THE OPTIMAL EXIT DATE FROM A DEFINED BENEFITS PENSION SCHEME

December 2010

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Abstract

This paper examines the issue of the optimal exit date for an employee in a defined benefits pension scheme, whether through retirement or job shifting. The optimal decision depends upon a number of parameters including the cost of shifting jobs. If the employee enters the scheme at the traditional age of about 25, and job shifting is costless, the optimal course of action is to shift jobs at 45-60, which is significantly before the traditional retirement age of 65. However, if job switching incurs an effective salary reduction of 20% and the salary reduction that would prompt retirement from all work is in excess of this, then job switching should be delayed by about 10 years.

1. Introduction

Pension schemes are broadly of two types: those in which both employers and employees explicitly contribute to a savings scheme which is invested in a portfolio chosen by the employee and accessible by them at some point (defined contributions scheme), and those in which employees contribute some proportion of their salary in return for a stream of pension entitlements from their retirement date until their death based upon their retiring salary and the years for which they have contributed (defined benefits scheme). In respect of the latter schemes, deferral of the retirement date from the job reduces the employee's residual life expectancy whilst raising the contribution period and the real final salary, and the net effect of these forces must eventually reduce the present value of the pension entitlements net of the contributions. Consequently, in such schemes, the optimal retirement date from the job will tend to be accelerated, either through switching to another job or by retirement from all work. If it is possible to costlessly switch jobs, then it will be optimal to do so and at the age that maximises the present value of the pension entitlements net of contributions providing that this is not earlier than the planned retirement date from all work. If job switching is costly, in the form of a lower salary and/or a less appealing job, then the benefits from job switching in the form of avoiding the unfavourable impact upon the present value of the pension entitlements must be balanced against the adverse salary and/or job conditions from doing so.

In view of these considerations, this paper seeks to determine the following. Firstly, if job shifting is costless, this paper seeks to determine the optimal time for job switching. Secondly, if job shifting is costly, this paper seeks to determine the salary reduction that is equivalent to the adverse impact upon the present value of the pension entitlements when working in the current job beyond the "optimal" point, so as to assist the employee in deciding whether or not to shift jobs or possibly even retire from all work. To do both of these, it is necessary to characterise a defined benefit scheme and we do so in a relatively simple fashion, i.e., the employee contributes a fixed proportion of their salary over the period in which they are employed, the pension benefits constitute a stream of payments to the contributor from their retirement date until their death, death before retirement leads to a refund of the employee's contributions to their estate, and the payments received during the employee's post-retirement life are tax free, inflation adjusted, and proportional to the final salary and the number of years for which contributions were made.

2. The Model

2.1 Analysis at the Entry Date

We start by analysing the problem as at the date the employee enters the pension scheme. The employee commences contributions towards a defined benefit scheme from some age (year 0) for a period of N years, at which point they either retire or switch jobs. The first step is to determine the relationship between N and the present value of the pension entitlements net of contributions, as at time 0. We let S denote the employee's initial salary, p the proportion of their salary that they contribute, i the expected CPI inflation rate, g the expected real growth rate in their salary, and q the (real) pension payment per \$1 of final salary and per year of contributions. For analytical convenience, we also characterise their death as occurring on one of their birthdays. So, if they are alive at a particular age, they will be alive over the following year and therefore will make the contributions from their salary over the entire course of that year. Consistent with standard practice in present valuing, such contributions are assumed to be made at the end of the year.

As at time 0, the net present value of the investment to the employee is the present value of the pension entitlements commencing in N years time plus the present value of the refunded contributions arising in the event of the employee dying within N years less the present value of the salary contributions made until the earlier of time N or their death. We let P_t^a denote the conditional probability that the employee is alive in t years given that they were alive one year earlier, P_t^d the conditional probability that the employee is dead in t years given that they were alive one year earlier, V_N the expectation of the present value at time N of the subsequent pension entitlements conditional upon the employee surviving until time N, R_t the expected refund paid in t years conditional upon the employee dying at that point, C_t the expected contributions paid in t years time conditional upon the employee being alive at the beginning of that year, and k_n the nominal discount rate relevant to V_N, R_t and C_t. The net present value now of the employee's investment for N years is then as follows:

$$NPV_0^N = \frac{P_1^a P_2^a ... P_N^a V_N}{(1+k_n)^N} + \frac{P_1^d R_1}{1+k_n} + ... + \frac{P_1^a P_2^a ... P_{N-1}^a P_N^d R_N}{(1+k_n)^N}$$

$$- \frac{C_1}{1+k_n} - \frac{P_1^a C_2}{(1+k_n)^2} - .... - \frac{P_1^a P_2^a ... P_{N-1}^a C_N}{(1+k_n)^N} \qquad (1)$$

