the US. Borrowers also have a predilection for variable rate loans, in contrast to US mortgagors, who generally choose to fix their loans. Daniel (2008) notes that in Australia approximately 80% of borrowers choose a variable rate loan over a fixed rate loan.[2] When a bank makes a fixed rate loan, it typically handles its interest rate risk by entering into an amortising swap so as to convert the borrower's fixed payments into a floating payment, which better matches the bank's funding sources (being mostly deposits or short maturity bills).

The last major distinction from US mortgages is the main focus of this paper. In all the countries

[1]This is not to say that default losses do not exist, as even with the right to pursue a household, the household's limited liquid wealth may lead to the mortgagee being unable to recover the shortfall between the house's value and the mortgage's principal.

[2]Miles (2004) finds comparable numbers for the UK, with 25% of borrowers choosing a fixed rate in 2003. As at December 2009, 16% of Irish mortgages (by value) had fixed terms of more than one year. In New Zealand, 36% of loans (by value) were fixed for more than one year (although 38% were fixed for one year or less). Canada is slightly anachronistic: Breslaw, Irvine, and Rahman (1996) find 72% of Canadian mortgages are fixed rate (in 1988). More recently, the FIRM Residential Mortgage Survey (2009) finds 69% of Canadian loans are fixed rate.

considered here, prepayment is associated with a financial penalty, which we refer to as a *break fee*.

In the case of the UK, as discussed in Azevedo-Pereira, Newton, and Paxson (2002), this break fee is generally a constant proportion of the principal outstanding on the loan. The analysis of refinancing options is thus fairly similar to that in the US. Even after paying a fee to refinance, Azevedo-Pereira et al. (2002) find that households can have a valuable refinancing option. More recently, a number of UK banks have begun offering break fees which decline as the mortgage's fixed term declines.[3]

Canada and Ireland, in contrast to the UK, both feature break fees which are calculated more consistently with the bank's actual loss from the borrower's prepayment. Canadian and Irish banks will calculate a break fee for a household based on the income lost if the bank relent the money prepaid for a fixed term equal to the remaining fixed term of the original mortgage, but at interest rates prevailing at the time of the mortgage being broken. Since in most cases, the household is choosing to prepay their loan because rates have fallen, the notional new loan would be at a lower rate than the prepaid loan, and a fee is levied. We refer to such a break fee as a *retail* break fee, since its size will be critically contingent on movements in retail interest rates. In Canada, this fee is normally subject to a minimum break fee of three months interest. Canadian banks also often offer customers the choice of a break-fee free mortgage (an *open* mortgage as opposed to the conventional *closed* mortgage) but at a substantial spread over the usual fixed rate, and generally for a short fixed term (one year or shorter).

A retail break fee accurately represents a lender's loss in the event that a new customer is found to replace the prepaying borrower. However, if no new customer can be found, an alternative response

[3]Much of the reason for the simplicity of UK break fees is due to the difficulty of enforcement of a more complex fee (see Miles (2004)). Fees are required by the Banking Ombudsman to be "calculatable in a way which is understandable to an ordinary borrower who has no understanding of how money markets work"(Banking Ombudsman's annual report 1998-1999).

would be to unwind the swap which was used to align the mortgage's payments with the bank's funding sources. If a bank calculates its break fee to compensate for this loss, we refer to this as a *wholesale* break-fee, since the profit or loss on the swap will be determined by wholesale rates.[4] This form of break fee is most prevalent in Australia and New Zealand. In these countries, the market is divided between banks using a wholesale break-fee policy, and those using a retail break-fee policy. Table 1 summarises the different break-fee regimes across countries.

[Table 1 about here.]

We note that retail and wholesale break fees can vary wildly as interest rates change. Figure 1 shows a history of five year retail and wholesale rates in New Zealand over the last ten years, along with proportional break fees for a five year fixed rate loan prepayed after one year. In many instances, rates were higher at the break date than at inception, resulting in no break fee. However, during 2009, retail break fees rose to be as high as 8.7%, and wholesale break fees peaked at 15.2% of principal prepaid.

[Figure 1 about here.]

Much empirical work has been undertaken on mortgage behaviour in these countries.[5] However, with the exception of the work by Azevedo-Pereira, Newton, and Paxson (2002, 2003) on the UK mortgage system, very little theoretical work has been done to understand the effect of the refinancing option for households. In particular, it is unclear the extent to which mortgages subject to retail or wholesale break-fees contain embedded refinancing options, if at all. The stylised fact (see Breslaw

[4]Since the bank's swap position will be paying a fixed rate and receiving a floating rate, the bank will make a loss when wholesale rates are low, and make a profit when wholesale rates are high.

[5]See, for example: Leece (2000) for the UK, Breslaw et al. (1996) and Zorn and Lea (1989) for Canada, and Daniel (2008) for Australia.

et al. (1996) and Miles (2004)) has been one wherein households make a decision between uncertain payments and a fixed payment - a decision which is largely contingent on risk preferences.[6]

This paper aims to perform a similar option theoretic analysis to that offered for US mortgages. We develop a two-factor model for mortgage interest rates, where retail rates are composed of a wholesale rate plus a spread. We explore the optimal exercise policy for mortgage refinancing under the various break-fee regimes prevalent in our collection of countries. We find scope for mortgagors to profit by managing their refinancing decision particularly under mechanical break fees (as found in the UK) and under wholesale break fees. We then explore, in the spirit of Stanton and Wallace (1998), whether there exists an optimal mortgage for a household, based on interest rates and the household's expected tenure in the property. We find that mortgagors can, in some circumstances, achieve substantial economic gains through mortgage selection. We also find that in the UK and in Australasia, mortgagors may have an optimal bank to approach, based on break-fee policies.

The layout of the remainder of the paper is as follows: section 1 discusses our theoretical model for mortgage prepayment decision, section 2 discusses the estimation of model parameters and section 3 presents the findings of our work, both in terms of exercise strategies and mortgage selection. Lastly, section 4 concludes.

[6] See also the discussion of Campbell and Cocco (2003) and Koijen, Van Hemert, and Van Nieuwerberger (2009) for the US.

1 Theory

1.1 Mortgage and hedge rates

We assume that mortgage rates are determined by two state variables: an instantaneous wholesale rate r_t, and an instantaneous credit spread s_t. Under the real world probability measure ($\mathbb{P}$), the two variables follow the processes:

$$dr_t = a_r(\mu_r - r_t)dt + \sigma_r r_t^{\gamma_r} d\tilde{W}_{rt} \tag{1}$$

$$ds_t = a_s(\mu_s - s_t)dt + \sigma_s s_t^{\gamma_s}(\rho d\tilde{W}_{rt} + \sqrt{1-\rho^2}d\tilde{W}_{st}) \tag{2}$$

where a_r, a_s, μ_r, μ_s, γ_r, γ_s, ρ, σ_r and σ_s are constants, and $d\tilde{W}_{rt}$ and $d\tilde{W}_{st}$ are independent Brownian motions. We assume constant market prices of risk (ξ_r and ξ_s for the two Brownian motions respectively) so that under the risk-neutral probability measure ($\mathbb{Q}$), wholesale rates and spreads follow:

$$dr_t = \left[a_r(\mu_r - r_t) - \xi_r\sigma_r r_t^{\gamma_r}\right]dt + \sigma_r r_t^{\gamma_r} dW_{rt} \tag{3}$$

$$ds_t = \left[a_s(\mu_s - s_t) - \hat{\xi}_s\sigma_s s_t^{\gamma_s}\right]dt + \sigma_s s_t^{\gamma_s}(\rho dW_{rt} + \sqrt{1-\rho^2}dW_{st}) \tag{4}$$

where dW_{rt} and dW_{st} are the increments of independent Brownian motions under $\mathbb{Q}$, and $\hat{\xi}_s \equiv \rho\xi_r + \sqrt{1-\rho^2}\xi_s$. The *instantaneous* retail mortgage rate is given by $r_t + s_t$. Longer term discount factors can be inferred from these processes. We denote the *retail discount factor* for time T, observed at time t as $d_t^R(T) = E_{\mathbb{Q}}\left(e^{-\int_t^T r_\theta + s_\theta d\theta}\right)$, and the *wholesale discount factor* as $d_t^W(T) = E_{\mathbb{Q}}\left(e^{-\int_t^T r_\theta d\theta}\right)$.[7]

[7] Our treatment of the pricing of default risk can be seen to be similar to that of Duffie and Singleton (1999), where our parameter s is the product of the intensity of default for the security and the expected portion of market value

Closed form solutions are available for longer term zero coupon wholesale rates, provided $\gamma_r = 0$ or $\gamma_r = 0.5$, and under the condition $\rho = 0, \gamma_r = \gamma_s = 0.5, \xi_r = \xi_s = 0$, longer term retail zero rates (see Duffie and Kan (1996)). For more general parameter cases (as explored in this paper) numerical solution is required.[8]

Given (3-4), standard asset pricing results allow us to characterise the value of securities as the solutions of partial differential equations. Denoting the value of a security as g per unit of principal, and the contract's time t principal as P_t, we can write:

$$\begin{aligned} & \frac{1}{P_t}\frac{\partial}{\partial t}\left[gP_t\right] + \left[a_r(\mu_r - r_t) - \xi_r\sigma_r r_t^{\gamma_r}\right]\frac{\partial g}{\partial r_t} + \left[a_s(\mu_s - s_t) - \hat{\xi}_s\sigma_s s_t^{\gamma_s}\right]\frac{\partial g}{\partial s_t} \\ & + \frac{1}{2}\frac{\partial^2 g}{\partial r_t^2}\sigma_r^2 r_t^{2\gamma_r} + \frac{1}{2}\frac{\partial^2 g}{\partial s_t^2}\sigma_s^2 s_t^{2\gamma_s} + \frac{\partial^2 g}{\partial r_t \partial s_t}\rho\sigma_r r_t^{\gamma_r}\sigma_s s_t^{\gamma_s} \\ = \; & A(r_t, s_t)g - K_t. \end{aligned} \tag{5}$$

Here $A(r_t, s_t) = r_t$ for a security which is valued by discounting at wholesale rates (such as the fixed of floating legs of a swap) and $A(r_t, s_t) = r_t + s_t$ for an instrument whose value should be discounted at retail rates (such as when valuing the remaining payments of a mortgage). K_t is the payment which the security makes at time t (per unit of principal) which may itself be a function of r_t and s_t (as would be the case in valuing the floating leg of a swap).

