Investing in vertical integration

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Abstract

Electricity industries are frequently characterised by a high degree of vertical integration. We describe how vertical integration can be seen to be similar to entering into hedging derivative contracts. We explore the option for a gentailer to enlarge its participation in the retail market, and show that the firm will choose to delay if market demand is low or high. Increased volatility can make the firm more inclined to invest, contrary to what is found in most real options studies.

Electricity firms frequently participate in both the generation (upstream) and retail (downstream) markets simultaneously, which is referred to in the economic literature as vertical integration. No such thing as a unified theory of vertical integration exists so far. There is an array of theories to explain this rather usual phenomenon in electricity markets. However, as posed by Joskow (2005), virtually all theories of vertical integration arise from the recognition of market imperfections of some type.

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Traditional approaches have mostly focused on vertical integration as an arrangement associated either with a strategic move to increase market power (downstream and/or upstream) or a response to market power problems already in existence.[1] Other alternative approaches focus on frictions such as transaction costs and incompleteness of contracts to study vertical integration from an organizational perspective. Unlike most the traditional approaches that consider the act of vertical integration as theoretically costless, alternatives such as the "Property Rights" approach and Transaction Cost Economics see vertical integration as a reaction to costs and benefits of internal organization and to inefficiencies of market transactions.[2]

The cost structure of the electricity retailers has several components. Retail businesses can be divided into two distinct groups: the low voltage regional distribution (lines business) and the commercialization of energy (energy business). On one hand, retailers operate and maintain copper and/or aluminium wires, power poles and connection points.[3] On the other hand, retailers undertake community/social initiatives and marketing campaigns in order to sell more electricity and strengthen their reputation. They also invest in call centres for account and fault management. For simplification, we assume in our equilibrium model that the marginal costs associated with both the distribution O&M expenditures and selling expenses are negligible in comparison to the marginal costs of power generation.

Gentailers can raise their retail market share by either taking consumers from rival gentailers or

[1] In this case, as explained by Joskow (2005), vertical integration could be an optimal response to costs of successive monopolies (Tirole (2000) chapter 4), it could enhance price discrimination (Perry (1978)), or it could be used to soften competition by increasing rivals' costs and/or foreclosure potential competitors(Aghion and Bolton (1987), Ordover, Saloner, and Salop (1990) and Hart, Tirole, Carlton, and Williamson (1990)). For a more didactic approach, see Schmalensee, Armstrong, and Porter (2007) chapter 33.

[2] See, for example, Williamson (1971), Klein, Crawford, and Alchian (1978), Klein and Leffler (1981), Williamson (1983), Williamson (1985), Grossman and Hart (1986), Hart and Moore (1990), Hart and Hart (1995), Klein (2000) and Williamson (2000).

[3] In New Zealand, lines companies do this job and retailers contract for delivery service over lines.

from pure retailers. A gentailer's decision to increase his vertical integration in the short-term (say, 4 years), involves undertaking projects for either expanding his regional distribution coverage or for increasing his marketing expenses. A considerable proportion of the costs associated with marketing campaigns and the expansion of lines coverage is irreversible. In particular, advertising in marketing campaigns entail totally sunk expenditure. It may also include a situation of a retailer entering a region as starting up, incurring transactions costs that are sunk.

Electricity markets are characterized by volatile spot prices and uncertainty about the demand. Both aspects directly affect the retail business. In an uncertain scenario with sunk costs, the value of flexibility is relevant to the decision of a generator to expand his participation in the retail market. In other words, the option to wait for undertaking the project of increasing his vertical integration has a positive value.

Precisely, we consider the specific issue of a gentailer's decision to irreversibly invest in a marketing campaign (and/or in an expansion of the distribution coverage) to increase his retail market share. For simplification, we assume that i) the effect of this campaign is instantaneous and has a pre-determined and certain duration, ii) the sunk costs associated with such campaign are one shot and instantaneous iii) demand is the only source of uncertainty, iv) the retail price is fixed for the investment horizon, v) competitors do not react strategically to this campaign and vi) forward contracts are given.

The reason for the assumptions i) and ii) is to focus on the determinants of vertical integration, avoiding complications arisen from more practical considerations about the nature of either the investments or the marketing campaign. The assumption iii) of only one state variable (demand) allows for the derivation of a closed-form solution for the problem. The rigidity of retail prices given by iv) is a reasonable approximation for the short and medium-term in markets that do not face abnormal changes in demand.[4] The assumption v) of no predictable strategic reaction to a increase in a gentailer's vertical integration simplifies the model considerably and enables for a *ceteris paribus* analysis.[5] Since the retail price is assumed to be rigid, changes in demand that increases the spot prices have an initial negative impact on the retail profit margin. Lastly, the assumption vi) implies that we take a partial equilibrium approach since forward contracts do not react to increases in vertical integration. That is, this exercise analyzes the value of vertical integration taking a specific K-dimensional vector of generators' quantity contracted $\vec{QC}$ as given.[6]

The layout of the remainder of the paper is as follows. Section 1 outlines an equilibrium model for wholesale market clearing. Section 2 explains the vertical integration expansion option. Section 3 presents a numerical example. Finally, section 4 concludes.

1 Equilibrium in the wholesale market with vertical integration

Assume N total firms made up of K generators, R retailers and I gentailers, where $I = K + R - N$. That is, gentailers are included in both retailers and generators groups. The retailers' revenues are determined by an exogenous retail price and by their market shares. The consumers' aggregate

[4] The timing and velocity of the retail price adjustments depend on the nature of the market and the retailers' expectation about future demand. For example, the more elastic is the demand, the greater is the sensitivity of retail prices to changes in the expected demand. The length of contracts also affects how fast changes in expected demand are translated into variations of equilibrium retail prices. Last, retail prices are also frequently subject of price regulation (price cap or rate of return). The more binding are these price constraints, the less retail prices are enabled to vary. A preliminary work allowing for retail prices to react to demand periodically is currently being conducted and achieves similar qualitative results.

[5] The results of this paper must be viewed as a best case scenario, since the rivals' reaction to the gentailer's project of increasing his retail market share would almost certainly decrease the value of his project. The extension of the framework here developed to enable for the possibility of strategic interactions among firms in the market is a promising avenue for future research.

[6] It is reasonable to expect that the optimal hedging decreases as a result of an increase in vertical integration. In the end of this paper, we conduct simulations to show how the results are affected by changes in QC_i.

demand at time t is defined by $\tilde{D}_t$. Since the participation of generators in the retail market also drives their supply decision, the characterization of each retailer's individual demand becomes relevant.

Definition 1 *Retailers' demand (gentailer or pure retailer) is defined as* $m_i\tilde{D}_t(p_t^R, \vec{W}_t), \forall i = 1, 2, \ldots, R$. *Here* $m_i > 0$ *is the given market share of retailer* i *and* $\sum_{i=1}^{R} m_i = 1$, *since gentailers are included in retailers. By construction,* $\sum_{i=1}^{R} m_i\tilde{D}_t(p_t^R, \vec{W}_t) = \tilde{D}_t(p_t^R, \vec{W}_t)$.

A retailer's demand is assumed to be a fixed proportion of the total consumers' demand and, by construction, the total retailers' demand must be equal to the aggregate consumers' demand. The exogeneity of m_i reflects the idea that the retail market shares are relatively fixed. It is certainly reasonable to assume that at the moment of an auction the retail market share is known and exogenous. In reality, contract arrangements between retailers and final consumers are relatively stable in comparison to the strong variations observed in both demand and generation inputs. Therefore, this assumption is a good approximation for the short or medium-term.

Definition 2 *The market clearing wholesale price* p_t^c *must equate aggregate demand and aggregate supply.* $\sum_{i=1}^{K} S_{it}(p_t^c, QC_{it}, \vec{W}_t) = \sum_{i=1}^{R} m_i\tilde{D}_t(p_t^R, \vec{W}_t) = \tilde{D}_t(p_t^R, \vec{W}_t)$

Again, firms simultaneously submit continuous supply schedules $\hat{S_{it}}$ and the auctioneer computes the equilibrium price p_t^c that satisfies the market clearing condition.

Definition 3 *Generator* i*'s ex-post profit upon the realization of the market clearing price is (where* $m_i = 0$ *for pure generators and* $m_i > 0$ *for gentailers):*

$$\begin{aligned} \pi_{it} &= S_{it}(p_t^c, QC_{it}, \vec{W}_t)p_t^c - C_{it}(S_{it}(p_t^c, QC_{it}, \vec{W}_t), \vec{W}_t) \\ &\quad + m_i(p_t^R - p_t^c)\tilde{D}_t(p_t^R, \vec{W}_t) + (PC_{it} - p_t^c)QC_{it} \end{aligned} \tag{1}$$

There are three possible sources of payoff for electricity companies $i = 1 \ldots N$: operating profit from generation activity $(S_{it} - C_{it})$, operating profit from retail activity $m_i(p_t^R - p_t^c)\tilde{D}_t$ and financial revenue $(PC_{it} - p_t^c)QC_{it}$ from forward market transactions. The differences between financial and physical transactions were explained in the introduction. As defined before, gentailers are characterized by participating in both generation and retail markets. Therefore, they have operating profits (or losses if negative) in both activities.

Definition 4 *Pure retailer i's ex-post profit upon the realization of market clearing price is:*

$$\pi_{it} = m_i(p_t^R - p_t^c)\tilde{D}_t(p_t^R, \vec{W}_t) - (p_t^c - PC_{it})QC_{it} \tag{2}$$

Notice that QC_{it} may be negative. For example, if pure retailers solely buy electricity in the forward market, they have a negative contract position by our definition. Definition 4 assumes retailers as passive players in the instantaneous wholesale spot market. That is, a retailer's purchase is totally determined by his exogenous retail market participation $m_i\tilde{D}_t(P_t^R, W_t)$. It also means that there are no strategic alternatives considered by pure retailers and the spot market equilibrium is fully determined by supplier strategies and the exogenous aggregate demand. This is a reasonable approximation for most uniform-price auctions in electricity markets, where only suppliers bid and markets are cleared by an auctioneer responsible for matching supply curves to particular electricity demands.