The expected contributions for year *t* conditional upon the employee being alive at the beginning of that year are proportion *p* of the expected salary for that year and hence

$$C_t = S(1+g)^t (1+i)^t p \qquad (2)$$

The expected refund for year *t* conditional upon the employee dying at the end of that year is the sum of the expected contributions until the end of year *t*, as follows:

$$R_t = Sp\left[(1+g)(1+i) + ... + (1+g)^t (1+i)^t\right]$$

$$= \frac{Sp(1+g)(1+i)\left[1-(1+g)^t (1+i)^t\right]}{1-(1+g)(1+i)} \qquad (3)$$

We turn now to V_N, which is the expectation now of the present value at time *N* of the subsequent pension entitlements conditional upon the employee surviving until time *N*. Under this condition, they will receive the pension payment over the first year of their retirement and this is treated as being received at the end of that year.[1] However, they receive the payment one year later only if they are still alive at time *N*+1, and they receive a payment one year after that only if they are still alive at time *N*+2, etc. Letting B_R denote the expectation now of the annual pension entitlement (prior to the post-retirement inflation adjustment), and *k* the discount rate on the real stream, it follows that

$$V_N = B_R\left[\frac{1}{1+k} + \frac{P_{N+1}^a}{(1+k)^2} + \frac{P_{N+1}^a P_{N+2}^a}{(1+k)^3} + ...\right] \qquad (4)$$

In addition

$$B_R = S(1+g)^N (1+i)^N Nq \qquad (5)$$

[1] The assumption of receipt at year end is standard practice in financial models.

In respect of the discount rate k, given that the pension entitlements are inflation protected and risk free (in real terms) except for the uncertainty surrounding the date of death of the retiree (which is not systematic in the CAPM sense), the most suitable asset to use for determining this discount rate is inflation-protected government bonds. However, both the real yield and the inflation component on these bonds are generally taxable and therefore we cannot discount the real tax-free pension benefits at the real pre-tax yield on these bonds. Recalling that B_R is the real tax-free pension benefit over the first year following retirement, and letting P denote its present value at the beginning of the year, y the real yield on inflation-protected bonds at retirement date, T the (average) tax rate on the nominal return on these bonds, and i the expected CPI inflation rate at that time, then P must be such that its investment into inflation-protected bonds for one year would yield the expected post-tax payoff equal to $B_R(1 + i)$, i.e.,

$$P\{1+[(1+y)(1+i)-1](1-T)\} = B_R(1+i)$$

It follows that this present value P can be expressed as follows:

$$P = \frac{B_R(1+i)}{1+[(1+y)(1+i)-1](1-T)} = \frac{B_R}{\frac{1+[(1+y)(1+i)-1](1-T)}{1+i}}$$

So, the appropriate discount rate k on the real tax-free pension benefit B_R is[2]

$$k = \frac{1+[(1+y)(1+i)-1](1-T)}{1+i} - 1 \quad (6)$$

Finally, in respect of the discount rate k_n applicable to V_N, R_t and C_t, these quantities are inflation adjusted as shown in equations (2) to (5) and they are also tax exempt. This suggests that the appropriate discount rate k_n is the nominal counterpart to k, i.e.,

$$k_n = [(1+y)(1+i)-1](1-T) \quad (7)$$

[2] This analysis follows Lally (2000).