We denote the time t_0 mortgage rate for a new mortgage with maturity date T and end of fixed period $\tau < T$ as $R_{t_0}(\tau, T)$. Mortgage rates must be consistent with the process for retail rates. This

which is lost in the event of default.

[8]Our analysis implicitly assumes that markets are complete, and therefore mortgages are redundant securities. In so far as households are unable to actually hedge their exposure to wholesale and retail interest rate shocks, this may be incorrect. The alternative, incomplete markets, approach (see for example, Koijen et al. (2009) and Campbell and Cocco (2003)) generally requires strong assumptions regarding household investments, and fairly abstract treatment of the mortgages. Since our analysis is largely focused on the technical details of mortgages, we consider the complete markets approach more appropriate here.

implies that the present value of the outstanding principal at the end of the mortgage's fixed period, plus the value of the mortgage's fixed payments must be equal to the initial principal of the loan (here assumed to be one). Mathematically, assuming continuous payment of interest and principal:

$$\frac{1 - e^{-R_{t_0}(\tau,T)(T-\tau)}}{1 - e^{-R_{t_0}(\tau,T)(T-t_0)}} d^R_{t_0}(\tau) + \frac{R_{t_0}(\tau, T)}{1 - e^{-R_{t_0}(\tau,T)(T-t_0)}} \int_{t_0}^{\tau} d^R_{t_0}(\theta) d\theta = 1. \tag{6}$$

If we know the value of a zero coupon bond maturing at the end of the mortgage's fixed term, and the annuity factor ($\int_{t_0}^{\tau} d_{t_0}(t)dt$) we can find the mortgage rate $R_{t_0}(\tau, T)$ by solving (6). These two components can be priced by solving (5). In the case of the zero coupon bond, we set $P_t = 1$, $A(r_t, s_t) = r_t + s_t$ and $K_t = 0$, with terminal condition $g(T) = 1$. For the annuity factor, we set $P_t = 1$, $A(r_t, s_t) = r_t + s_t$, $K_t = 1$, and set the terminal condition such that $g(T) = 0$.

We also define the associated *hedge rate* for a new mortgage set up at time t_0, with maturity T and fixed period ending at time τ as $r_{t_0}(\tau, T)$. The hedge rate is defined as the fixed rate which would be paid on a $\tau - t_0$ year fixed-floating swap which has declining principal identical to that of the associated mortgage. To calculate the hedge rate, we note that $P_\theta = \frac{1-e^{-R_{t_0}(\tau,T)(T-\theta)}}{1-e^{-R_{t_0}(\tau,T)(T-t_0)}}$. Hence the hedge rate will be the solution of

$$r_{t_0}(\tau, T) \int_{t_0}^{\tau} P_\theta d^r_{t_0}(\theta) d\theta - E_{\mathbb{Q}} \left(\int_{t_0}^{\tau} P_\theta r_\theta e^{-\int_{t_0}^{\theta} r_z dz} d\theta \right) = 0. \tag{7}$$

We can evaluate the second integral by solving (5), with $A(r_t, s_t) = r_t$, $K_t = r_t$. The first integral is a wholesale annuity factor (albeit with declining principal) and can be solved similarly to the retail case, with $A(r_t, s_t) = r_t$ and $K_t = 1$. For both valuing the floating leg of the swap, and for valuing the annuity factor, the terminal condition is $g(T) = 0$.

1.2 Mortgage valuation

We assume that a borrower may need to sell their house for reasons exogenous to the model (examples of these could be an inability to service the loan due to income loss, lifestyle considerations causing the borrower to need a larger/smaller home, or moving locations for employment reasons). In this case, the borrower could either exit the housing market, or could buy a new house and require a new mortgage. For simplicity, we assume that in the event of buying a new house, the new mortgage is of identical (or larger) size or to the existing loan, so that we can treat this as a refinancing of existing debt.[9] These two outcomes (exiting the housing market and buying a new house) occur with constant Poisson intensities of $\lambda_1 dt$ and $\lambda_2 dt$ respectively. In both cases, the household must pay any break fees associated with their existing mortgage.[10]

We assume that payments are made on the mortgage continuously, so that for a mortgage incepted at time t_0 and observed at a later time t, the continuous coupon payment on the mortgage per unit of principal is given by:

$$C_t(t_0, \tau, T, R_{t_0}(\tau, T)) = \frac{R_{t_0}(\tau, T)}{1 - e^{-R_{t_0}(\tau,T)(T-t)}}.$$

Outstanding principal at time t is given by

$$P_t = P_{t_0} \frac{1 - e^{-R_{t_0}(\tau,T)(T-t)}}{1 - e^{-R_{t_0}(\tau,T)(T-t_0)}}.$$

A mortgage's value at any point in time will depend on its structure, the current level of r and s, and also the mortgage's interest rate. For mortgages with wholesale break fees, the mortgage's associated

[9]The model could potentially be augmented to allow borrowers to downsize by adding a third type of suboptimal prepayment, in which the household's new mortgage has lower principal than the outstanding loan.

[10]We note that in some cases, a household may be able to transfer their existing mortgage to a new house, hence avoiding any break fees. Where this is a possibility, we would regard λ_2 as reflecting the probability of moving with a mismatch between selling one property and buying the new one, which would require creation of a new mortgage.

hedge rate will also affect its value, since it will affect break fees, if they occur. Hence we may write the value of a mortgage, observed at time t, created at time $t_0 \leq t$, per unit of principal as being:

$$f(t, r_t, s_t, \tau, T, R_{t_0}(\tau, T), r_{t_0}(\tau, T)).$$

A borrower choosing a new mortgage at time t, where maturities of $t, t+1, \ldots t+\bar{\tau}$ are available, solves the problem:

$$\min_{\substack{\tau=t,\ldots,t+\bar{\tau} \\ \tau \leq T}} f(t, r_t, s_t, \tau, T, R_t(\tau, T), r_t(\tau, T)) \equiv f^*(t, T, r_t, s_t).$$

Note that $\tau = t$ is a possibility, since the household could choose a floating rate loan instead of a fixed loan.

Since the mortgage's value depends on the two state variables r_t and s_t, standard no-arbitrage arguments show that f satisfies the differential equation:

$$\begin{aligned} & \frac{1}{P_t}\frac{\partial}{\partial t}[P_t f] + [a_r(\mu_r - r_t) - \xi_r \sigma_r r_t^{\gamma_r}]\frac{\partial f}{\partial r_t} + \left[a_s(\mu_s - s_t) - \hat{\xi}_s \sigma_s s_t^{\gamma_s}\right]\frac{\partial f}{\partial s_t} \\ & + \frac{1}{2}\frac{\partial^2 f}{\partial r_t^2}\sigma_r^2 r_t^{2\gamma_r} + \frac{1}{2}\frac{\partial^2 f}{\partial s_t^2}\sigma_s^2 s_t^{2\gamma_s} + \frac{\partial^2 f}{\partial r_t \partial s_t}\rho\sigma_r r_t^{\gamma_r}\sigma_s s_t^{\gamma_s} \\ & + \lambda_1(1 + B_t - f) + \lambda_2(f^* - f + B_t) \\ \leq \; & (r_t + s_t)f - C_t(t_0, \tau, T, R_{t_0}(\tau, T)) \end{aligned} \tag{8}$$

where B_t is the full break fee associated with the mortgage, which potentially depends on the current level of interest rates, as well as the mortgage's characteristics (see below).

Equation (8) is defined in conjunction with either one or two complementary slackness conditions, associated with the refinancing options available to the borrower. The first condition applies to all mortgages:

$$f \leq f^* + B_t, \tag{9}$$

meaning that a household can always choose to completely repay their mortgage. However, in some circumstances, a household may be forgiven a certain portion of their loan prepayment, as a fraction of their initial principal (denoted ψ). In this case, it is also potentially optimal for the household to prepay only the forgiven portion of their loan, in which case

$$f \leq \max(0, 1 - \frac{\psi P_{t_0}}{P_t})f' + \min(1, \frac{\psi P_{t_0}}{P_t})f^* \tag{10}$$

where f' is the value of an otherwise identical mortgage where $\psi = 0$.

Lastly, the terminal condition for (8-10) is given by $f(\tau) = f^*(\tau)$, since on conclusion of the mortgage's fixed term, the mortgagor can freely choose a new fixed term.

1.3 Calculating break fees

As discussed in the introduction, we consider three types of break fee: those which do not depend on interest rates, those based on retail rates, and those based on the cost of breaking the swap a bank has entered into. We focus here on a household who prepays their entire principal outstanding. Similar calculations hold for partial prepayments, scaled accordingly. We express break fees relative to current principal outstanding, as per (8).

1.3.1 Non-rate dependent break fees

The break fees found in the United Kingdom are the most straightforward to describe. We consider two possible break fees here. The first break fee is simply a flat portion of the principal outstanding, as considered by Azevedo-Pereira et al. (2002) and Chen, Connolly, Tang, and Su (2009).

$$\text{Break fee (flat)} = \zeta_1$$

where ζ_1 is a constant. The second type of break fee considered is a *declining* fee. Here the fee is proportional to the time remaining of the loan:

$$\text{Break fee (declining)} = \zeta_2(\tau - t)$$

where ζ_2 is a constant. In general $\zeta_2 > \zeta_1$. The rationale for charging a declining fee is that the bank's loss, either due to having to relend the prepaid principal at lower rates, or break a hedging swap, declines as the mortgage approaches the end of its fixed term. We can thus see the declining fee as a (very) rough approximation of the retail/wholesale break fee schemes.

1.3.2 Retail break fees

Here the fee is calculated according to the assumption that once the household pays off its loan, the bank will proceed to relend the money, under the same terms (τ and T) as the existing loan. We outline two possible ways in which this can be implemented.

Single discount rate (SDR) In this case, the bank calculates a rate for a new loan set up to mature at time T, and conclude its fixed period at time τ. The household's break fee is the value of their remaining payments discounted at this rate ($R_t(\tau, T)$), less the principal they are repaying:

$$\begin{aligned}
\text{Break fee (SDR. retail)} &= \frac{P_\tau}{P_t} e^{-R_t(\tau,T)(\tau-t)} \\
&\quad + C_t(t_0, \tau, T, R_{t_0}(\tau, T)) \int_t^\tau e^{-R_t(\tau,T)(\theta-t)} d\theta - 1 \qquad (11) \\
&= \frac{1 - e^{-R_{t_0}(\tau,T)(T-\tau)}}{1 - e^{-R_{t_0}(\tau,T)(T-t)}} e^{-R_t(\tau,T)(\tau-t)} \\
&\quad + \frac{1 - e^{-R_t(\tau,T)(\tau-t)}}{1 - e^{-R_{t_0}(\tau,T)(T-t)}} \frac{R_{t_0}(\tau, T)}{R_t(\tau, T)} - 1. \qquad (12)
\end{aligned}$$

Note that for the case where $R_t(\tau, T) = R_{t_0}(\tau, T)$, the rate the bank will (notionally) relend the money at is identical to the borrower's current rate, and the break fee will be zero. If the break-fee is positive (the new rate is lower than the existing rate) the borrower will pay the fee. If the break fee is negative, the borrower will pay nothing.