Assume that generator/gentailer i's bidder when deciding the bid schedule $\hat{S}_{it}(p)$ has utility maximizing behavior. The bidder i expected utility maximization problem is:

$$\max_{\hat{S}_{it}(p)} \int_{\underline{p}}^{\overline{p}} \quad U[\hat{S}_{it}(p)p - C_{it}(\hat{S}_{it}(p), \vec{W}_t) + m_i(p_t^R - p_t^c)\tilde{D}_t(p^R, \vec{W}_t)$$
$$+(PC_{it} - p)QC_{it}]dH_{it}(p, \hat{S}_{it}(p); QC_{it}), \tag{3}$$

The integral is taken over all possible realizations of the market clearing price, weighted by the probability density dH_{it}.

Consider that the following definitions hold.

Definition 5 *The consumers' aggregate demand at time t is defined by the function* $\tilde{D}_t = D_t(p_t^R, \vec{W}_t)$. *Retail price* p_t^R *is assumed to be exogenous.*

Definition 6 *The other firms' correspondences* $(QC_{jt}, PC_{jt})\forall j \neq i$ *are unknown by firm i.*

Aggregate demand is only affected by the state variables $\vec{W}_t$ and the retail price p_t^R. At the time of the auction, the demand function is deterministic. In terms of the model analyzed in the previous section, this definition is equivalent to assuming that ε is negligible. In fact, uniform-price auctions used to clear electricity spot markets have a very short-term horizon. Bids into uniform price electricity auctions are made for delivering energy close to dispatch. In markets such as the NZEM, the bid can be modified until two hours to the delivery time. The more significant source of uncertainty for a specific bidder at the time of the auction is the hedging position of his rivals.

Lemma 1 *In equilibrium, assuming again that supply schedules are continuously differentiable and that* $S_{it}^*(p)$ *is the optimal supply curve of firm i at time t, the first order condition for the bidder's (gentailer/generator) maximization problem is:*

$$p - MC_{it}(S^*_{it}(p), \vec{W}_t) = [S^*_{it}(p) - QC^*_{it} - m_i D_{it}(p^R_t, \vec{W}_t)] \frac{H_S(p, S^*_{it}(p); QC^*_{it})}{H_p(p, S^*_{it}(p); QC^*_{it})} \tag{4}$$

where

$$H_p(p, S^*_{it}(p); QC^*_{it}) = \frac{\partial}{\partial p} Pr(p^c_t \leq p \mid QC^*_{it}, S^*_{it}(p))$$
$$H_S(p, S^*_{it}(p); QC^*_{it}) = \frac{\partial}{\partial S} Pr(p^c_t \leq p \mid QC^*_{it}, S^*_{it}(p))$$

Proof: see appendix A.

This result follows from the deterministic nature of all non-control variables and from the fact that the bidder is able to choose an optimal supply for each possible state of nature.

Assume that generator/gentailer i's net supply is the difference between his supply and his retail position $(S^*_{it}(p) - m_i D_{it})$. Equation (4) shows that a gentailer's bid behavior is driven *ceteris paribus* not only by contracts but also by his net supply. In the absence of forward contracts, the bigger the net supply, the bigger the incentive to exert market power in the wholesale market. The generator's participation in the retail market reduces his incentives to bid above marginal cost. If the gentailer is a net retailer $S^*_{it}(p) - m_i D_{it} < 0$ he may even bid below his marginal cost.

Lemma 2 *For a range of prices $p \in [\underline{p}, \overline{p}]$ the first order condition at time t becomes:*

$$p_t - MC_{it} = \frac{S_{it} - QC^*_{it} - m_i D_t}{\frac{\partial \sum_{j \neq i} S_{jt}}{\partial p_t}} \tag{5}$$

Alternatively,

$$\frac{p_t - MC_{it}}{p_t} = \frac{1}{\varepsilon'_{it}(q'_{it})} \tag{6}$$

Where $\varepsilon'_{it}(q'_{it})$ is the elasticity of the net residual demand q'_{it}, here defined as $q'_{it} = D_t - \sum_{j\neq i} S_{jt} - QC^*_{it} - m_i D_t$.

Proof: see appendix B.

Under the hypothesis of quadratic cost functions and linear demand, the gentailer's supply curve obeys the usual pointwise Lerner index as shown by equation (5).

Specifically, in equilibrium, gentailer i's supply S_i is such that his Lerner index $\frac{p_t - MC_{it}}{p_t}$ corresponds to the inverse of the elasticity $\frac{1}{\varepsilon_{it}(q_i)}$ of his residual demand $D_t - \sum_{j\neq i} S_{jt}$ net of his equilibrium forward position QC^*_{it} and his participation in the retail market $m_i D_t$. In other words, the elasticity of the net demand q_i fully explains wholesaler i's market power. This result comes though from the additional assumption of instantaneous perfect inelasticity of aggregate demand D_t to wholesale spot prices p_t at time t.

Proposition 1 *If (i) there are a fixed number $K > 2$ generators/gentailers in the market, (ii) marginal cost functions are linear and symmetrical between firms in the market ($MC_{it}(S_{it}, \vec{W}_t) = a + bS_{it} + \sum_{j=1}^{L} \rho_j w_{jt}$ $\forall i = 1, 2, \ldots N$) and (iii) the aggregate demand is linear with constant retail price ($D_t(p^R_t, \vec{W}_t) = c - \kappa_o p^R + \sum_{j=1}^{L} \kappa_j w_{jt}$) then there is a simple symmetric Bayesian-Nash equilibrium where the clearing wholesale spot price can be rewritten as the following:*

$$p^c_t = A - B\sum_{i=1}^{K} QC^*_{it} + \sum_{j=1}^{L} C_j w_{jt} \tag{7}$$

Where

$$
\begin{aligned}
A &= a + b\frac{(c-\kappa_o p^R)\left(K - (1+\sum_{i=1}^{K} m_i)\right)}{K(K-2)} \\
B &= \frac{b}{K(K-2)} \\
C_j &= \rho_j + b\frac{\left(K - (1+\sum_{i=1}^{K} m_i)\right)}{K(K-2)}\kappa_j
\end{aligned}
$$

Proof: see appendix C.

Positive shifts in generators' costs and in aggregate demand increase the spot price. An increase in the retail price decreases spot price. The sum of generators' contracts $\sum_{i=1}^{K} QC_{it}$, play an important role in price formation.[7]

Equation (7) also shows that, holding forward contracts constant, an increase in the degree of vertical integration ($\sum_{i=1}^{K} m_i$) in the market implies a decrease in spot prices. The reason is that more vertically integrated firms have a smaller net supply $S_{it} - m_i D_{it}$ and therefore less incentives to exert market power in the wholesale market taking contracts as fixed.[8]

Corrolary 1 *If $K \to \infty$ then $p \to MC$.*

[7]The aggregate position of generators ($\sum_{i=1}^{K} QC_{it}$) is close to zero and do not affect spot prices in two basic situations: (i) electricity markets with a poorly developed forward market and (ii) fully vertically integrated markets. In particular, markets made exclusively of gentailers with the same market share in both the retail and generation markets have little reason to develop forward markets in a large scale, since their wholesale transactions are internally hedged.

[8]Hogan (2010) finds similar result in a different and deterministic framework, addressing the incentives of gentailers and pure retailers. He finds that the vertically integrated firm has an incentive to compete more aggressively in the retail market than pure retailers. Gans, Wolak, and Carlton (2008) find opposite results considering the role of passive vertical integration. They find that an increase in vertical integration would decrease quantity contracted that would in turn increase spot prices. This result relies strongly that the wholesale and retail businesses are completely separated (independent). This means that the gentailers do not necessarily make a first best decision. Specifically, the forward contract aspects of vertical integration are not considered in the gentailers' supply decision.

There are two exceptions where the hedging decision does not matter for spot price modeling purposes, notwithstanding the size of the electricity hedging market. The first, as posed by the corollary above, refers to the perfect competition case. If the number of generators in the market goes to infinity, the mark-up component of the spot price formation tends to zero. In the limit, we have the competitive result of spot price being equal to generators' marginal cost. In other words, if generators in an electricity market were atomized, wholesale prices would be primarily driven by their marginal costs. Perfect competition is not feasible in many electricity wholesale markets around the world. So this is unlikely to be observed in practice.

Corrolary 2 *If* $K = N$ *then* $\sum_{i=1}^{K} m_i = \sum_{i=1}^{N} m_i = 1$ *and we have:*

$$p_t^c \quad = \quad a + \frac{bc - \kappa_o p^R}{K} + \sum_{j=1}^{L} \left(\frac{b}{K} \kappa_j + \rho_j \right) w_{jt} \tag{8}$$

The second concerns the case where forward contracts are fully cleared by generators ($\sum_{i=1}^{K} QC_{it} = 0$). From corollary 2, this fact applies to markets where $K = N$. That is, where all the firms in the market are generators or gentailers (i.e. all retailers are also generators). In such a case, contracts do not affect the aggregate supply and, consequently, the clearing spot price.

Since this model approximates demand and marginal costs by linear functions, by equation (5) the optimal individual supplies are also linear. In particular, they are positively affected by the quantity contracted (QC_{it}). Gentailers can be net wholesalers, net retailers or have the same share in both markets. Intuitively, in order to hedge risks, they are expected to have $QC_{it} > 0$, $QC_{it} < 0$ and $QC_{it} = 0$ respectively. Therefore, if all the players are gentailers and the aggregate supply is linearly affected by the sum of the generators' outstanding contracts, it is reasonable to expect that the oversupply of net wholesalers will offset the undersupply of net retailers and the aggregate outstanding contracts will have no effect on the aggregate demand.