However, the sums being discounted by k_n are subject to real risk arising from the future real growth rate in the employee's salary and this raises the question of whether they are subject to systematic risk in the CAPM sense. Theory offers no clear guidance on this matter: unexpected productivity increases in the economy will tend to raise both market returns and real salaries, implying a positive beta, whilst some real salary increases may be at the expense of profits implying a negative beta. Accordingly, empirical evidence is considered. The relevant beta here involves the relationship between changes in two "stock" variables: the present value of the pension entitlements and the market value of equity. Since the former is not observable, one must resort to "flow" variables, involving a regression of real salaries on real dividends. Exley et al (1997, Figure 7.2) present time series graphs on each of these two variables for each of the UK, France and Germany over long time spans, and they reveal regression coefficients that are close to zero for each country. This suggests that the beta is close to zero and therefore k_n should be determined in accordance with equation (7).

We now turn to the optimal strategy for the employee. If the employee can costlessly shift jobs, then they should do so after N^* years, where N^* maximises the NPV shown in equation (1), assuming this point occurs before retirement would have otherwise occurred. By contrast, if job switching is costly, we determine the salary reduction proportion Z that is equivalent to the reduction in the present value of the pension entitlements from continued contribution to the scheme for one further year, so as to assist the employee in deciding whether or not to shift jobs or possibly even retire from all work. Letting the investor's marginal tax rate on the salary change be denoted T_m, this value Z satisfies the following equation:

$$NPV_0^N - NPV_0^{N+1} = \frac{ZS(1+i)^{N+1}(1+g)^{N+1}(1-T_m)}{(1+k_n)^{N+1}}$$

and hence

$$Z = \frac{NPV_0^N - NPV_0^{N+1}}{S(1-T_m)\left[\frac{(1+i)^{N+1}(1+g)^{N+1}}{(1+k_n)^{N+1}}\right]} \tag{8}$$

Thus, the employee should switch jobs at the point N where Z as determined in the last equation first exceeds the effective salary reduction from job shifting (or the salary reduction in the present job that would provoke early retirement if it is smaller than that from job shifting), unless retirement from all work is optimal at an earlier point.

2.2 Analysis at the Possible Exit Date

The preceding analysis is conducted as at the time the employee first enters the pension scheme, i.e., the point at which the employee exits the pension scheme is determined at the time of entry. However, it is suboptimal to act in this way because it sacrifices the opportunity to take account of subsequent information. The better approach is to compare, at each point in time, the consequences of retirement from the current job with continued employment there. If job switching is costless, then the employee should continue working at the current job until the present value of the pension benefits from immediate retirement is superior to the present value of benefits (net of incremental contributions) from deferring retirement from this job for one or more further years, and then switch to an alternative job. By contrast, if job switching is costly, then the employee should consider the salary reduction that they would suffer in the course of switching to an otherwise equivalent job, or the salary reduction in the present job that would prompt early retirement from all work if this is less than the salary reduction from job shifting, and balance this against the pension benefit from retirement from the current job.

Suppose N years have passed since the employee entered the pension scheme. If they retire from the current job at this point, then the pension benefits have the present value V_N given by equation (4) except that the first pension payment reflects the employee's actual salary at this time (S_N):

$$V_N = S_N N q \left[\frac{1}{1+k} + \frac{P_{N+1}^a}{(1+k)^2} + \frac{P_{N+1}^a P_{N+2}^a}{(1+k)^3} + ... \right] \qquad (9)$$

By contrast, if retirement from the current job is delayed by one year to year N+1, then the NPV at time N of the future pension consequences will be as follows:

$$NPV_N^{N+1} = \frac{P_{N+1}^a V_{N+1}}{(1+k_n)} + \frac{P_{N+1}^d R_{N+1}}{1+k_n} - \frac{C_{N+1}}{(1+k_n)} \tag{10}$$

Similarly, if retirement from the current job is delayed for two years to year N+2, then the NPV at time N of the future pension consequences will be as follows:

$$NPV_N^{N+2} = \frac{P_{N+1}^a P_{N+2}^a V_{N+2}}{(1+k_n)^2} + \frac{P_{N+1}^d R_{N+1}}{1+k_n} + \frac{P_{N+1}^a P_{N+2}^d R_{N+1}}{(1+k_n)^2} - \frac{C_1}{1+k_n} - \frac{P_{N+1}^a C_2}{(1+k_n)^2} \tag{11}$$