Full valuation (FV) In this case, the bank values the household's remaining payments, discounting each at the appropriate zero coupon rate for that date, using (6). Given knowledge of the retail annuity factor, and of the retail discount factor for the maturity date of the mortgage, we can easily calculate this value. The break fee is then given by the difference between this market value and the principal the household repays:

$$\begin{aligned}
\text{Break fee (FV. retail)} &= \frac{1 - e^{-R_{t_0}(\tau,T)(T-\tau)}}{1 - e^{-R_{t_0}(\tau,T)(T-t)}} d_t^R(\tau) \\
&\quad + \frac{R_{t_0}(\tau, T)}{1 - e^{-R_{t_0}(\tau,T)(T-t)}} \int_t^\tau d_t^R(\theta) d\theta - 1. \qquad (13)
\end{aligned}$$

In general the break fee calculated by the single discount rate method will be *close* to the full valuation method. The two numbers will not exactly match (except for the case where $R_t(\tau, T) = R_{t_0}(\tau, T)$), since the new retail rate is calculated based upon amortising occurring consistent with the new rate, whereas the existing cash flows are amortising consistent with the old rate. The difference between these two numbers (the SDR break fee and the profit for the household from refinancing) are generally small, except in situations where the change in interest rates has been very large. In our analyses in this paper, for brevity, we use only the full valuation technique in our retail rate calculations.[11]

1.3.3 Wholesale break fee

In this case, the bank assumes that in the event of prepayment, no new loan is entered into, so that the mortgagee's hedge position must be unwound. The swap in question would have a notional principal on any date equal to the amortised principal of the actual mortgage. The hedge swap can be valued after its initial creation, at time t, per unit of principal, by evaluating:

$$r_{t_0}(\tau, T) \int_t^\tau \frac{P_\theta}{P_t} d_t^r(\theta) d\theta - E_{\mathbb{Q}} \left(\int_t^\tau \frac{P_\theta}{P_t} r_\theta e^{-\int_t^\theta r_z dz} dt \right). \tag{14}$$

There are certain similarities between the retail break fee and the wholesale break fee. In both cases, if interest rates decline, the household will be penalised (either because the household's remaining payments will be more valuable, or because the bank will have lost money on its swap). The sensitivity of this penalty will also decline as the loan approaches the end of its fixed term. However the critical difference between the two fees is the *rate* which determines the break fee. In the case of the wholesale break fee, the rate which is critical for the household's valuation of the mortgage is the *retail* rate, while its break fee is being calculated based on *wholesale* rates. As such, the household

[11] This exact system of break-fee exists in Denmark, where mortgagors can discharge their mortgage by delivering mortgage backed bonds to the mortgagee, effectively purchasing another borrower's mortgage in the secondary market. See Frankel, Gyntelberg, Kjeldsen, and Persson (2004).

may profit substantially from refinancing in situations where wholesale rates have risen (the bank has made a profit on its swap) but retail rates have fallen (presumably due to a decline in s).

1.4 Solution technique

We proceed by first deriving term structures of interest rates for different levels of state variables r and s. Using these, we are then able to solve for mortgage values, conditional on the mortgage's remaining life, the initial fixed term, the current fixed term, and term structures of interest rates. We price these in tandem, allowing us to consider the refinancing options available to the household. Further details are given in appendix A.

2 Parameter estimation

Data for wholesale interest rates are obtained from Datastream interbank rates. We proxy instantaneous wholesale rates using the one month interbank rates for our collection of countries. Floating mortgage rates are generally available from the central bank of each country, with the exception of Ireland and the UK, where these data are provided by the Central Statistics Office. Our spread is assumed to be the difference between floating mortgage rates and the interbank rate. Data cover the period February 1995 to January 2010, except for Ireland, where data are only available from January 1999.

For the UK mortgage rates, we use standard variable rate data. Banks in the UK generally offer two different types of floating rate loan. The first rate, a "standard variable rate" loan, has payments based on the bank's own floating rate. A "tracker" loan, in contrast, has payments directly pegged to the Bank of England's interest rates. Banks often "buffer" their customers against changes in the

Bank of England rates, lowering their rates less in response to a cut, and raising retail rates less in response to a rate hike by the central bank. The tracker loan is thus more similar to an American ARM rate. However, the standard variable rate loan is more comparable to floating rate loans in the other countries we consider here, so we use standard variable rate data to estimate our model.

We estimate the parameters for (1) and (2) using Generalised Method of Moments estimation, as described in Chan, Karolyi, Longstaff, and Sanders (1992). For most of the countries considered, we obtain sensible parameter estimates. However, for the UK, we have difficulty obtaining convergence to stable parameter values. For the UK, we use Quasi-Maximum Likelihood estimation to obtain parameter estimates, where we assume that monthly innovations to the interest rate are normally distributed.

[Table 2 about here.]

Table 2 contains the results of our estimation. Perhaps the most striking observation is the negative correlation between spreads and the wholesale rate. This suggests that in most of our countries, as is the case in the UK, a rise in wholesale rates is not completely passed through to customers, nor is a decline. This effect in New Zealand is documented by Liu, Margaritis, and Tourani-Rad (2008).[12]

The second empirical observation is the marked difference between a_r and a_s. The countries all exhibit a much more rapid mean reversion for credit spreads than is the case for wholesale rates. The implication of this for our model is that fluctuations in credit spreads will lead to marked changes in the slope of the retail mortgage yield curve, whereas changes in wholesale rates will generally result

[12]Given empirical evidence from the US corporate bond market, we also should not be surprised to find that default intensities and losses (measured by s) are negatively correlated with interest rates (measured by r). See, for example, Duffee (1999).

in more parallel shifts in the curve.

Lastly, we note a reasonable dispersion in γ_r and γ_s. Most countries have values considerably less than 0.5, suggesting that their rate process lies somewhere between a Vasicek process ($\gamma = 0$) and a Cox-Ingersoll-Ross process ($\gamma = 0.5$). Ireland is the outlier here, having $\gamma_r = 1.784$ and $\gamma_s = 0.822$. Ireland's wholesale rate process thus seems most similar to the treasury rate process estimated for the US by Chan et al. (1992). Since Ireland operates on the Euro currency, the wholesale rate process estimated here should be reasonably comparable with that of other European nations. In contrast, the spread process is liable to differ considerably across countries, since it will be heavily affected by changes in house prices.

In order to implement our model, we need to convert the real world ($\mathbb{P}$) processes into the risk-neutral world ($\mathbb{Q}$) processes (see equations (3-4)). To do this, we use observations from December 2009 for each country, and calibrate the selection of ξ_r and $\hat{\xi}_s$ so that longer maturity plain vanilla (i.e. non-amortising) swaps are correctly priced, along with longer maturity mortgages. For most cases, we are able to obtain five year fixed mortgage rates and also use five year swap rates. For Australia, the central bank only makes three year fixed rates available, and for Ireland a composite 1-5 year rate is available, which we use as being approximately equal to the three year rate.[13]

The estimates of ξ_r and $\hat{\xi}_s$ are universally negative. The result of this is that the steady state for retail and wholesale rates is higher in the risk-neutral world ($\bar{r}$ and $\bar{s}$) than in the real world (μ_r and μ_s). Hence the countries considered will have persistent upward sloping wholesale curves and increasing credit spreads with time to maturity, as generally observed in practice.

[13]We note that in Ireland at the time, the mortgage yield curve is almost completely flat, so any approximation error in this is negligible.

3 Results

To understand the optimal selection of mortgages, it is important to begin by understanding when a mortgage should be prepaid. Since our model, by construction, offers borrowers long term rates which are consistent with the short interest rate processes, a borrower who takes out a fixed rate loan and services it until the end of the fixed term will have created a debt instrument whose value is identical to the initial principal of the loan. Similarly, a borrower who takes out a floating rate loan will also create a debt instrument which is valued at par. However, a borrower who exercises their option to prepay optimally, will produce an instrument whose value is below par. By examining the extent to which this is the case for a particular borrower for different mortgage structures, we can ascertain both the best choice of mortgage for the borrower, and also measure the economic gain from selection and timing of mortgage prepayment.

We thus begin by discussing the optimal prepayment of mortgages, exploring how this behaviour changes as the fee structure of the loan changes. Next, we explore the optimal selection of mortgages, both in terms of fixed horizon, and break fee, and show how this varies across countries and levels of the state variables of our model. Lastly, we examine the economic significance of our results, both for borrowers and lenders.

3.1 Optimal prepayment strategies

For this section, we focus our attention on the United Kingdom, Canada and New Zealand. Since Irish banks make use of retail break fees, and Australian banks use retail and wholesale break fees, the results for New Zealand provide most of the intuition for these two countries. The United Kingdom case explores the effect of formulaic break fees, while the Canadian case highlights the effect of open versus closed mortgages. We return attention to Ireland and Australia in our subsequent discussion of mortgage selection and value gains.

For each country, we consider two cases. In the first, the borrower is *intransient*, with $\lambda_1 = \lambda_2 = 0$. This borrower is assumed, therefore, to remain in their house for the entire life of the mortgage in question. The second borrower has $\lambda_1 = \lambda_2 = 0.1$. This borrower thus has a 20% probability of selling their house each year. Conditional on selling the house, with probability 50%, the borrower buys a new house (and therefore takes out a new mortgage), while with probability 50%, they do not buy a new house, and therefore do not require a new mortgage. Both borrowers are assumed to have taken on a 30 year amortising loan, and to have fixed their loan for five years. In all cases, the first 20% of prepayments are taken to be forgiven ($\psi = 0.2$).

To evaluate the optimal exercise frontier, we require details of the borrowers' interest payments. For each country, we perturb the short rates observed on December 2009 so that selection of a five year fixed rate mortgage (with declining break fee in the UK, retail break fee in New Zealand and closed structure in Canada) is optimal for the intansient borrower, when compared with other maturities (see section 3.2).[14] Our mortgage and hedge rates considered for each country are given in table 3.

[Table 3 about here.]

[Figure 2 about here.]