Define markets where $K = N$ as fully vertically integrated markets. Notice that this definition is broader than the usual definition of full vertical integration in the literature, as it admits mismatch between the participation of an individual gentailer in the generation and retail markets.[9] Our definition comprises (but it is not limited to) either (i) markets where all the generators are gentailers ($K = I = N$) or, more strictly, (ii) markets where each generator sells all his production directly to consumers through his retail business (individual full vertical integration).

The gentailer dominated electricity markets of Spain, New Zealand or Germany, for example, fit closely to this definition. In New Zealand, the market is dominated by gentailers but some firms present mismatch between their wholesale and retail market shares. In other words, there are big net wholesalers and big net retailers.

Notice that the clearing price is equal to the average marginal cost in fully vertically integrated electricity markets since, in equilibrium, the average supply $\overline{S}$ is equal to the aggregate demand divided by the number of gentailers ($\overline{S} = \frac{D}{K}$). This means that individual firms may have market power when $K = N$ but the average mark-up in the market is equal to zero.

2 Investing in Vertical Integration

Formally, assume a particular electricity market that meets the assumptions of model I of section 1. We have K generators in the market and a specific generator/gentailer i is considering to irreversibly invest I to expand his retail market share by Δm_i for h years. Assume that $\omega \Delta m_i$ of this increase is taken from the rival gentailers j (where $j \neq i$ and $j = 1, 2, \ldots, K-1$), changing their average retail

[9] As for example Dixit (1983).

market share by $\Delta \overline{m}_j = -\frac{\omega}{K-1}\Delta m_i$. Notice that $\omega \in [0,1]$. Obtaining retail share from pure retailers will have different effects than obtaining retail share from rival generators. As an illustration, we retain an oligopoly in generation ($K = 5$) and assume that a generator has the option to irreversibly invest 100 million dollars in a marketing campaign to increase his retail market share by 2% for the next 4 years. Presume that $\omega = 60\%$, which implies that 1.2% of the market share is taken from rival gentailers and 0.8% is taken from pure retailers. In this case, the average market share of firm i's rival gentailers would change by $\Delta \overline{m}_j = -0.30\%$.

Lemma 3 *Presume that the assumptions of section 1 hold. Assume that demand itself is the only state variable and has no effect on firms' marginal costs ($\vec{W}_t = D_t$, $\rho = 0$ and $\kappa = 1$). A* ceteris paribus *increase at t in generator/gentailer i's retail market share by Δm_i, accompanied by a decrease of $\omega \Delta m_i$ in rival generators' retail market share, implies the following change in his profit level:*

$$\Delta \pi_{it} = \hat{A} D_t - \hat{B} D_t^2 \tag{9}$$

where,

$$\begin{aligned}
\hat{A} &= \Delta m_i \left(p^R - a + \frac{b}{K(K-2)} \left(1 - \frac{\omega}{(K-1)^2} \right) QC - \frac{b\omega}{(K-1)^2} QC_i \right) \\
\hat{B} &= \frac{b \Delta m_i}{K(K-2)} \left((K-1-M) \left(1 - \frac{\omega}{(K-1)^2} \right) + \frac{\omega K (K-2)}{(K-1)^2} m_i - \right. \\
&\quad \left. \left(1 + \frac{\omega}{(K-1)^2} \right) \frac{\Delta m_i}{2} \right)
\end{aligned}$$

Proof: see appendix D.

Equation (9) shows the impact on profit at time t of a change in firm i's market share. This equation reflects fixed retail prices for all players in our model and the absence of demand effects except those arising from exogenous shocks. Assuming everything else constant, an increase in demand increases the clearing spot price p_t^c which decreases the retail margin $(p^R - p_t^c)$ and increases the amount of electricity sold by gentailer i in the retail market $(m_i D_t)$. Therefore, an increase in demand has an ambiguous effect on retail profit. The retail margin can become negative for high demand scenarios so that a marginal increment of the demand can decrease firm i's profit.

If ω is equal to zero (i.e firm i expands his retail positions solely out of pure retailers' market shares), the first order effect of changes in gentailer i's retail share on his profit is equal to $\Delta\pi_{it} = \Delta m_i(p^R - p_t^c)D$. In equilibrium, all the indirect effects of changes in firm i's market share cancel out. That is, an increase in gentailer i's retail share (Δm_i) causes an increase in his equilibrium supply S_i^* which in turn increases his marginal costs MC_i and reduces the price mark-up $p^c - MC_i$. As shown in appendix D, the overall impact of these indirect effects on firm i's profit is zero, i.e. $\frac{\partial \pi_i}{\partial p^c}\frac{\partial p^c}{\partial m_i} = -\frac{\partial \pi_i}{\partial S_i^*}\frac{\partial S_i^*}{\partial m_i}$. An isolated increase in m_i linearly increases the profit for a given demand.

If ω is different from zero (i.e firm i steals retail market share from rival gentailers), we have additional first order effects that originate from firm i's decision to increase his market share. Despite the fact that a decrease in the market shares of rival gentailers do not affect firm i's profits directly, the indirect effects of $\Delta\overline{m}_j$ are not canceled out. A decrease in $\overline{m_j}$ increases generator i's supply, which in turn increases his marginal costs but also increases his wholesale margin (spot price mark-up) and unambiguously increases π_i. The demonstration of these results and the derivation of the second order effects are in appendix D.

Assume that demand follows a geometric brownian motion, where μ is the drift, σ is the demand volatility and dW_t is an increment of a wiener process.

$$dD_t = D_t\mu dt + D_t\sigma dW_t \tag{10}$$

This is a standard assumption in the real options literature that generally results in closed-form solutions.

Lemma 4 *Define* $\sum_{k=1}^{K} m_k$ *as* M. *Consider that* $\sum_{k=1}^{K} QC_{kt}$ *is a constant* QC *and that the marginal cost is equal to* $MC_i = a + bS_i$. *Define* h *as the horizon of the investment. Assume that* $r > 2\mu + \sigma^2$. *The present value* V_t *at* t *of the project's cash inflows is equal to:*

$$V(D_t) = \tilde{A}D_t - \tilde{B}D_t^2 - I \tag{11}$$

where:

$$\begin{aligned}\tilde{A} &= \hat{A}\frac{\left(1 - e^{-(r-\mu)h}\right)}{r-\mu} \\ \tilde{B} &= \hat{B}\frac{\left(1 - e^{-(r-2\mu-\sigma^2)h}\right)}{r - 2\mu - \sigma^2}\end{aligned}$$

and $\tilde{A} > 0$ *and* $\tilde{B} > 0$. Proof: see appendix E.

The value of undertaking the project is a quadratic concave function of demand D. This value depends on initial market parameters (K, M, m_i, QC, QC_i, a and b), on the characteristics of the project (Δm_i, ω, I and h), the risk-free rate r and on the stochastic process for demand (μ and σ).[10]

Define $F(D)$ as the value of generator/gentailer i's investment opportunity to increase his participation in the retail market for a given realization of D. Expanding dF through Ito's Lemma and

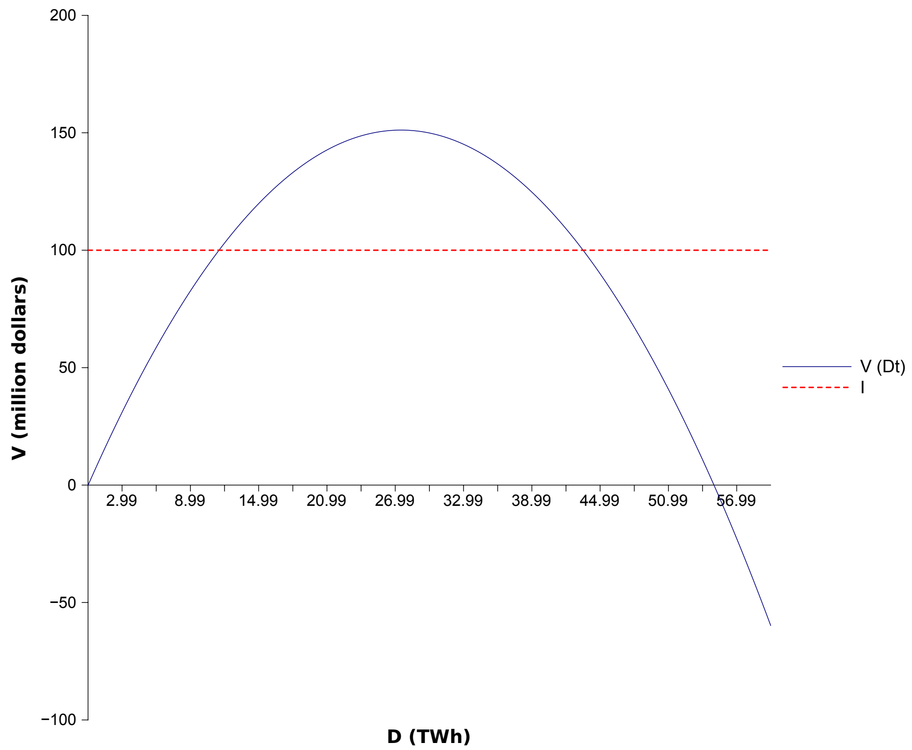
[10] Since $E_t^T(D_t^2) = e^{(2\mu+\sigma^2)(T-t)}D_t^2$, the condition $r > 2\mu + \sigma^2$ guarantees that the discounted value of the expected demand squared converges.

using standard contingent claims analysis, we have the following quadratic differential equation:[11]

$$\frac{1}{2}\sigma^2 D^2 F''(D) + \mu D F'(D) - rF = 0 \tag{12}$$

The solution of this partial differential equation above is not trivial since $V(D_t)$, as defined by equation (11), is a quadratic function of D_t. This means that there is a point where the demand is so high that the value of the project is increased by reduced demand.