We now turn to the optimal strategy for the employee. If the employee can costlessly shift jobs, then the employee should do so at the point N where V_N as shown in equation (9) exceeds each of the NPVs shown in equations (10) and (11)[3], unless retirement from all work is optimal at an earlier point. By contrast, if job switching is costly, we determine the salary reduction proportion Z that is equivalent to the reduction in the present value of the pension entitlements from continued contribution to the scheme for one further year, so as to assist the employee in deciding whether or not to shift jobs or possibly even retire from all work[4]. This value Z satisfies the following equation:

$$V_{N^*} - NPV_N^{N+1} = \frac{ZS(1+i)^{N+1}(1+g)^{N+1}(1-T_m)}{1+k_n}$$

and hence

$$Z = \frac{V_N - NPV_N^{N+1}}{S(1-T_m)\left[\dfrac{(1+i)^{N+1}(1+g)^{N+1}}{1+k_n}\right]} \tag{12}$$

[3] It is sufficient to consider only the NPVs shown in equations (10) and (11) on the grounds that, if V_N exceeds the NPV from deferring retirement from the current job for both one and two years, it will almost certainly exceed the NPVs from deferring retirement from the current job for more than two years.

[4] Consideration of deferring the job switch for one year presumes that deferral for one year is the best deferral option. If deferral for two years were superior to deferral for one year, then deferral for two years would displace deferral for one year. However, the subsequent analysis shows that deferral for one year always dominates deferral for two years.

If the effective salary reduction from job switching (or the salary reduction that would prompt early retirement from all work, if this is smaller than that from job shifting) is considered to be less than *Z*, the employee should immediately retire from the present job. Otherwise, the employee should remain the present job for one further year and then reconsider the decision at that point following the same process.

3. Parameter Estimates

In order to implement these models, we require estimates for a number of parameters, comprising the real yield *y* on inflation-protected bonds, the conditional probabilities for survival P_t^a, the expected CPI inflation rate *i*, the expected real growth *g* in the employee's salary over the contribution period, and the tax rate *T*. In order to offer conclusions that are not specific to a particular moment in time, we will consider the average situation over the last ten years in conjunction with the most extreme situations in the same period. New Zealand data are used. The employee's initial salary *S* is immaterial and is therefore treated as $1.

In respect of *y*, which is the yield on inflation-protected government bonds at the retirement date from the job, this has averaged 3.9% in New Zealand over the last ten years (Dec 1999 to Dec 2009) and ranged from 2.7% (December 2009) to 5.3% (Feb 2000 average) within that period (Reserve Bank, 2009b).

In respect of *i*, which is the expected CPI inflation rate per year from the time of retirement, we use consensus forecasts presented by the New Zealand Institute of Economic Research (NZIER). These have averaged 2.3% over the last ten years and have ranged from 1.9% to 2.8% (NZIER, Dec 1999 to Dec 2009).

In respect of the set of conditional probabilities for survival P_t^a, the most recent estimates provided by the New Zealand Statistics Department are for 2005-2007, preceded by those for 2000-2002. Accordingly, we use the 2005-2007 Life Tables as representative of the last ten years (Statistics Department, 2009b). These tables provide figures for both men and women, and we use the figures for males.

Finally, in respect of the real growth rate in the employee's salary, this comprises the real growth rate for salaries in general coupled with the promotional effect for an individual. Regarding real growth for salaries in general, this has averaged 1% per year in New Zealand over the last 20 years (March 1988 to Sept 2009, Reserve Bank of New Zealand, 2009a).[5] The promotional effect is much harder to estimate, and we therefore consider values of up to 2% per year (equivalent to a salary growth of 120% over 40 years). Accordingly, values for g ranging from 1% to 3% are considered.

4. Results

4.1 Analysis at the Entry Time

We start by considering the optimal employee action based upon information available at the time the employee enters the pension scheme. Consistent with the analysis in section 3, suppose that the expected real growth rate in the employee's salary over the contribution period is $g = .02$, the expected CPI inflation rate over the same period is $i = .025$, and the real yield on inflation-protected bonds at the retirement point is $y = .04$. Furthermore, the tax rate T is 30%. Following equations (6) and (7), the discount rate on the real benefits is then $k = .0207$ and its nominal counterpart is $k_n = .0462$. In addition, we assume that the employee's pension contribution rate is $p = .06$ and the (real) pension payment per \$1 of final salary and per year of contributions is $q = .01$. Finally, in order to approximate what we understand to be the typical situation, we suppose that the employee enters the pension scheme at the age of 25.