Results for the UK are given in figure 2. We assume $\zeta_1 = 0.03$ while $\zeta_2 = 0.01$ so that the household either pays a flat 3% break fee (fixed fee) or a 1% fee per year remaining in the loan

[14]For the UK case, the transient borrower finds a 5 year fixed fee mortgage optimal, while shorter term mortgages are optimal for the transient borrower in New Zealand and Canada. For these latter two countries, setting a rate high enough for both borrowers to find five year fixed rates optimal would result in unrealistically high wholesale rates (see section 3.2).

(declining fee). Each line represents the locus of points which would cause the borrower to prepay their mortgage. Hence points below and to the left of the line are situations where the mortgage would be refinanced, and those above and to the right are where the mortgage should continue to be serviced. For each mortgage type, there are two lines. The first (solid) line represents the states where the household should prepay the forgiven portion of their loan. The second (dashed) line represents the states where the unforgiven portion of the loan should be prepaid. For each mortgage type and each borrower, we show frontiers for 1 year and 4 years remaining of the mortgage's fixed term.

In each case, there is a negative relationship between the level of wholesale rate and spread which trigger prepayment. This is because new rates the borrower can access depend on a combination of both state variables. For a low wholesale rate, even for quite a high credit spread, the household may be better off to refinance. Conversely, for a high wholesale rate, even a very low level of credit spread may still render refinancing suboptimal. The relationship is not *exactly* linear, because the mortgage depends on a combination of longer maturity rates, each of which depend non-linearly on the state variables (due to mean-reversion).

Comparing across the time dimension, we note that for the forgiven portion of the loan, the household becomes more willing to prepay as the end of the fixed term approaches (the exercise frontier shifts upwards). This is caused by the diminishing time value of their option. For the portion of their mortgage which attracts a break-fee, we see that for borrowers with a fixed break fee, exercise is (relatively) more attractive with four years of fixed term remaining, while for the declining fee, later exercise is more attractive.

Lastly, comparing the two borrowers, we see very similar behaviour. In our later examples, we note a slight tendency for the transient borrower toward prepayment. Again, this follows from the time value of the option. Since the transient borrower faces the prospect of suboptimal prepayment, their option time value is lower, and they are more inclined to prepay.

[Figure 3 about here.]

In the Canadian case (see figure 3) we see the distinction between a closed and an open mortgage. For the closed mortgage, a retail break fee is calculated, with a minimum of three months interest charged. For the open mortgage, no break fee is charged, however, the borrower pays a 3% premium for this privilege.

Focusing first on the closed mortgage, we again note the rising exercise frontiers as the mortgage approaches the end of its fixed term, and a downward sloping frontier, due to the tradeoff between r and s in determining retail rates. We note that for longer maturities, a borrower is liable to break both sections of his mortgage in fairly rapid succession. Once rates have fallen low enough to motivate prepayment of the forgiven section, even a small decline will trigger prepayment of the remainder of the loan. However, as maturity approaches, the three months interest minimum break fee becomes more important (similar to the fixed fee case in the UK) and renders prepayment less attractive.

For the open mortgage, there is no distinction between parts of the loan, since no fee is paid on prepayment. However, since the mortgage has a rate in excess of the regular retail yield curve, prepayment will be optimal at even higher levels of r and s than is the case for the forgiven portion of a closed loan.

[Figure 4 about here.]

Lastly, figure 4 contains the optimal policies for our New Zealand borrowers. For the retail break fee case, we note that the optimal exercise frontiers are actually slightly *lower* for the forgiven portion of the loan than is the case for the fee-bearing portion. The reason for this is that the retail break-fee represents a lose-lose situation for the borrower: if the loan is prepaid when rates have fallen, the household loses all their potential gains to break fees. However, if rates have *risen*, the household is uncompensated. Once the free portion of the loan has been prepaid, it is therefore *immediately* optimal to prepay the remainder of the loan, since retail rates must be sufficiently low to place the household in the break-fee bearing region (by virtue of having just triggered prepayment of the free portion).

The wholesale break-fee case generates the most qualitatively different exercise frontiers. In this case, because the break fee is based on the level of wholesale rates, whereas the household's profit from refinancing is based upon retail rates, we actually see a hump shaped relationship between the levels of r and s which trigger prepayment. For high levels of wholesale rates, the household faces no fee, and therefore prepays the unforgiven portion of the mortgage similarly to the forgiven portion. However, if wholesale rates are low, the household faces a break fee which grows larger, the lower wholesale rates have become. Hence the household requires a much lower level of *retail rate* to trigger prepayment. Note that if wholesale rates are low, this can result in situations where the household prepays their forgiven portion, and does not prepay the remainder of the loan, in contrast to the behaviour of a retail break fee borrower.

3.2 Optimal selection of mortgage type

We next turn our attention to the optimal selection of a mortgage contract. Here we consider a range of different levels for the instantaneous wholesale rate and spread.

We again present results for both intransient ($\lambda_1 = \lambda_2 = 0$) and transient ($\lambda_1 = \lambda_2 = 0.1$)

borrowers. Clearly, this exercise could be repeated using a different level for λ_1 and λ_2, and would lead to different preferred mortgage types for different short wholesale rates and spreads. Our two sets of results provide a fairly clear indication of the qualitative effects of household transience on mortgage selection. In all cases, we allow the mortgagor to fix their loan for a maximum of five years, consistent with common lending practices. As in section 3.1, we assume $\psi = 0.2$ for all mortgages.

[Figure 5 about here.]

In many ways, the UK mortgage system presents the simplest case of mortgage selection. The household chooses between a situation where early exercise of the prepayment option may be cheap at the expense of late exercise (a flat fee) versus a fee which is liable to be more in line with option exercise values (a declining fee). Figure 5 presents the optimal choice of mortgage. We consider $\zeta_1 = 0.006(\tau - t_0)$ and $\zeta_2 = 0.01$, so that the flat fee is 0.6% per year of fixed term, while the declining fee remains 1% per remaining year of the loan.

For low levels of r and s (and thus low levels of retail rates) the optimal choice of mortgage is to float. Taking on a fixed rate loan (with either type of fee) has little potential for profitable refinancing, since rates are unlikely to decline from present levels. Furthermore, for a transient borrower, there exists the potential for suboptimal exercise to result in a fee being paid. As rates increase, the potential for a profitable refinancing becomes possible, and eventually exceeds the risk due to suboptimal prepayment. As this occurs, longer fixed terms become optimal. For the intransient borrower, there is no risk of suboptimal prepayment, and therefore even for modest levels of r and s, a fixed rate contract becomes the preferred choice.

The two borrower types also have a preferred habitat for break-fees. For the tansient borrower, uncertainty about their tenure in the loan reduces the value of later optionality, and therefore choosing a flat fee (which offers better prepayment opportunities early in the loan's life) is preferable to a declining fee. Conversely, the intransient borrower has a preference for a declining fee.

[Figure 6 about here.]

Canada and Ireland (figure 6), in contrast to the UK situation, both face mortgage choices where the fee is based on the gain the household could achieve from refinancing their loan. The Irish case shows the effect of a retail break fee with no minimum charge (similar to those found in New Zealand and Australia). As noted in section 3.1, this eradicates any potential for profitable refinancing of the unforgiven portion of the loan, restricting optionality to the forgiven portion. The standard tradeoff exists, where borrowers trade off the risk of suboptimal exercise against optimal exercise. Not surprisingly, the results are qualitatively similar: for low interest rates, floating is optimal, and as rates increase, fixing for longer terms becomes preferred. Again, intransient borrowers have a preference for longer terms.

Examining the case of the intransient Canadian borrower, we see similar behaviour. Low rates encourage fixing for short terms. We note that the open mortgage is never optimal: all loans are closed. For the transient borrower, however, results are quite stark. For almost all levels of interest rates, floating is the optimal choice. The enforcement of a lower bound on break-fees almost completely eliminates option value for our transient borrower. Clearly given that an intransient borrower can find fixing optimal, for some modest levels of λ_1 and λ_2 fixing may still be optimal.

[Figure 7 about here.]

Figure 7 contains the optimal selection for a household in New Zealand (top panel) and Australia (bottom panel). For both countries, the figures can be essentially broken into three regions. For low interest rates (both low wholesale rates and low spreads), a floating rate mortgage is most valuable, for identical reasons to the UK, Irish and Canadian situation. As retail rates increase, fixing the mortgage becomes valuable, since the possibility of using the refinancing option becomes feasible. Generally speaking, the higher the retail rate, the longer the optimal fixing term becomes, since the refinancing option becomes more and more valuable.

The selection of whether to choose a retail or wholesale break-fee is a function of the level of the credit spread. As credit spreads increase, there becomes a possibility of a mortgagor with a wholesale break fee being able to costlessly refinance, in a situation where retail rates fall, but wholesale rates rise (i.e. the decline in credit spread exceeds the rise in wholesale rates). This considerably increases the option value for wholesale break fees. In contrast, for retail break fees, the household has no ability to profit in the presence of break fees, and can only gain option value from the portion of their loan which is exempt from break fees.

However, this option value must be weighed against the risk to the household of suboptimal exercise. With a wholesale break fee, the household not only faces a loss if they prepay into a higher retail rate, but also in the case when they prepay and the retail rate has *fallen*, but the wholesale rate has *fallen by more* so that the break-fee outweighs the mortgagor's profit.[15] This effect is most likely to occur when credit spreads are low, and so in these situations, the mortgagor finds it optimal to choose a retail based break-fee.

[15]Many households in Australasia found themselves in exactly such a situation during the recent credit crunch. See figure 1.

3.3 Value gains from prepayment

Lastly, we turn our attention to the economic significance of the mortgage selection and optimal refinancing decision. As noted previously, a household who either takes a floating loan or holds a fixed loan until maturity creates a par value debt instrument. In contrast a household who exercises their prepayment option optimally will reduce the value of their loan. Since floating is always an option, this will result in mortgage values below the loan's principal. By taking the difference between the mortgage's value and the loan's principal, we can ascertain the value gained from optimal prepayment and mortgage selection. Note that this gain to the household is a *loss* to the lender of exactly the same amount. This can be contrasted to the situation in American mortgages, where *default* on a loan could lead to different losses for the two counterparties, if the household may face future credit costs associated with the default. Here the situation is far simpler: a dollar gained by the mortgagor is a dollar lost to the mortgagee.

[Figure 8 about here.]

Figure 8 gives results for the United Kingdom. Clearly, the intransient investor achieves greater gains through mortgage timing. In both cases, it is the high interest rate environments, where fixing is optimal, where gains are greatest. We further note that in high credit spread environments, the gains for the borrower can be quite substantial.