Figure 1: Concavity of $V(D_t)$



Therefore, we have a demand interval where the project is undertaken. Define the the critical lower and upper bounds that trigger the investment as D_1^* and D_2^*, respectively. The solution of the differential equation above can be attained by separating the problem into three different regions. The first one, given by $[0, D_1^*]$, concerns the low demand region where waiting for an increase in

[11]See Dixit, Pindyck, and Davis (1994) pages 114-117 and 150-151.

demand has equal or greater value than undertaking the project. The second is given by $[D_1^*, D_2^*]$ and reflects the exercise region where the project is undertaken and $F(D) = V(D)$. Lastly, in the region where the demand is greater than D_2^* (D_2^*, ∞) the firm's optimal strategy is to wait for the demand to decrease before initiating the project.

The solution also requires the determination of suitable boundary conditions for the first and third regions. If $D \in [0, D_1^*]$,

$$F(0) = 0 \tag{13}$$

$$F(D_1^*) = \tilde{A}D_1^* - \tilde{B}D_1^{*2} - I \tag{14}$$

$$F'(D_1^*) = \tilde{A} - 2\tilde{B}D_1^* \tag{15}$$

If demand, D, goes to zero then the value of undertaking the project will remain at $-I$.[12] Therefore, as stated by equation (13), it is reasonable to assume that the value of the investment opportunity is zero in such a case ($F(0) = 0$). The other two conditions come from optimality considerations. Equation (14) establishes that the gentailer i receives a net payoff of $\tilde{A}D_1^* - \tilde{B}D_1^{*2} - I$ upon investment, which is equal to the value of the project if it is undertaken ($F(D_1^*)$). This is sometimes called the value matching condition. As is usual in the real options literature, equation (15) assumes that $F(D)$ is continuous and smooth at the critical exercise demand D_1^*, i.e. F(D_1^*) and V(D_1^*) have the same derivatives. This is also known as the smooth pasting condition.[13]

[12]Notice that, in this case, $F(0 \leq D \leq D_1^*) = 0$ because the investment will not be taken.
[13]See Dixit et al. (1994).

Conversely, If $D \in [D_2^*, \infty)$,

$$F(D_2^*) = \tilde{A}D_2^* - \tilde{B}D_2^{*2} - I \tag{16}$$

$$F'(D_2^*) = \tilde{A} - 2\tilde{B}D_2^* \tag{17}$$

$$F(\infty) = 0 \tag{18}$$

In this case, the value function is decreasing in D. For sufficiently high levels of demand the project is not executed unless demand decreases to the trigger point D_2^*. Again we assume that $F(D)$ is continuous and smooth at this critical point (equations (16-17)). The greater is the demand from D_2^* onwards, the smaller is the value of the project opportunity. As stated by equation (18), in the limit this value goes to zero. This is often referred to as the transversality condition.

Proposition 2 *The particular solution of the PDE given by equation (12) and bounded by equations (13-18) is the function $F : \Re^+ \to \Re^+$ below:*

$$F(D) = \begin{cases} \alpha_{11} D^{\beta_1} & \text{if } D \in [0, D_1^*] \\ \tilde{A}D - \tilde{B}D^2 - I & \text{if } D \in [D_1^*, D_2^*] \\ \alpha_{22} D^{\beta_2} & \text{if } [D_2^*, \infty[\end{cases}$$

$$\beta_1 = \frac{1}{2} - \frac{(r-\mu)}{\sigma^2} + \sqrt{\left(\frac{(r-\mu)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} \tag{19}$$

$$\beta_2 = \frac{1}{2} - \frac{(r-\mu)}{\sigma^2} - \sqrt{\left(\frac{(r-\mu)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} \tag{20}$$

$$D_1^* = \frac{(1-\beta_1)\tilde{A} + \sqrt{(1-\beta_1)^2\tilde{A}^2 + 4(2-\beta_1)\beta_1\tilde{B}I}}{2(2-\beta_1)\tilde{B}} \tag{21}$$

$$D_2^* = \frac{(1-\beta_2)\tilde{A} + \sqrt{(1-\beta_2)^2\tilde{A}^2 + 4(2-\beta_2)\beta_2\tilde{B}I}}{2(2-\beta_2)\tilde{B}} \quad (22)$$

$$\alpha_{11} = \frac{\tilde{A}}{\beta_1} D_1^{*1-\beta_1} - 2\frac{\tilde{B}}{\beta_1} D_1^{*2-\beta_1} \quad (23)$$

$$\alpha_{22} = \frac{\tilde{A}}{\beta_2} D_2^{*1-\beta_2} - 2\frac{\tilde{B}}{\beta_2} D_2^{*2-\beta_2} \quad (24)$$

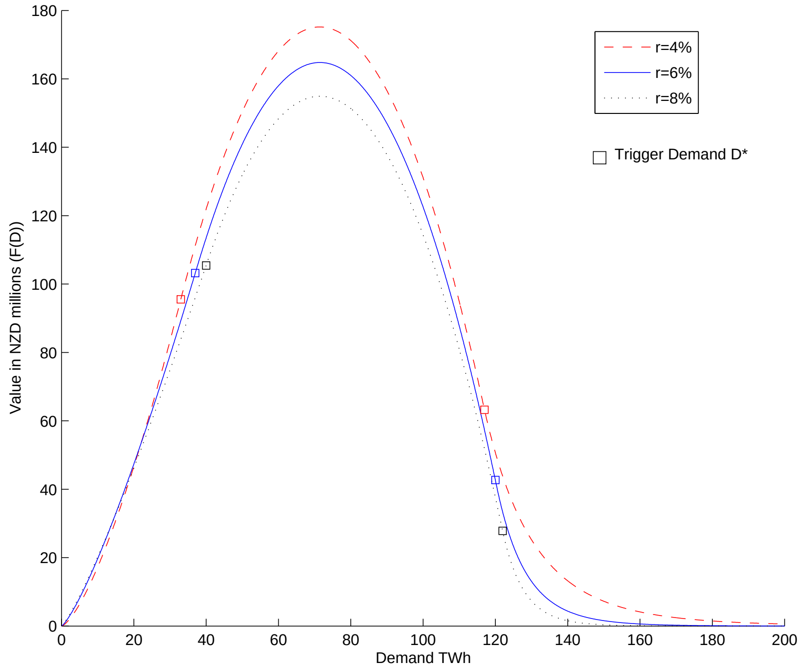
Proof: see appendix F.

3 Numerical Example

We now consider a numerical example, approximately based on actual NZEM numbers. We use this example to illustrate possible consequences and results of our theoretical model.

Our base scenario corresponds to a vertically integrated market composed of initially symmetric gentailers with 50% of the retail market ($K = 5$, $M = 0.5$ and $m_i = M/K \ \forall i = 1, 2, \ldots, K$). Assume the same parameter as used in our illustration ($K = 5$, $I = 100$ million, $\Delta m_i = 2\%$, $h = 4$ and $\omega = 60\%$). Consider that the annual aggregate demand at t is equal to 37.5 TWh (the approximate annualized average of the NZEM offtake between 22/01/2004 and 30/11/2010). Suppose that 10 TWh is hedged through forward contracts ($QC = 10$) and remains fixed during the project's horizon. The base scenario considers average levels of vertical integration and hedge coverage. We analyze the project value under different scenarios. In equilibrium, the average annual supply is equal to 7.5 TWh ($\overline{S} = D/K$). Consider a spot price of approximately NZD72.50/MWh (Haywards node average between 22/01/2004 and 30/11/2010). We know by equation (5) that marginal costs are equal to spot prices ($p = MC$) in fully vertically integrated economies. In this example, we presume that $a = 50$ and $b = 3$ to keep the same average marginal cost ($p = MC = a + b * 7.5 = 72.5$). As shown

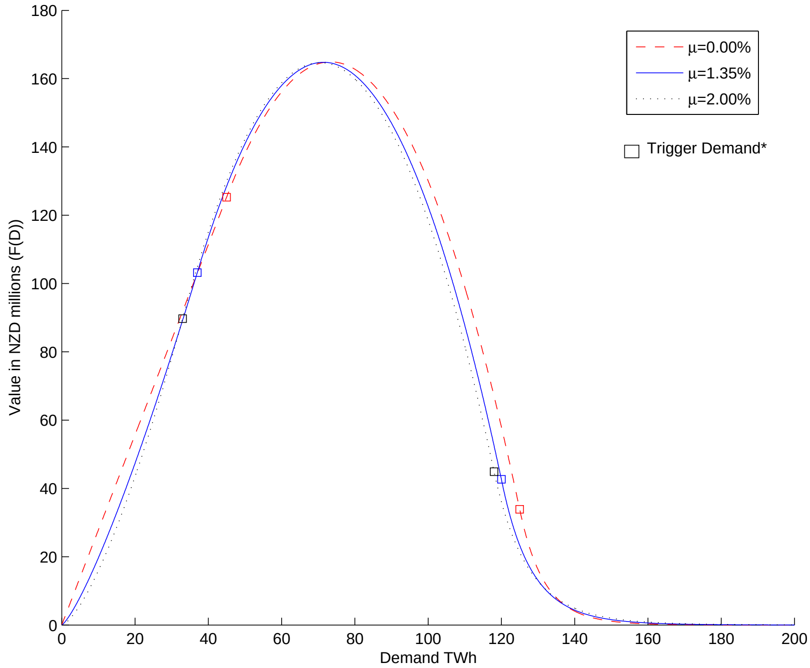
Figure 2: Project value and investment policy (different risk free rates)



by Figure 2, the risk-free rate has the standard impact on the value of the investment opportunity. An increase in r raises the opportunity cost of the project and diminishes the value of the investment opportunity F. However, the impact on the timing of the project is more ambiguous. The triggers (D_1^* and D_2^*) in each scenario are represented by the squares. An inspection of equations (21-22) and Figure 3 shows that the impact on the timing of the investment differs. This happens because the impact of r on D_1^* and D_2^* is not monotonic (r affects not only the characteristic roots but also the parameters $\tilde{A}$ and $\tilde{B}$). In this example, an increase in r slightly postpones investments in both high ($D > D_2^*$) and low ($D < D_1^*$) demand scenarios.