We start with a contribution period of $N = 40$ years providing the employee lives that long. Following equation (5), the expected annual pension payment to the employee (before the post-retirement inflation adjustment) would then be

$$B_R = (1.025)^{40}(1.02)^{40}(40)(.01) = \$2.37$$

[5] This is based upon an average growth rate in hourly earnings of 3.7% and average inflation of 2.7%. The Reserve Bank table does not provide data on the former growth rate prior to March 1988, and hence the calculation is limited to the period since then.

Following equation (4), the expectation now of the value in 40 years time of the subsequent pension entitlements per $1 of initial salary, and conditional upon the employee being alive in 40 years, would be

$$V_{40} = \$2.37\left[\frac{1}{1.0207} + \frac{0.9871}{(1.0207)^2} + \frac{(0.9871)(0.9858)}{(1.0207)^3} + \ldots.\right] = \$34.92$$

Following equation (3), the expected refunds at the end of years 1, 2, ...40 per $1 of initial salary, and conditional upon the employee dying at those points, would be as follows:

$$R_1 = \frac{.06(1.02)(1.025)\left[1-(1.02)(1.025)\right]}{1-(1.02)(1.025)} = \$0.0627$$

$$R_2 = \frac{.06(1.02)(1.025)\left[1-(1.02)^2(1.025)^2\right]}{1-(1.02)(1.025)} = \$0.1282$$

etc

Following equation (2), the expected contributions at the end of years 1, 2, ...40 per $1 of initial salary, and conditional upon the employee being alive at the beginning of those years, would be as follows:

$$C_1 = (1.02)(1.025)(.06) = \$0.0627$$

$$C_2 = (1.02)^2(1.025)^2(.06) = \$0.0656$$

etc

Following equation (1), the net present value now of the employee's investment in the pension scheme over the next 40 years is then as follows:

$$NPV_0 = \frac{(0.9990)\ldots(0.9883)\$34.92}{(1.0462)^{40}}$$

$$+\frac{(0.0010)\$0.0627}{1.0462} + \ldots. + \frac{(0.9990)\ldots(0.9894)(0.0117)\$6.795}{(1.0462)^{40}}$$

$$-\frac{\$0.0627}{1.0462} - \frac{(0.9990)\$0.0656}{(1.0462)^2} - \ldots. - \frac{(0.9990)\ldots(0.9894)\$0.356}{(1.0462)^{40}}$$

$$= \$5.00 + \$0.13 - \$2.29$$
$$= \$2.84$$

So, for N = 40 years, the NPV is $2.84 per $1 of initial salary, comprising a present value of $5 for the pension entitlements received subsequent to the retirement point, plus $0.13 for the present value of the refunds received prior to retirement (arising in the event of the employee dying before retirement), less the present value of the contributions of $2.29. Thus the present value of the benefits ($5.13) is over twice that of the contributions. These results are shown in Table 1 for a range of alternative values for the contribution period N.

Table 1: Present Value of Pension Benefits

N	Benefits	Conts	NPV	Z
10	$2.81	$0.60	$2.21	
20	$4.72	$1.18	$3.54	
25	$5.28	$1.47	$3.81	
27	$5.42	$1.58	$3.84	0.02
30	$5.55	$1.75	$3.80	0.09
35	$5.48	$2.02	$3.46	0.17
40	$5.13	$2.29	$2.84	0.27
45	$4.48	$2.53	$1.95	
50	$3.64	$2.76	$0.88	

The present value of the contributions is (naturally) monotonically increasing in N whilst the present value of the benefits is concave in N because the monotonic increase in the present value of the annual (real) retirement entitlement (B_R) is eventually outweighed by the reduction in the expected period over which this entitlement is received. Consequently the NPV is concave in N. As indicated by the table, the NPV is maximised at $3.84 by retiring after 27 years (which is at age 52) rather than the traditional figure of 40 years (at age 65).

This analysis is based upon the values for various parameters, most particularly g, y, p and q. So, the optimal value for N is determined for various alternative combinations of values for these parameters and the results shown in Table 2. The results range from 18 to 36 years. So, even at the upper end of the range, the optimal value for N is still less than the figure of 40 years corresponding to retirement at the traditional figure of 65 years.