[Figure 9 about here.]

Figure 9 shows results for Canada and Ireland. Given that break fees in both these countries are based on the retail methodology, the prepayment option is considerably less valuable than in the UK. For the case of Canada, in particular, we find that the gains for transient borrowers are negligible. Even for intransient borrowers, mortgage choice and prepayment is of small significance, rarely exceeding 0.1% of the loan's principal. Irish results are more substantial, with intransient borrowers frequently able to reduce loan value by 1%. Nevertheless, transient borrowers' gains are far more subdued.

[Figure 10 about here.]

Figure 10 shows results for Australia and New Zealand. For both countries, we observe the largest gains to be achieved in situations where the wholesale break fee loan is optimal (high credit spreads and high wholesale rates). Here the household can profit both from prepaying the forgiven portion of the loan, and also from prepaying the unforgiven portion.

Interestingly, comparing across the two countries, we observe markedly larger gains for Australian borrowers. This is driven by the difference in the process followed by s in the two countries. In New Zealand, mean reversion is quite rapid in credit spreads, resulting in deviations in credit spreads often being small and short lived. In contrast, Australia sees much larger and longer lived credit spread movements. These movements create an ideal environment for borrowers facing wholesale break fees to exercise their prepayment options.

4 Conclusion

In this paper, we have established a framework for modelling the prepayment timing and mortgage selection problem of borrowers in a collection of countries whose lending regimes are characterised by potentially substantial fees associated with unwinding of a fixed rate contract. Our findings suggest that for all the countries considered excepting Canada, there is economically significant gain to be achieved by borrowers at the expense of lenders.

Two comments regarding why this might occur are in order. First, it is difficult to see how banks can price this out of the market. The problem for the lender is the unobservability of λ_1 and λ_2. Under the US mortgage system, the use of points (see Stanton and Wallace (1998)) allows lenders to generate a finely graded separating equilibrium where borrowers of varying degrees of transience pay different up-front fees in order to reduce their mortgage rates. Without a points system in place, the countries in our sample have only a relatively coarse set of products available, and therefore to price contracts so as to eliminate borrowers' option values is difficult.

The second point relates to credit risk. As mentioned in the introduction of this paper, the banking systems in our collection of countries are characterised by recourse lending regimes, meaning that lenders can prosecute defaulting borrowers for the full value of their loan, rather than being limited to seizing the property which secures the mortgage (as is the case in many US states). This eliminates the default optionality which is common in US mortgages. However, it does not mean that default is costless for the borrower. Legal and administrative costs involved with default are still liable to be substantial, resulting in dead-weight losses in the event of default. Our model suggests that the households who best exploit prepayment opportunities are intransient households, who are also probably those households who are *less likely* to default on their loans. Indirectly, these "good risk" borrowers borrow more cheaply, since they extract gains from prepayment and loan selection. Prepayment options thus provide a mechanism for attracting more creditworthy borrowers.

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A Solving the mortgage valuation problem

Assuming a constant unit P_t, we solve equations (5) and (8) using the Craig and Sneyd (1988) Alternating Direction Implicit (ADI) finite difference algorithm. This technique works by splitting the operator in equation (8) across the dimensions of the problem (r and s). The technique then alternates which dimension is solved for implicitly, so that instead of requiring one huge system of equations to be solved, many smaller systems must be solved. Craig and Sneyd (1988) improve upon the regular Douglas (1962) style splitting by updating the cross-partial term between substeps of the algorithm. This maintains the second order accuracy of the method, allowing us to take larger time steps than would normally be possible when solving a problem with cross partial terms using ADI methods. The finite difference method calculates security values at discrete grid points $t = 0, \Delta t, 2\Delta t, \ldots, T$, $r = r_0, r_0 + \Delta r, \ldots, r_0 + N_r \Delta r$, $s = s_0, s_0 + \Delta s, \ldots, s_0 + N_s \Delta s$, where N_r and N_s are constant integers. Solution proceeds by backward induction, solving for f_t using the solution at $f_{t+\Delta t}$.[16]

When dealing with a security with declining principal (as is the case for mortgage valuation or hedge swap valuation), we value the security by rescaling the subsequent time step to account for the security's declining principal, i.e. working with a rescaled solution $\hat{f}_{t+\Delta t} = \frac{P_{t+\Delta t}}{P_t} f_{t+\Delta t}$ when evaluating the security value at time t (f_t).

We begin by using our algorithm to solve (5) in order to determine, for each point on our finite difference grid, $R_t(\tau, T)$ and $r_t(\tau, T)$ for $\tau = t + \Delta t, t + 1, t + 2, \ldots, \min(t + \bar{\tau}, T)$. This allows us to determine retail rates and associated hedge rates for new mortgages available to the borrower at any date.

Next, we work backwards from time T. At each time step, we solve (8), with $\lambda_1 = \lambda_2 = 0$ for all possible mortgages a household could hold. For a given type of break fee, this consists of: all possible levels of r_{t_0} and s_{t_0}, all possible current fixed terms $(\tau - t)$, and all possible initial fixed terms $(\tau - t_0)$. We repeat this exercise for each type of break fee available to the borrower, and also consider both the case where the mortgage has $\psi > 0$ and $\psi = 0$ (i.e. we consider mortgages who have already had their forgiven portion prepaid).

In tandem with solving for all the mortgages' values, we also solve (5) to value the annuity factors

[16] An accessible reference for PDE solution is Ames (1992). See in particular, pages 349–354.

and zero coupon bond prices needed to evaluate equations (12 - 14). For the valuation of the hedge swaps, we use a similar trick to the mortgages, by rescaling the solution to account for declining principal.

Having solved (8) for each possible mortgage, we find f^* for each point in the grid. This allows us to evaluate whether it is optimal to prepay each of our possible mortgages. Since we know the value of mortgages with $\psi = 0$, we can evaluate whether it would be optimal to partially repay, if this option has not been exercised yet. Finally, we adjust the solution to allow probability $\lambda_1 \Delta t$ of prepayment without refinancing and probability $\lambda_2 \Delta t$ of prepayment with refinancing.

Our solution consists of matrices of mortgage values for each time step in our solution. We can also produce matrices of the optimal fixing period to generate f^*. Lastly, by examining the points where the complementary slackness conditions (9-10) hold with equality, we can observe when a household should prepay their mortgage.

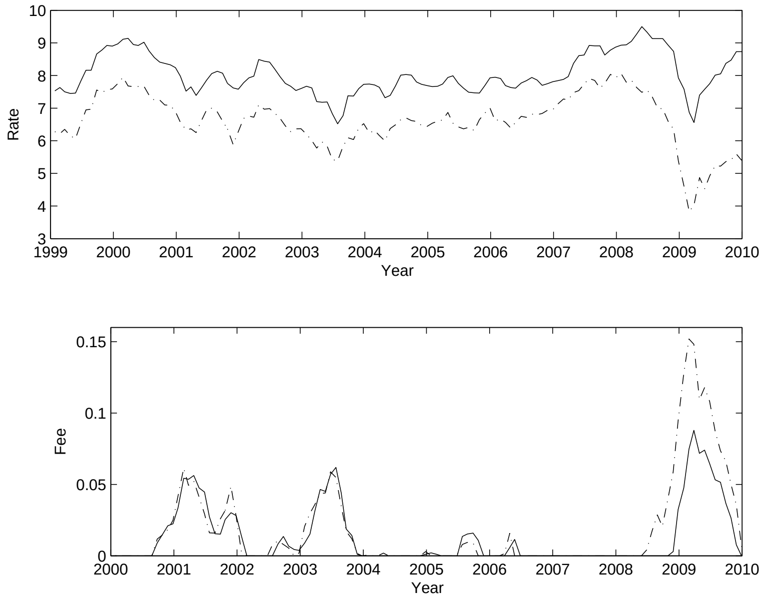


Figure 1: Top graph shows five year mortgage rates (solid) and corresponding hedge swap rates (dashed) for New Zealand. Both are measured in percent. Lower graph shows retail break fees (solid) and wholesale break fees (dashed) for the same period, assuming a mortgagor took out a five year fixed rate loan (with 30 year amortisation) and broke this one year later, as fraction of initial principal. Data sourced from Datastream and Reserve Bank of New Zealand.

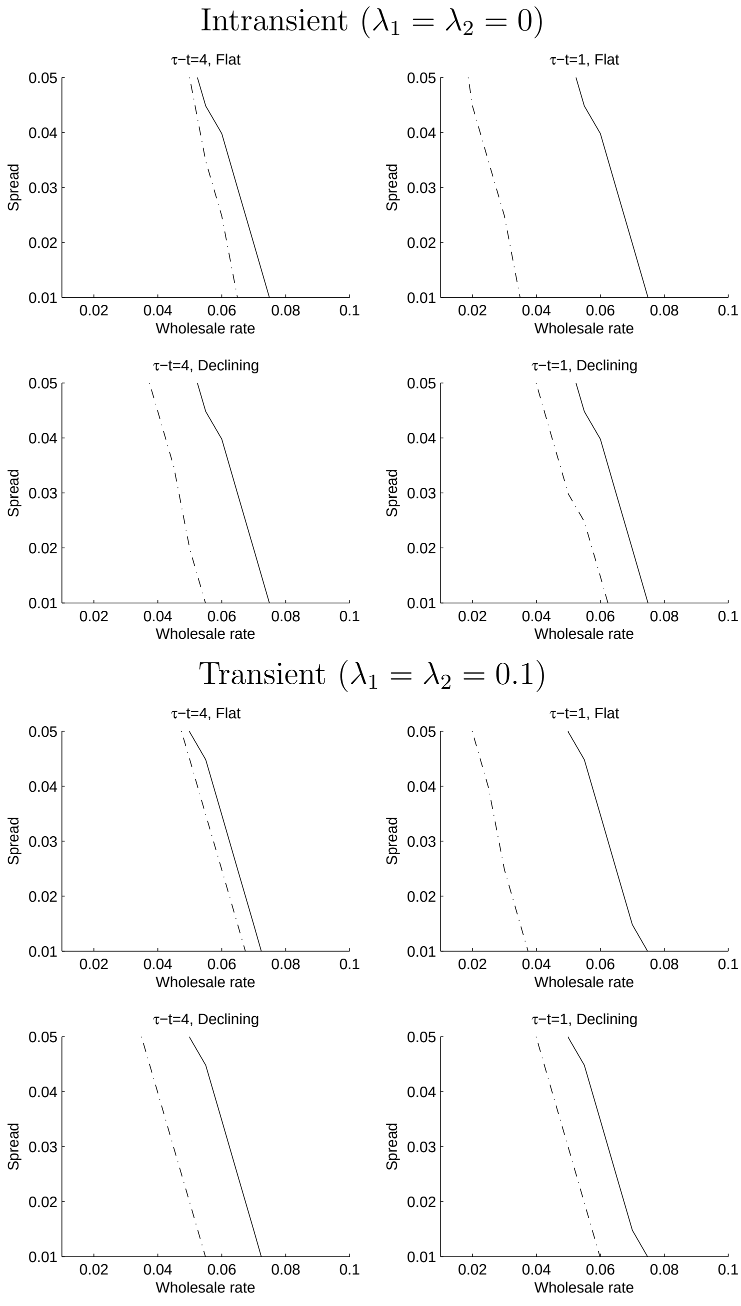


Figure 2: Optimal mortgage refinancing for the United Kingdom. Solid line is for break-fee exempt part of mortgage, while dashed line denotes exercise frontier for mortgage once forgiven portion has been exhausted. Mortgage is initally a thirty year loan, with five year fixed period.