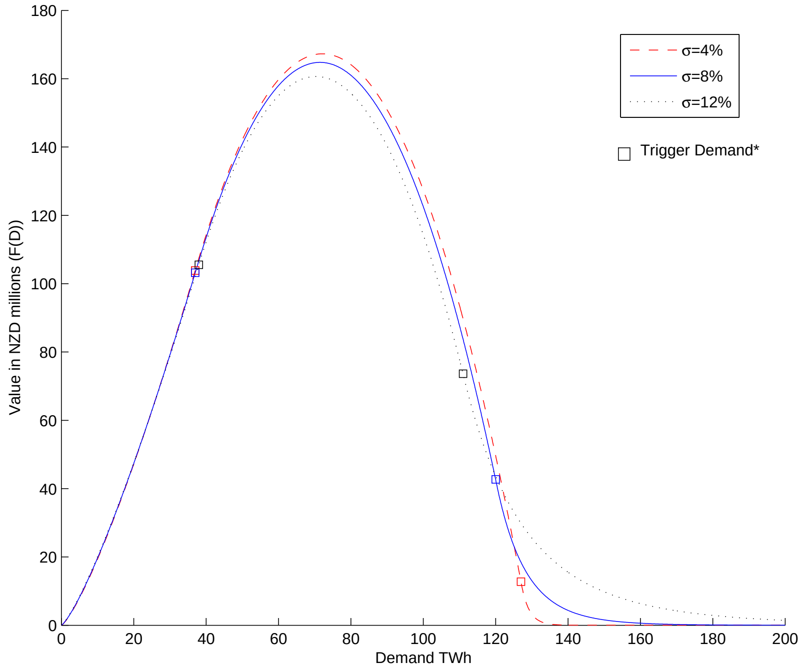
Figure 3 shows that changes in the demand growth rate μ have an ambiguous effect on both the value of the investment opportunity and the timing of the investment. An increase in μ, increases the expected demand which in turns raises the expected amount of electricity sold. On the other hand,

Figure 3: Project value and investment policy (different drifts)



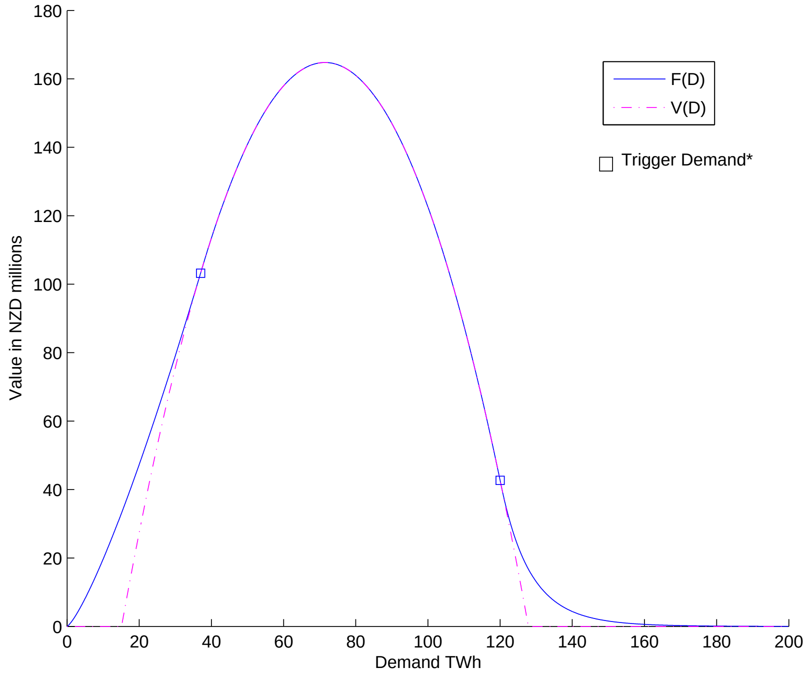
since the retail price p^R is fixed, an increase in μ, increases the expected spot price and decreases the expected margin. The overall net effect on the value of undertaking the project (payoff) depends on the market parameters. In our example, decreases in μ postpone the investments for low values of demand and anticipate it for high values of demand. The effect on the value of the investment depends on the level of demand. For example, for sufficiently extreme values of the demand (low or high), the highest value of the project opportunity occurs when $\mu = 0$. The opposite happens for intermediate values of demand. This happens because the quadratic term of the payoff function $V(D)$ decreases since $B \geq 0$. For longer investment horizons, it is reasonable to expect that drifts greater than zero $\mu > 0$ would significantly affect the retail price. Figure 4 shows that volatility has negative impact on the value of the investment in the exercise range. An increase in σ decreases the value of the option to wait when the demand is smaller than D_1^* and also decreases the value of the

Figure 4: Project value and investment policy (different volatilities)



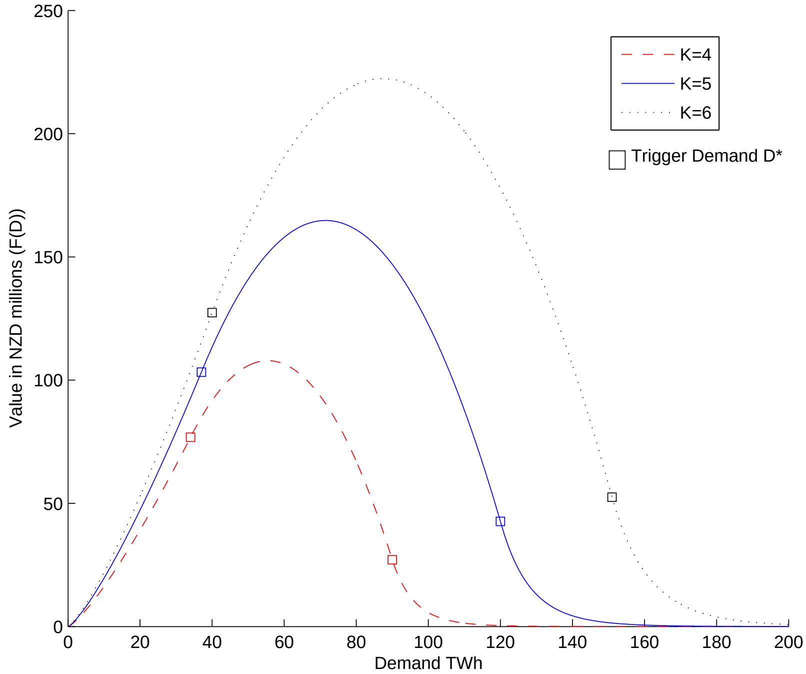
undertaken project in the exercise region ($D_1^* < D < D_2^*$). However, the effect of volatility on $F(D)$ is positive for a sufficiently high demand level. These results reflect the functional form of the payoff function ($\max(V(D_t) - I, 0)$). Unlike standard single variable American call options whose payoffs are convex in terms of their underlying state variable, here, the payoff of undertaking the project presents both convex and concave regions with regards to demand D. Inspection of the dash-dot line in Figure 5 shows that the payoff function is typically convex in the non-exercise regions ($D < D_1^*$ and $D > D_2^*$) and concave in the exercise region. This means that the effect of volatility on the call option F depends in which region the average demand lies. Clearly, the concave region dominates in the given example. We observe from Figure 6 that an increase in the number of firms increases the value of the opportunity to invest in retail expansion. The main reason is that the more competitive is the market, the greater is the retail margin for a given retail price. In the considered example, D_1^*

Figure 5: Concavities of F(D) and V(D)



that triggers the decision to undertake the investment is greater in markets with smaller numbers of generators (say, $K = 4$ *versus* $K = 6$). The same happens to D_2^*. This means that, in this example, the investments will be undertaken in competitive markets later than in non-competitive markets when the demand is low ($D < D_1^*$) and sooner when the demand is high ($D > D_2^*$). Figures 7 and 8 respectively show that the effect of the generators' forward contracts (QC) and the degree of vertical integration (M) over both the value and timing of the investment in retail expansion follow a similar pattern. Given p^R and QC_i (firm i's hedging position), Markets with greater QC have *ceteris paribus* a smaller electricity spot price and a bigger retail margin (see equation (7)). The same happens for M for a given firm i's market share (m_i). In particular, *ceteris paribus*, Figure 7 shows that a decrease of forward contracts QC would decrease the project value but would not affect significantly the decision to undertake the project. In the most extreme (and unrealistic) case, where the retail

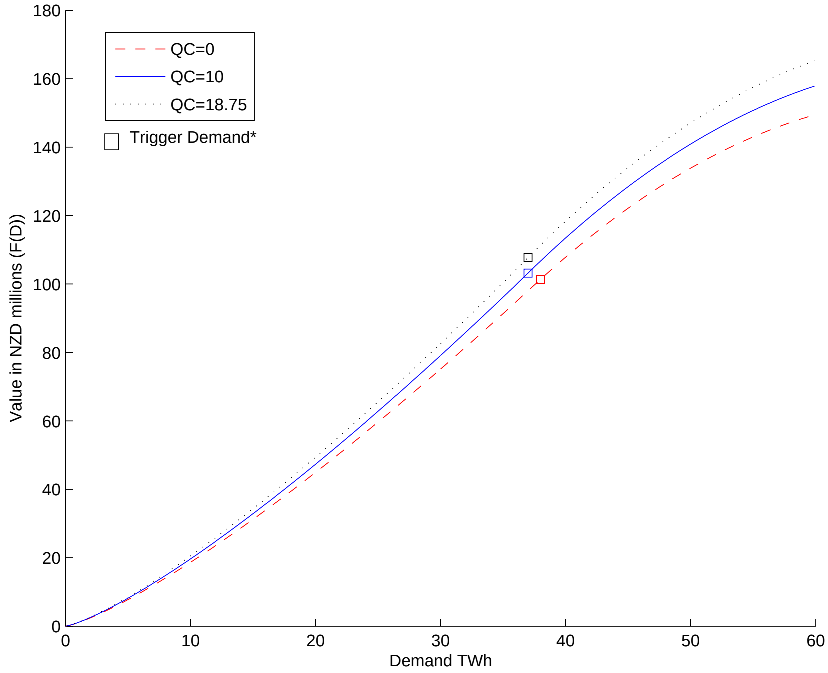
Figure 6: Project value and investment policy (different number of firms)



expansion of 2% decreases the gentailers' financial hedges to zero ($QC = 0$), the changes on both timing and value of the project are very small. Last but not least, Figure 9 shows that changes in the retail price have the expected result on both the value and timing of increasing vertical integration. Projects with smaller retail prices are postponed and have smaller values. Since the retail price is fixed in our framework, a decrease in p^R directly decreases the profitability of the investment (with no short-term effect on demand). In this example, the demand interval which triggers the gentailer i's retail market expansion is considerably shorter for $p^R = 120$ than for $p^R = 170$.