Table 2: Optimal Value of N

		$g = .01$		$g = .03$	
		$y = .03$	$y = .05$	$y = .03$	$y = .05$
$p = .04$	$q = .007$	25	21	34	29
	$q = .013$	27	23	36	31
$p = .08$	$q = .008$	22	18	30	24
	$q = .013$	25	21	34	29

We now turn to the optimal strategy for the employee, using the parameter values underlying Table 1. If it is costless to switch jobs then, and as shown in Table 1, the optimal strategy is to do so after 27 years, so as to initiate the pension payments at that point. If there are costs to switching jobs, in the form of a lower effective salary, we determine the salary reduction that is equivalent to the reduction in the present value of the pension benefits from continuing to contribute towards the scheme for one further year. As shown in the last column of Table 1, and following equation (8) with a marginal tax rate of 40%, these salary reductions range from 2% after 27 years (age 52) to 27% after 40 years (age 65). Thus, if job switching incurs an effective salary reduction of 20%, and the salary reduction that would prompt retirement from all work is 25%, then job switching should occur after 36 years (age 61). Alternatively, if job switching incurs an effective salary reduction of 20%, and the salary reduction that would prompt retirement from all work is 15% then the employee should retire from all work after 34 years (age 59).

4.2 Analysis at the Possible Exit Time

We now turn to assess the optimal course of action based upon information available at the time of the proposed "retirement" from the current job. The model has been presented in section 2.2. If job shifting is costless, the employee should switch jobs at the time when the present value of the pension benefits arising from that point exceeds the present value of benefits (net of incremental contributions) from deferring job switching for one or more further years. By contrast, if job switching is costly, we determine the salary reduction proportion Z that is equivalent to the reduction in the present value of the pension entitlements from continued contribution to the scheme for one further year, so as to assist the employee in deciding whether or not to shift jobs or possibly even retire from all work.

We illustrate this process, using the same parameter values as those used in the previous section. Thus, any difference in decisions will be due solely to the act of survival until the time of the decision. So, we assume that an employee enters the scheme at age 25, and that at each subsequent point in time the expected real growth rate in the employee's salary is $g = .02$, the expected CPI inflation rate is $i = .025$, the real yield on inflation-protected bonds is $y = .04$, and the tax rate T is 30%. Following equations (6) and (7), at any point in time, the discount rate on the real benefits is then $k = .0207$ and its nominal counterpart is $k_n = .0462$. In addition, as before, we assume that the salary contribution rate is $p = .06$ and the (real) pension payment per \$1 of final salary and per year of contributions is $q = .01$. In addition, the realised inflation and real salary growth rates are assumed to match the expected rates.

We start by assuming that job shifting is costless. After one year, the employee's salary would be as follows:

$$S_1 = \$1(1.02)(1.025) = \$1.0455$$

Following equation (9), the value at that point of the future pension benefits contingent upon the employee being alive and job shifting at that point would be as follows:

$$V_1 = \$1.0455(1)(.01)\left[\frac{1}{1.0207} + \frac{0.99899}{(1.0207)^2} + \frac{(0.99899)(0.99902)}{(1.0207)^3} + ...\right] = \$0.329$$

If job shifting is deferred for one year, then the employee's salary at that subsequent point will be expected to be \$1.093. Following the same process, the expectation at time 1 of the value of the pension entitlements commencing one year later, and contingent upon the employee being alive and job shifting at that later point, would be as follows:

$$V_2 = \$1.093(2)(.01)\left[\frac{1}{1.0207} + \frac{0.99902}{(1.0207)^2} + \frac{(0.99902)(0.99905)}{(1.0207)^3} + ...\right] = \$0.686$$

Following equation (10), the NPV at time 1 from deferring job shifting for one year is as follows:

$$NPV_1^2 = \frac{(0.99899)\$0.686}{1.0462} + \frac{(0.00101)\$\$0.128}{1.0462} - \frac{\$0.066}{1.0462} = \$0.618$$

Since this NPV at time 1 exceeds the present value of job shifting at time 1 (\$0.329), it is optimal to continue working in this job for at least one further year and therefore it is unnecessary to consider the possibility of deferring job shifting for more than one year.