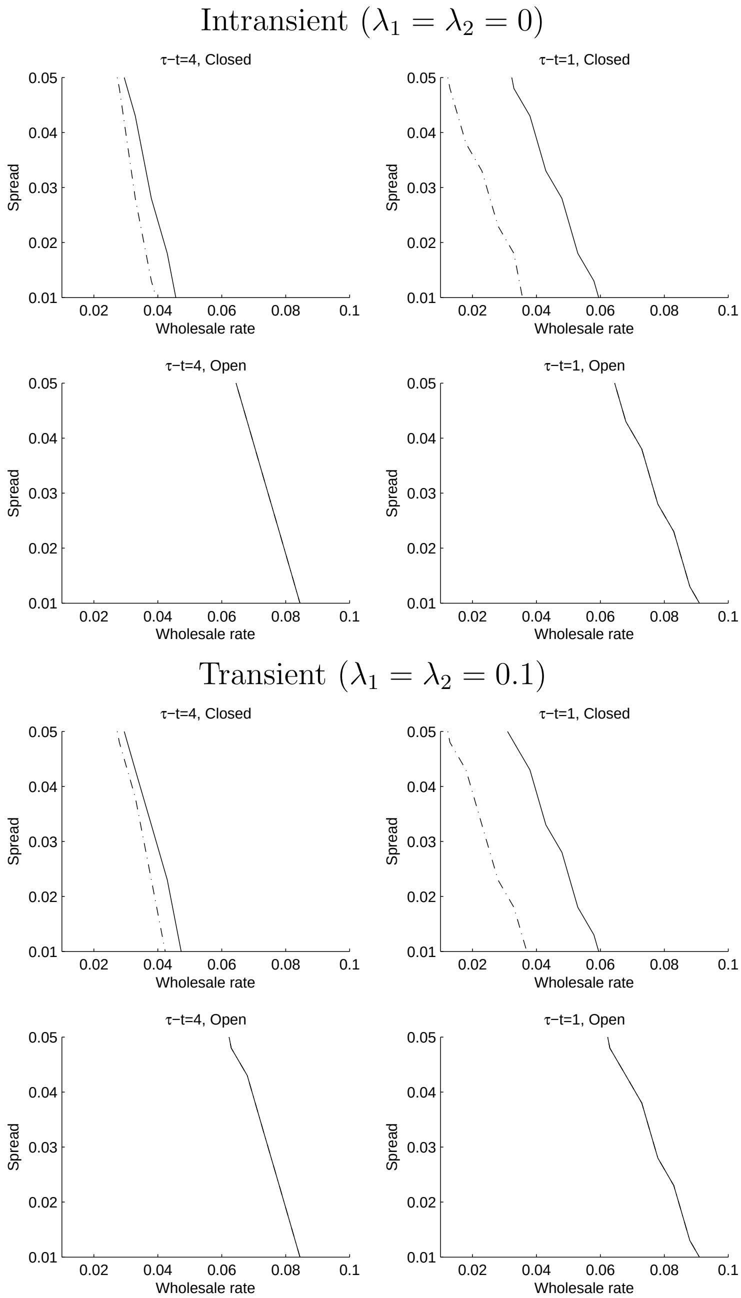


Figure 3: Optimal mortgage refinancing for Canada. Solid line is for break-fee exempt part of mortgage, while dashed line denotes exercise frontier for mortgage once forgiven portion has been exhausted. Mortgage is initally a thirty year loan, with five year fixed period.

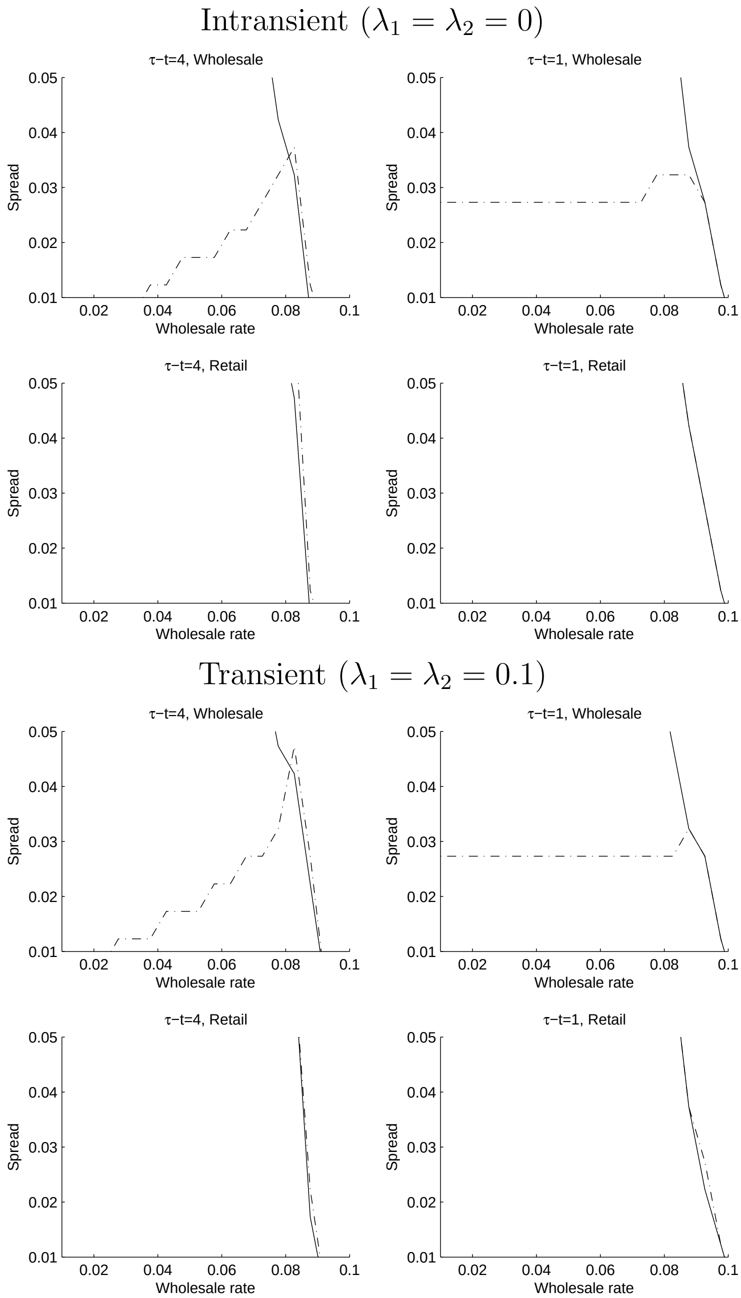
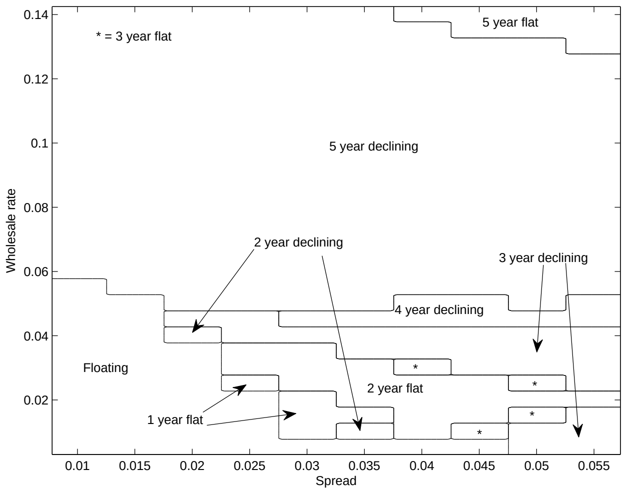


Figure 4: Optimal mortgage refinancing for New Zealand. Solid line is for break-fee exempt part of mortgage, while dashed line denotes exercise frontier for mortgage once forgiven portion has been exhausted. Mortgage is initally a thirty year loan, with five year fixed period.

United Kingdom



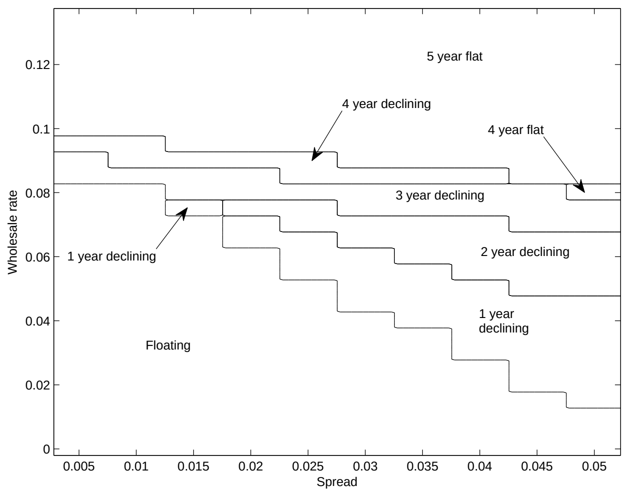


Figure 5: Optimal mortgage selection in the UK, for intransient (left) and transient (right) borrowers. Spread and wholesale rate are instantaneous (s and r respectively).