In summary, in electricity markets where retail prices are relatively fixed, the gentailers' decision to undertake sunk investments (e.g marketing campaigns) to increase their participation in the retail market is affected both by market equilibrium parameters and the level of demand ($D < D_1^*$ or $D > D_2^*$). In particular, because of the rigidity of p^R, the project's payoff is potentially concave and

Figure 7: Project value and investment policy (quantity contracted)

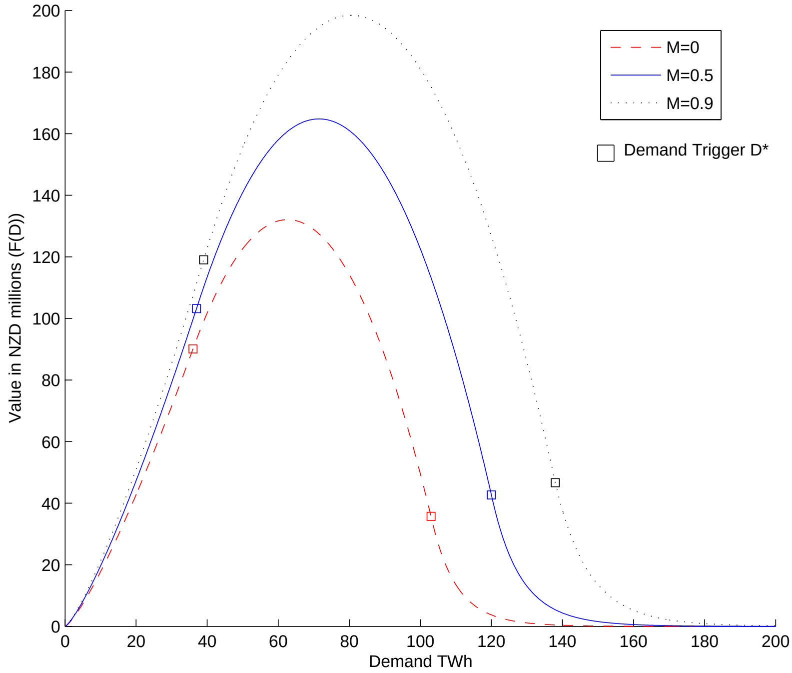


can imply an unusual negative relationship between volatility and the option to wait. In addition, aspects such as market power and the degree of vertical integration play an important role in the retail expansion decision. While the example, is somewhat stylised, it does suggest that market frictions arisen from unrecoverable costs can be an important factor in market/region entry decision for retailers.

4 Conclusion

This paper shows that vertical integration can serve as a hedge in an electricity market. Both financial contracts and vertical integration remove risk due to fluctuations in wholesale prices. A critical difference between vertical integration and financial hedges, however, is that vertical integration results in uncertainty regarding the actual size of the hedge due to variations in consumer demand

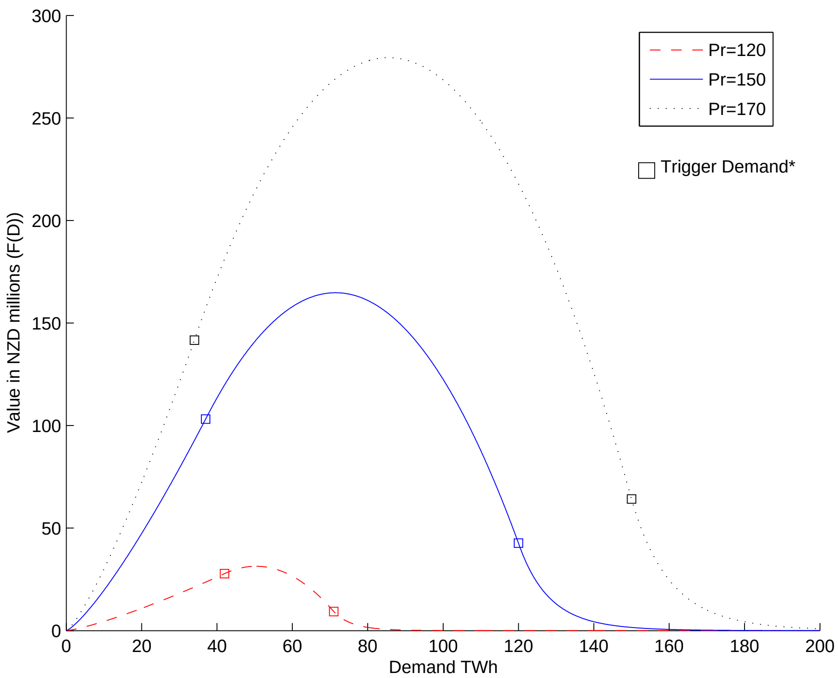
Figure 8: Project value and investment policy (degree of vertical integration)



for electricity.

When we examine a firm choosing to expand its retail operations, we find that the firm will be unwilling to do so when demand is low (since this may result in little business relative to costs incurred) but also when demand is very high (since the firm may end up having to supply electricity at a price which is too low relative to the firm's marginal production costs). The resulting option value is hump shaped. Contrary to most real options problems, it is also locally concave in some regions, resulting in lower value as volatility increases.

Figure 9: Project value and investment policy (Retail prices)



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A Proof of lemma 1

To solve this maximization problem we need first to integrate by parts the objective function. Suppressing the i and t indices we have, modulo a constant term:

$$-\int_{\underline{p}}^{\overline{p}} \quad U'\left(S(p)p - C(S(p),\vec{W}) - (p-PC)QC - m_i(p-p^R)D(p_t^R,\vec{W}_t)\right) \times (S'(p)p - C'(S(p),\vec{W})S'(p) - QC)H(p,S(p);QC)dp.$$

Labeling the integrand:

$$\begin{aligned} F &= -U'\left(S(p)p - C(S(p),\vec{W}) - (p-PC)QC - m(p-p^R)D(p_t^R,\vec{W}_t)\right) \\ &\quad \times (S'(p)p + S(p) - C'(S(p),\vec{W})S'(p) - QC - mD(p_t^R,\vec{W}_t)) \\ &\quad \times H(p,S(p);QC). \end{aligned}$$

From the calculus of variation, the Euler-Lagrange necessary condition for the optimal $S(p)$ is given by:

$$\frac{d}{dp}F_{S'} = F_S.$$

Evaluating the derivatives:

$$\begin{aligned} -F_S &= H_S U'(.)[pS' + S - C'(S,\vec{W})S' - QC - mD(p_t^R,\vec{W}_t))] \\ &\quad + HU''(.)(p - C'(S,\vec{W}))[pS' + S - C'(S,\vec{W})S' - QC - mD(p_t^R,\vec{W}_t))] \\ &\quad + HU'(.)[1 - C''(S,\vec{W})S'] \end{aligned}$$

$$-F_{S'} \quad = \quad HU'(.)[p - C'(S, \vec{W})].$$

And taking the total derivative of $F_{S'}$ with respect to p:

$$\begin{aligned} -\frac{d}{dp}F_{S'} \quad &= \quad H_pU'(.)[p - C'(S, \vec{W})] + H_S S' U'(.)[p - C'(S, \vec{W})] \\ &\quad +HU''(.)[pS' + S - C'(S, \vec{W})S' - QC - mD(p_t^R, \vec{W}_t))](p - C'(S, \vec{W})) \\ &\quad +HU'(.)(1 - C''(S, \vec{W})S'). \end{aligned}$$

Equating and canceling terms we get:

$$H_S U'(.)(S - QC - mD(p_t^R, \vec{W}_t))) = H_p U'(.)(p - C'(S, \vec{W})).$$

Considering again the i and t indices we have:

$$p - MC_{it}(S_{it}^*(p), \vec{W}_t) = [S_{it}^*(p) - QC_{it} - m_i D(p_t^R, \vec{W}_t))] \frac{H_S(p, S_{it}^*(p); QC_{it})}{H_p(p, S_{it}^*(p); QC_{it})}.$$

B Proof of lemma 2

Suppose that,

$$S_i(p, QC_i, \vec{W}) = \alpha_i(p) + \beta_i(QC_i) + \sum_{j=1}^{L} \delta_{ki}(w_k).$$

Given that we can use the market clearing condition to represent the event $Pr(p_t^c \leq p \mid QC_i, S_i(p))$:

$$\sum_{j \neq i} \beta_j(QC_j) + \sum_{j \neq i} \sum_{k=1}^{M} \delta_{kj}(w_k) \geq D(p^R, \vec{W}_t) - S_i - \sum_{j \neq i} \alpha_j(p).$$