We therefore shift forward one further year, to time 2, and repeat the process. If the employee is alive at that point, job shifting at that point yields V_2 = \$0.686 as noted above whilst deferring job shifting for one year yields NPV_2^3 = \$0.950. So, again, deferring job shifting for at least one year is warranted.

We continue in this way, with the benefit from deferral relative to immediate job shifting progressively declining. If the employee survives for 27 years, then job shifting at that point yields V_{27} = \$19.07 whilst deferral for one year yields NPV_{27}^{28} = \$19.05. Since deferral for one year is inferior to immediate job shifting, then the consequences of deferral for two years are examined. Following equation (11), this yields NPV_{27}^{29} = \$18.99. Deferral for even longer than two years yields even lower values. So, for the first time, job shifting is the optimal course of action.

All of these results are shown in Table 3, with the benefit from deferring job shifting for one year shown in the penultimate column. So long as the employee survives until that point, job shifting in 27 years (age 52) is optimal, which matches the result from the analysis in the previous section. However, such a match in results is not certain even if the same values for *g*, *i* and *y* are used and the realised values for *i* and *g* match the expected values because the analysis in the previous section takes place at the employee's age of 25 whilst the analysis in the present section is performed at subsequent ages; in the first analysis, death prior to retirement at age 52 is possible and is formally recognised through the survivorship probabilities, whilst in the second analysis the employee is presumed to have survived till the age of 52. The fact that the results match can be attributed to the low death probabilities from age 25 to 52.

Table 3: Implications of Job Shifting at Various Ages with *g* = .02

Year	V_t	NPV_t^{t+1}	NPV_t^{t+2}	V_t - NPV_t	Z
1	\$0.33	\$0.62		-\$0.29	
2	\$0.69	\$0.95		-\$0.26	
.	.	.		.	
.	.	.		.	
26	\$17.96	\$17.98		-\$0.02	
27	\$19.07	\$19.05	\$18.99	\$0.02	0.01
30	\$22.55	\$22.38	\$22.17	\$0.17	0.07
35	\$28.71	\$28.16	\$27.56	\$0.55	0.18
40	\$34.91	\$33.81	\$32.64	\$1.10	0.28

We now turn to the situation in which job shifting is costly. After 27 years of contributions, remaining in the current job for one further year reduces the present value of the pension benefits by \$0.02, as shown in the penultimate column of Table 2. Following equation (12), this is equivalent to a salary reduction of 1% of the prevailing salary using a marginal tax rate of 40%, as shown in the final column of Table 3 and denoted *Z*. After 30 years, the equivalent salary reduction rises to 7%, and rises further to 28% after 40 years, as also shown in Table 3. Thus, if job switching incurs an effective salary reduction of 20% and the salary reduction that would prompt retirement from all work is 25%, then job switching should occur after 36 years (age 61). Alternatively, if job switching incurs a salary reduction of 20% and the salary reduction that would prompt retirement from all work is 15%, then the employee should retire from all work after 34 years (age 59). As with the case of costless job shifting, these results match those in the previous section.

4.3 Further Analysis at the Possible Exit Time

The previous section assesses the optimal course of action based upon information available at the time of the proposed job shifting, but subject to otherwise using the same parameter values as those estimated at the time of entering the pension scheme. We now consider the impact of various parameter values changing subsequent to the employee entering the pension scheme.

The salary growth rate over the period from entering the scheme until the proposed time of job shifting (inflation and real growth) has no impact upon the value of *Z* as shown in equation (12), and therefore on the optimal decision, because it raises the critical terms in that equation by the same proportion. In addition, the expected future inflation rate from the proposed time of job shifting has no impact upon the principal terms within equation (12), and therefore upon the value of *Z*, and hence upon the optimal decision.

However, the expected future real growth rate in the employee's salary significantly affects V_{N+1} and hence NPV_N^{N+1} in equation (10), with flow-through to equation (12). For example, if this expected real growth rate falls from 2% to 1% during the course of the first year following the employee's entry to the pension scheme, and remains at this lower level, with the realised rate matching this expected rate, then the results are as shown in Table 4. Relative to Table 3, the employee's critical actions are shifted forwards in time by about four years. For example, if job shifting is costless, it should occur after 23 years (age 48) rather than 27 years (age 52) and, if job shifting incurs a cost of 20%, it should occur after 32 years (age 57) rather than after 36 years (age 61).