Canada

Ireland

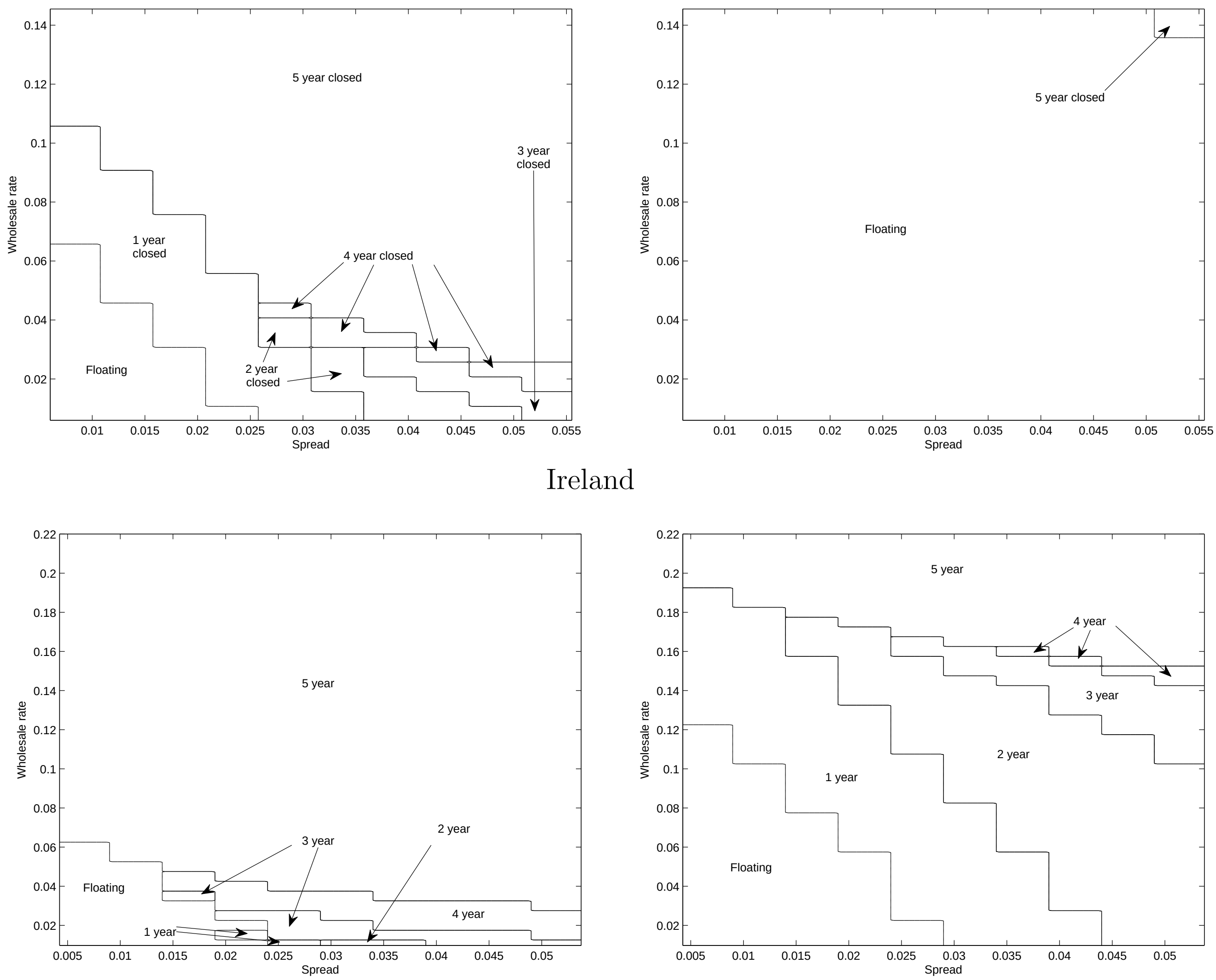


Figure 6: Optimal mortgage selection in Canada (top) and Ireland (bottom), for intransient (left) and transient (right) borrowers. Spread and wholesale rate are instantaneous (s and r respectively).

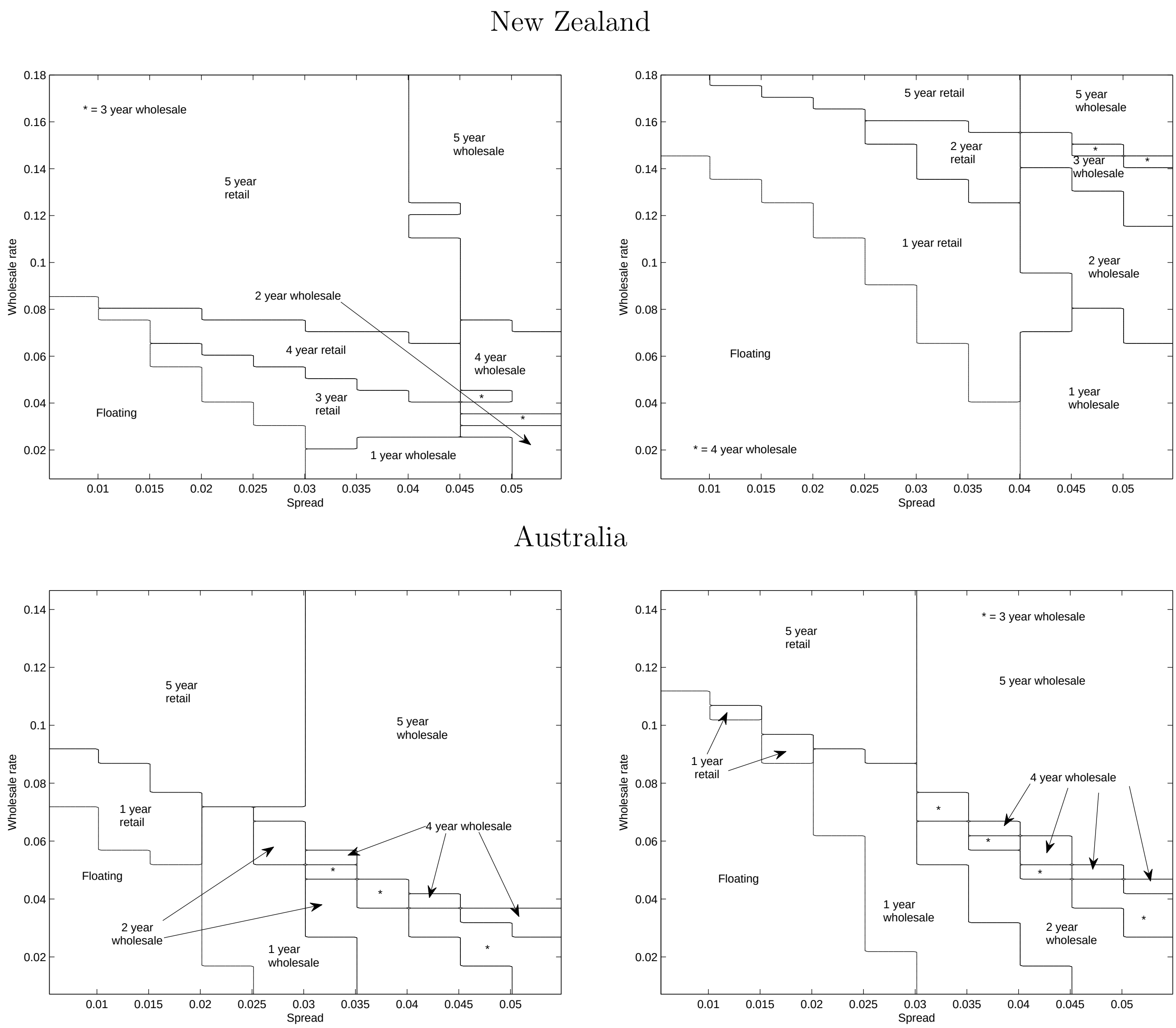


Figure 7: Optimal mortgage selection in New Zealand (top) and Australia (bottom), for intransient (left) and transient (right) borrowers. Spread and wholesale rate are instantaneous (s and r respectively).

United Kingdom

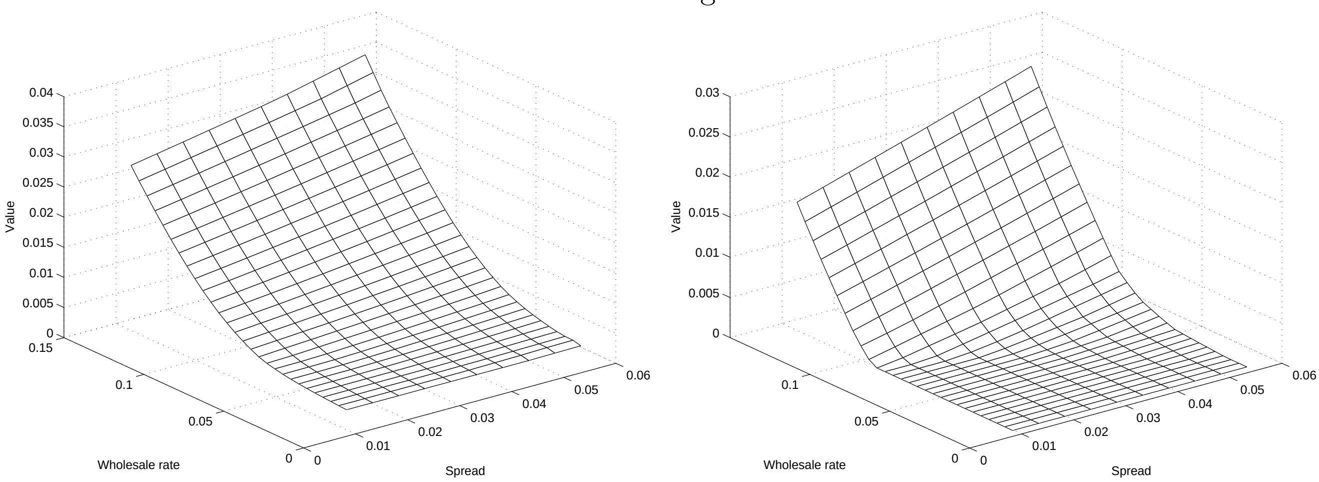


Figure 8: Value gains for investor (loss for bank) from prepayment exercise in the UK. Spread and wholesale rate are instantaneous (s and r respectively). Left graph represents the intransient case, while right graph represents the transient case.