The left hand side of this inequality can be labeled as a (bidder specific) random variable, θ_i that does not depend on p, and the right hand side is a deterministic function of price. Let $\Gamma_i(.)$ denote the cdf of θ_i and $\gamma_i(.)$ denote the pdf (both conditional on the bidder's contract quantity QC_i). Given these:

$$\begin{aligned}
H_p(p, S_i; QC_i) &= \frac{\partial}{\partial p} Pr(p_t^c \leq p \mid QC_i, S_i) \\
&= \frac{\partial}{\partial p} Pr(\theta_i \geq D(p^R, \vec{W}_t) - S_i - \sum_{j \neq i} \alpha_j(p)) \\
&= \frac{\partial}{\partial p}[1 - \Gamma_i(D(p^R, \vec{W}_t) - S_i - \sum_{j \neq i} \alpha_j(p))] \\
&= -\gamma_i \left(D(p^R, \vec{W}_t) - S_i - \sum_{j \neq i} \alpha_j(p) \right) \frac{\partial}{\partial p}(D(p^R, \vec{W}_t) \\
&\quad -S_i - \sum_{j \neq i} \alpha_j(p)),
\end{aligned}$$

and

$$\begin{aligned}
H_S(p, S_i; QC_i) &= \frac{\partial}{\partial S_i} Pr(p_t^c \leq p \mid QC_i, S_i) \\
&= -\gamma_i \left(D(p^R, \vec{W}_t) - S_i - \sum_{j \neq i} \alpha_j(p) \right) \frac{\partial}{\partial S_i}(D(p^R, \vec{W}_t) \\
&\quad -S_i - \sum_{j \neq i} \alpha_j(p)).
\end{aligned}$$

Evaluating the derivatives gives $\frac{H_p(p,S_i;QC_i)}{H_S(p,S_i;QC_i)} = -[\frac{\partial D(p^R,\vec{W}_t)}{\partial p} - \sum_{j \neq i} \alpha_j'(p)]$, which is the derivative in respect to price of the residual demand faced by firm i. By definition of the aggregate demand, $\frac{\partial D(p^R,\vec{W}_t)}{\partial p} = 0$.

Substituting these results in the equation (4), defining the marginal cost $C_i'(S_i(p),\vec{W})$ as $MC_i(S_i,\vec{W})$ and considering again the index t, we yield the following equation:

$$\begin{aligned} p_t - MC_{it} &= -\frac{S_{it} - QC_{it} - m_i D_t}{\frac{\partial D_t}{\partial p_t} - \frac{\partial \sum_{j\neq i} S_{jt}}{\partial p_t}} \\ p_t - MC_{it} &= \frac{S_{it} - QC_{it} - m_i D_t}{\frac{\partial \sum_{j\neq i} S_{jt}}{\partial p_t}}. \end{aligned}$$

Now let's consider $q_{it}' = D_t - \sum_{j\neq i} S_{jt} - QC_{it} - m_i D_t$. The elasticity ε_{it}' is equal to $-\frac{dq_{it}'}{dp_t}\frac{p_t}{q_{it}'}$. Observe that $\frac{dq_{it}'}{dp_t} = -\frac{\partial \sum_{j\neq i} S_{jt}}{\partial p_t}$ and, in equilibrium, $D_t - \sum_{j\neq i} S_{jt} = S_{it}$. Therefore $q_{it}' = S_{it} - QC_{it} - m_i D_t$ and,

$$\frac{p_t - MC_{it}}{p_t} = \frac{1}{\varepsilon_{it}'(q_{it}')}.$$

C Proof of proposition 1

Let's now assume that the general form of the symmetric Bayesian-Nash equilibria is a linear supply function of the form:

$$S_{it}(p_t, QC_{it}, \vec{W}_t) = \psi_i + \alpha p_t + \beta QC_{it} + \sum_{j=1}^{L} \delta_j w_{jt}.$$

Now substituting and suppressing time subscript t, we have:

$$p - a - \sum_{j=1}^{L} \rho_j w_j - bS_i = \frac{QC_i + m_i D - S_i}{-(K-1)\alpha}$$

$$(K-1)\alpha p - (K-1)\alpha a - (K-1)\alpha \sum_{j=1}^{L} \rho_j w_{jt} + QC_i + m_i(c - \kappa_o p^R + \sum_{j=1}^{L} \kappa_j w_{jt})$$

$$= S_i(p)[1 + (K-1)\alpha b].$$

Reorganizing equations we have:

$$m_i(c - \kappa_o p^R) - (K-1)\alpha a + (K-1)\alpha p - \sum_{j=1}^{L} \left((K-1)\alpha \rho_j - m_i \kappa_j\right) w_{jt} + QC_i$$

$$= (\psi_i + \alpha p + \beta QC_i + \sum_{j=1}^{L} \delta_{ij} w_j)[1 + (K-1)\alpha b],$$

which implies that the following must hold for $i = 1, 2, \ldots N$:

$$\frac{(K-1)\alpha}{1 + (K-1)\alpha b} = \alpha$$

$$\frac{+m_i(c - \kappa_o p^R) - a(K-1)\alpha}{1 + (K-1)\alpha b} = \psi_i$$

$$\frac{1}{1 + (K-1)\alpha b} = \beta$$

$$-\frac{\rho_1 (K-1)\alpha - m_i \kappa_1}{1 + (K-1)\alpha b} = \delta_{i1}$$

$$-\frac{\rho_2 (K-1)\alpha - m_i \kappa_2}{1 + (K-1)\alpha b} = \delta_{i2}$$

$$\vdots$$

$$-\frac{\rho_L (K-1)\alpha - m_i \kappa_L}{1 + (K-1)\alpha b} = \delta_{iL}$$

Solving the system and substituting back into the assumed general supply function,

$$
\begin{aligned}
S_{it}(p_t, QC_{it}, \vec{W}_t) &= -\frac{a(K-2)}{b(K-1)} + \frac{K-2}{b(K-1)} p_t + \frac{1}{K-1} QC_{it} \\
&\quad + \frac{m_i}{K-1} D_t(p_t^R, \vec{W}_t) - \frac{(K-2)}{b(K-1)} \sum_{j=1}^{L} \rho_j w_{jt} \\
D_t(p_t^R, \vec{W}_t) &= c - \kappa_o p^R + \sum_{j=1}^{L} \kappa_j w_{jt}.
\end{aligned}
$$

Clearing the market and putting in terms of p_t^c,

$$
\begin{aligned}
p_t^c &= a + b \frac{(c - \kappa_o p^R)\left(K - (1 + \sum_{i=1}^{K} m_i)\right)}{K(K-2)} - \frac{b}{K(K-2)} \sum_{i=1}^{K} QC_{it} \\
&\quad + \sum_{j=1}^{L} \left(\rho_j + b \frac{\left(K - (1 + \sum_{i=1}^{K} m_i)\right)}{K(K-2)} \kappa_j \right) w_{jt},
\end{aligned}
$$

which is the same as:

$$
p_t^c = A - B \sum_{i=1}^{K} QC_{it}^* + \sum_{j=1}^{L} C_j w_{jt},
$$

where

$$
\begin{aligned}
A &= a + b \frac{(c - \kappa_o p^R)\left(K - (1 + \sum_{i=1}^{K} m_i)\right)}{K(K-2)} \\
B &= \frac{b}{K(K-2)} \\
C_j &= \rho_j + b \frac{\left(K - (1 + \sum_{i=1}^{K} m_i)\right)}{K(K-2)} \kappa_j.
\end{aligned}
$$

D Proof of lemma 3

The profit function is equal to:

$$\pi_i \quad = \quad p^c S_i^*(p^c) - C(S^*(p^c)) + (PC - p^c)QC_i + m_i(p^R - p^c)D. \tag{25}$$

The optimal supply function is equal to:

$$\begin{aligned} S_{it}(p_t, QC_{it}, \vec{W}_t) \quad &= \quad -\frac{a(K-2)}{b(K-1)} + \frac{K-2}{b(K-1)}p_t + \frac{1}{K-1}QC_{it} \\ &+\frac{m_i}{K-1}D_t - \frac{(K-2)}{b(K-1)}\sum_{j=1}^{L}\rho_j w_{jt}. \end{aligned} \tag{26}$$

Now, consider $\overline{QC}_{jt} = \frac{\sum_{j\neq i} QC_{jt}}{K-1}$ and $\overline{m}_j = \frac{\sum_{j\neq i} m_j}{K-1}$. The clearing price p_t^c is described by the equation:

$$\begin{aligned} p_t^c \quad &= \quad a + \sum_{j=1}^{L}\rho_j w_{jt} + b\frac{(K - (1 + m_i + (K-1)\overline{m}_j))}{K(K-2)}D_t \\ &- \frac{b}{K(K-2)}\left(QC_i + (K-1)\overline{QC}_j\right). \end{aligned} \tag{27}$$

Substituting back in the firm i's supply equation, we have the following optimal equilibrium supply:

$$\begin{aligned} & S_{it}(p_t, QC_{it}, \vec{W}_t) \\ = \quad & \frac{K-2}{b(K-1)}\left(a + \sum_{j=1}^{L}\rho_j w_{jt} + b\frac{(K - (1 + m_i + (K-1)\overline{m}_j))}{K(K-2)}D_t\right) \\ & -\frac{1}{K(K-1)}\left(QC_i + (K-1)\overline{QC}_j\right) - \frac{a(K-2)}{b(K-1)} + \frac{1}{K-1}QC_{it} \\ & +\frac{m_i}{K-1}D_t - \frac{(K-2)}{b(K-1)}\sum_{j=1}^{L}\rho_j w_{jt} \end{aligned}$$

$$
\begin{aligned}
&= \frac{K-1-m_i-(K-1)\overline{m}_j}{K(K-1)}D_t - \frac{QC_i+(K-1)\overline{QC}_j}{K(K-1)} + \frac{1}{K-1}QC_{it} \\
&\quad + \frac{m_i}{K-1}D_t \\
&= \frac{K-1-(1-K)m_i-(K-1)\overline{m}_j}{K(K-1)}D_t - \frac{(1-K)QC_i+(K-1)\overline{QC}_j}{K(K-1)} \\
&= \frac{1+m_i-\overline{m}_j}{K}D_t + \frac{QC_i-\overline{QC}_j}{K}
\end{aligned}
$$

$$
S^*_{it} = \frac{(1+m_i-\overline{m}_j)D_t}{K} + \frac{QC_{it}-\overline{QC}_{jt}}{K}. \tag{28}
$$

Now suppose that the generator/gentailer i has the option to establish/expand his retail position by investing I to increase his retail market share by Δm_i.