Table 4: Implications of Job Shifting at Various Ages with g = .01

Year	V_t	NPV_t^{t+1}	NPV_t^{t+2}	V_t - NPV_t	Z
1	$0.33	$0.58		-$0.25	
.	.	.		.	
.	.	.		.	
22	$11.11	$11.13		-$0.02	
23	$11.80	$11.78		$0.02	0.01
30	$16.78	$16.49		$0.29	0.16
35	$20.33	$19.76		$0.57	0.26
40	$23.55	$22.58		$0.97	0.37

The results in Table 3 are also sensitive to the real yield on inflation-protected bonds (y) at the time of the proposed job shifting. For example, suppose that y rises from 4% to 5% during the first year following the employee's entry to the pension scheme, and remains at this higher level, the results are as shown in Table 5.

Table 5: Implications of Job Shifting at Various Ages with y = .05

Year	V_t	NPV_t^{t+1}	NPV_t^{t+2}	V_t - NPV_t	Z
1	$0.28	$0.50		-$0.22	
.	.	.		.	
.	.	.		.	
24	$14.24	$14.25		-$0.01	
25	$15.22	$15.20		$0.02	0.01
30	$20.56	$20.30		$0.26	0.10
35	$26.47	$25.83		$0.64	0.21
40	$32.55	$31.35		$1.20	0.32

Relative to Table 3, the employee's critical actions are shifted forwards in time by only about two years. For example, if job shifting is costless, it should occur after 25 years (age 50) rather than 27 years (age 52) and, if job shifting incurs a cost of 20% it should occur after 34 years (age 59) rather than after 36 years (age 61). These effects are about half as great as that from a change in the expected real growth rate in the employee's salary of the same amount.

5. Conclusions

This paper has analysed the question of when an employee in a defined benefits pension scheme should shift jobs or retire from all work in order to initiate the pension payments. The extent of job shifting costs is crucial to the decision. If the decision is made at the time of entering the scheme, and the employee enters the scheme at the traditional age of about 25, and job shifting is costless, the optimal course of action is to shift jobs at a point depending upon various parameter values, but all such points are well before the traditional retirement age of 65. However, if job switching incurs an effective salary reduction of 20% and the salary reduction that would prompt retirement from all work is in excess of this, then job switching should be delayed by about 10 years.

These conclusions are not materially altered if the decision is delayed until the point of possible job shifting or retirement, so long as the relevant parameter values do not change over time. The parameter value for which results are most sensitive to a change is the expected real growth rate in salary, with a reduction from 2% to 1% inducing relevant actions that shift forwards in time by about four years. In particular, job switching will occur about four years earlier for any specified level of job switching costs so long as job switching is superior to retirement from all work.

REFERENCES

Exley, C., Mehta, S. and Smith, A., 1997. 'The Financial Theory of Defined Benefit Pension Schemes', *British Actuaries Journal*, vol. 3 (IV), pp. 835-966.

Government Superannuation Fund, 2009. *Booklet GS6* www.gsf.govt.nz (accessed 15 January 2010).

Lally, M., 2000. 'The Valuation of GSF's Defined Benefit Pension Entitlements', *New Zealand Economic Papers*, vol. 34 (2), pp. 183-199.

Reserve Bank of New Zealand, 2009a. *Table A3*: *Incomes and Prices* www.rbnz.govt.nz (accessed 15 January 2010).

Reserve Bank of New Zealand, 2009b. *Table B2*: *Wholesale Interest Rates* www.rbnz.govt.nz (accessed 15 January 2010).

Reserve Bank of New Zealand, 2009c. *Table A1*: *Specified Loans on and off Balance Sheet-Registered Banks* www.rbnz.govt.nz (accessed 15 January 2010).

Statistics Department, 2009a. *Median Age at Marriage by Previous Marital Status* www.stats.govt.nz (accessed 15 January 2010).

Statistics Department, 2009b. *New Zealand Life Tables 2005-07* www.stats.govt.nz (accessed 15 January 2010).

Watt, B. and Reddell, M. 1997, 'Some Perspectives on Inflation-Indexed Bonds', *Reserve Bank Bulletin*, vol. 60 (4), pp. 322-331.