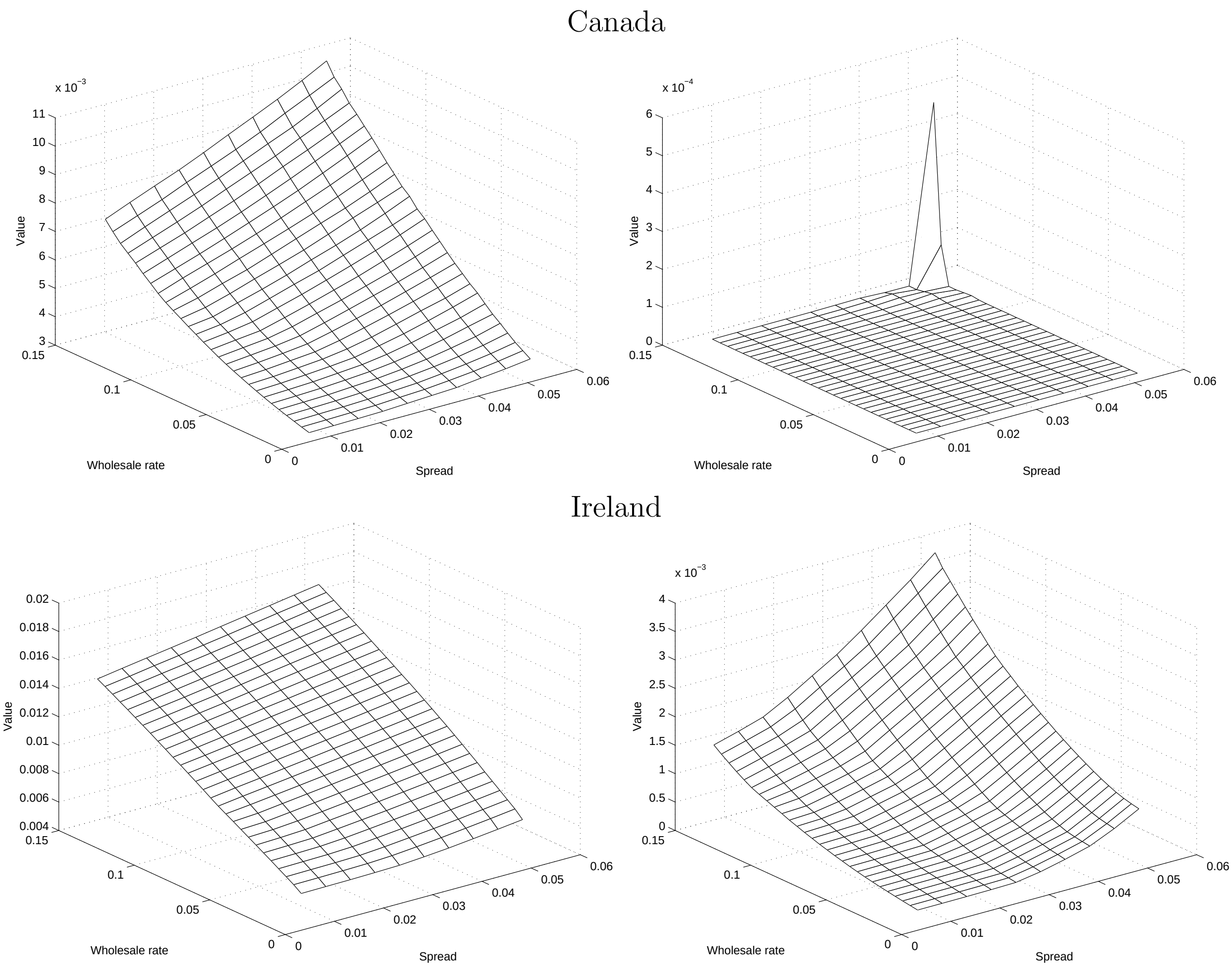


Figure 9: Value gains for investor (loss for bank) from prepayment exercise in Canada (top) and Ireland (bottom). Spread and wholesale rate are instantaneous (s and r respectively). In each case, left graph represents the intransient case, while right graph represents the transient case.

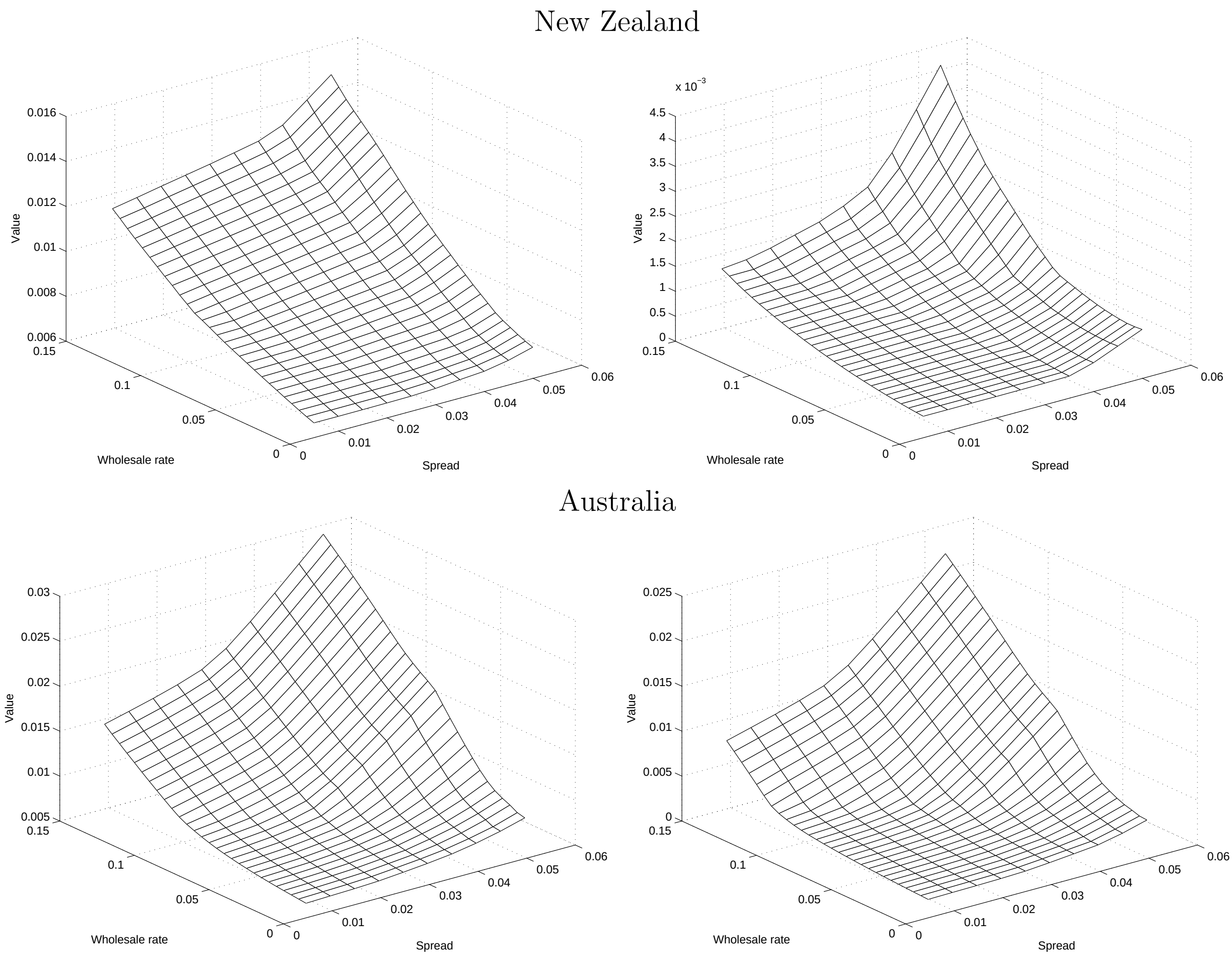


Figure 10: Value gains for investor (loss for bank) from prepayment exercise in New Zealand (top) and Australia (bottom). Spread and wholesale rate are instantaneous (s and r respectively). In each case, left graph represents the intransient case, while right graph represents the transient case.

Country	Break fee types
United Kingdom	Flat portion of principal, *or*
	Declining portion of principal.
Canada	Closed loan: retail break fee (generally with minimum charge), *or*
	Open loan: no break fee.
Ireland	Retail break fee.
New Zealand	Retail break fee, *or*
	Wholesale break fee.
Australia	Retail break fee, *or*
	Wholesale break fee.

Table 1: Summary of break fee regimes for countries in sample.

Parameter	Australia	Canada	Ireland	NZ	UK
μ_r	0.0481	0.0146	0.0263	0.0468	0.0414
a_r	0.2998	0.2129	0.7478	0.2024	0.4772
σ_r	0.0247	0.0149	2.1964	0.0257	0.0200
γ_r	0.4142	0.1627	1.7840	0.2209	0.2107
Short Wholesale	0.0391	0.0030	0.0048	0.0277	0.0050
Long Wholesale	0.0519	0.0245	0.0215	0.0544	0.0302
Wholesale Long Maturity	3	5	3	5	5
ξ_r	-0.7898	-1.3014	-1.4072	-0.8840	-0.2684
$\bar{r}$	0.0697	0.0743	0.1078	0.1166	0.0473
μ_s	0.0186	0.0228	0.0120	0.0207	0.0174
a_s	1.0587	1.3077	2.8515	3.4913	2.8360
σ_s	0.0184	0.0183	0.0449	0.0314	0.0144
γ_s	0.2597	0.1462	0.8222	0.2688	0.1072
ρ	-0.4962	-0.5352	-0.5581	-0.6780	-0.8813
Short Retail	0.0665	0.0360	0.0360	0.0600	0.0398
Long Retail	0.0760	0.0549	0.0359	0.0873	0.0571
Retail Long Maturity	3	5	(3)	5	5
$\hat{\xi}_s$	-1.4518	-0.9499	-1.9815	-3.6439	-2.7252
$\bar{s}$	0.0286	0.0308	0.0218	0.0339	0.0268

Table 2: Parameter estimates for the wholesale rate and spread processes. Estimates are generated by Generalised Method of Moments, except for the case of the UK, where Quasi-Maximum Likelihood is used. Short wholesale and short retail are the one month rates observed in December 2009. Long wholesale is the longer term (plain vanilla) swap rate observed at this date, while long retail is the longer maturity retail rate observed at this date. Maturity in each case refers to the maturity of rate observed. In the case of Ireland, the rate is an average of 1-5 year maturity rates, which we take to be representative of a 3 year rate. $\bar{r}$ and $\bar{s}$ are the steady state levels for r and s under the risk-neutral probability measure $\mathbb{Q}$.

Rate	UK	NZ	Canada
$R_0(5,30)$	0.0906	0.1261	0.0805
$r_0(5,30)$	0.0627	0.0926	0.0495
r_0	0.0850	0.0827	0.0380
s_0	0.0348	0.0333	0.0330

Table 3: Five year retail rates, five year hedge rates, short wholesale rates and short spreads for the analysis in section 3.1.