Using Taylor expansion, we know that the impact of a change in m_i on the gentailer/generator profit is equal to:

$$
\Delta\pi_i = \frac{d\pi_i}{dm_i}\Delta m_i + \frac{1}{2}\frac{d^2\pi_i}{dm_i^2}(\Delta m_i)^2 \tag{29}
$$

$$
\Delta\pi_i = \frac{d\pi_i}{d\overline{m}_j}\Delta \overline{m}_j + \frac{1}{2}\frac{d^2\pi_i}{d\overline{m}_j^2}(\Delta \overline{m}_j)^2. \tag{30}
$$

Calculating the derivatives of the first equation and skipping the time subscripts, we have:

$$
\begin{aligned}
\frac{d\pi_i(p^c(m_i), S^*(m_i))}{dm_i} &= \frac{\partial\pi_i}{\partial m_i} + \frac{\partial\pi_i}{\partial p^c}\frac{\partial p^c}{\partial m_i} + \frac{\partial\pi_i}{\partial S^*_i}\frac{\partial S^*_i}{\partial m_i} \\
\frac{\partial\pi_i}{\partial m_i} &= (p^R - p^c)D
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \pi_i}{\partial p^c}\frac{\partial p^c}{\partial m_i} &= -\frac{b(S-QC_i-m_iD)D}{K(K-2)} \\
\frac{\partial \pi_i}{\partial S_i^*}\frac{\partial S_i^*}{\partial m_i} &= (P-MC)\frac{D}{K}.
\end{aligned}
$$

Define $M=\sum_{k=1}^{K} m_k$ and $\sum_{k=1}^{K} QC_{kt} = QC$. Now we show that the first two terms are equal to zero:

$$
\begin{aligned}
p^c - MC_i &= p^c - (a+\sum_{j=1}^{L}\rho_j w_j) - bS_i \\
&= b\frac{(K-1-m_i-(K-1)\overline{m}_j))}{K(K-2)}D - \frac{b(QC_i+(K-1)\overline{QC}_j)}{K(K-2)} \\
&\quad -b\left(\frac{(1+m_i-\overline{m}_j)D}{K}+\frac{QC_{it}-\overline{QC}_j}{K}\right) \\
&= \frac{b}{K(K-2)}\left(K-1-m_i-(K-1)\overline{m}_j\right)D \\
&\quad -\frac{b}{K(K-2)}(K-2)(1+m_i-\overline{m}_j)D \\
&\quad -\frac{b(QC_i+(K-2)QC_i)}{K(K-2)} - \frac{b((K-1)\overline{QC}_j-(K-2)\overline{QC}_j)}{K(K-2)} \\
&= \frac{b\left(1-(K-1)m_i-\overline{m}_j\right)}{K(K-2)}D - \frac{b\left((K-1)QC_i+\overline{QC}_j\right)}{K(K-2)}
\end{aligned}
$$

$$
\begin{aligned}
S_i - QC_i - m_iD &= \frac{(1+m_i-\overline{m}_j)D}{K} + \frac{QC_i-\overline{QC}_j}{K} - QC_i - m_iD \\
&= \frac{(1-(K-1)m_i-\overline{m}_j)}{K}D - \frac{\left((K-1)QC_i+\overline{QC}_j\right)}{K}.
\end{aligned}
$$

Calculating the first order derivative we have:

$$
\frac{\partial \pi_i}{\partial p^c}\frac{\partial p^c}{\partial m_i} + \frac{\partial \pi_i}{\partial S_i^*}\frac{\partial S_i^*}{\partial m_i} = 0.
$$

We also have the following first order derivative:

$$\frac{d\pi_i(m_i, p^c(m_i), S^*(m_i))}{dm_i} = (p^R - p^c)D \tag{31}$$

The second order derivative is equal to:

$$\begin{aligned}\frac{d^2\pi_i}{dm_i^2} &= -D\frac{\partial p^c}{\partial m_i} \\ &= \frac{b}{K(K-2)}D^2,\end{aligned} \tag{32}$$

so that

$$\Delta\pi_{it} = (p^R - p^c)D\Delta m_i + \frac{1}{2}\frac{b}{K(K-2)}D^2(\Delta m_i)^2. \tag{33}$$

We have an analogous calculation for the impact of a decrease in rival gentailers' average $\overline{m}_j$.

Calculating the derivatives of the first equation and skipping the time subscripts, we have:

$$\begin{aligned}\frac{d\pi_i(\overline{m}_j, p^c(\overline{m}_j), S^*(\overline{m}_j))}{d\overline{m}_j} &= \frac{\partial\pi_i}{\partial\overline{m}_j} + \frac{\partial\pi_i}{\partial p^c}\frac{\partial p^c}{\partial\overline{m}_j} + \frac{\partial\pi_i}{\partial S_i^*}\frac{\partial S_i^*}{\partial\overline{m}_j} \\ \frac{\partial\pi_i}{\partial\overline{m}_j} &= 0 \\ \frac{\partial\pi_i}{\partial p^c}\frac{\partial p^c}{\partial\overline{m}_j} &= (K-1)\frac{\partial\pi_i}{\partial p^c}\frac{\partial p^c}{\partial m_i} \\ \frac{\partial\pi_i}{\partial S_i^*}\frac{\partial S_i^*}{\partial\overline{m}_j} &= -\frac{\partial\pi_i}{\partial S_i^*}\frac{\partial S_i^*}{\partial m_i}.\end{aligned}$$

Considering that $\frac{\partial\pi_i}{\partial p^c}\frac{\partial p^c}{\partial m_i} = -\frac{\partial\pi_i}{\partial S_i^*}\frac{\partial S_i^*}{\partial m_i}$, we have:

$$\begin{aligned}\frac{d\pi_i(\overline{m}_j, p^c(\overline{m}_j), S^*(\overline{m}_j))}{d\overline{m}_j} &= K\frac{\partial \pi_i}{\partial p^c}\frac{\partial p^c}{\partial m_i} \\ &= \frac{b\left((K-1)QC_i + \overline{QC}_j\right)}{K(K-2)}D \\ &\quad -\frac{b\left(1-(K-1)m_i - \overline{m}_j\right)}{K(K-2)}D^2.\end{aligned}$$

The second order effect is equal to:

$$\frac{d^2\pi_i(\overline{m}_j, p^c(\overline{m}_j), S^*(\overline{m}_j))}{d\overline{m}_j^2} = \frac{b}{K(K-2)}D^2. \tag{34}$$

Therefore, we have:

$$\begin{aligned}\Delta\pi_i &= \frac{b\left((K-1)QC_i + \overline{QC}_j\right)}{K(K-2)}D\Delta\overline{m}_j - \frac{b\left(1-(K-1)m_i - \overline{m}_j\right)}{K(K-2)}D^2\Delta\overline{m}_j \\ &\quad +\frac{1}{2}\frac{b}{K(K-2)}D^2(\Delta\overline{m}_j)^2.\end{aligned}$$

For simplicity, consider that $\sum_{k=1}^{K} QC_{kt}$ is a constant QC and that costs are not driven by any additional state variable. That is, $MC_i = a + bS_i$ and the clearing price is defined as:

$$p_t^c = a + b\frac{(K-1-M)}{K(K-2)}D_t. - \frac{b}{K(K-2)}QC$$

Define $\Delta\overline{m}_j = -\frac{\omega\Delta m_i}{K-1}$. Putting the direct ($m_i$) and indirect ($\overline{m}_j$) effects together, the investment has the following impact on profit:

$$\begin{aligned}
\Delta\pi_{it} &= \Delta m_i(p^R - p^c)D + \Delta\overline{m}_j \frac{b\left((K-1)QC_i + \overline{QC}_j\right)}{K(K-2)} D \\
&+ \frac{(\Delta m_i)^2}{2}\frac{b}{K(K-2)}D^2 - \Delta\overline{m}_j \frac{b\,(1-(K-1)m_i - \overline{m}_j)}{K(K-2)}D^2 \\
&+ \frac{(\Delta\overline{m}_j)^2}{2}\frac{b}{K(K-2)}D^2 \\
&= \Delta m_i\left(p^R - a + \frac{b}{K(K-2)}QC\right)D \\
&+ \Delta\overline{m}_j \frac{b\left((K-1)QC_i + \overline{QC}_j\right)}{K(K-2)} D \\
&- \Delta m_i b\frac{(K-1-M)}{K(K-2)}D^2 + \frac{(\Delta m_i)^2}{2}\frac{b}{K(K-2)}D^2 \\
&- \Delta\overline{m}_j \frac{b\,(1-(K-1)m_i - \overline{m}_j)}{K(K-2)}D^2 + \frac{(\Delta\overline{m}_j)^2}{2}\frac{b}{K(K-2)}D^2 \\
&= \Delta m_i\left(p^R - a + \frac{b}{K(K-2)}QC\right)D \\
&- \frac{\omega\Delta m_i}{K-1}\frac{b\left((K-1)QC_i + \overline{QC}_j\right)}{K(K-2)} D \\
&- \Delta m_i b\frac{(K-1-M)}{K(K-2)}D^2 + \frac{(\Delta m_i)^2}{2}\frac{b}{K(K-2)}D^2 \\
&+ \frac{\omega\Delta m_i}{K-1}\frac{b\,(1-(K-1)m_i - \overline{m}_j)}{K(K-2)}D^2 + \frac{\omega^2(\Delta m_i)^2}{2(K-1)^2}\frac{b}{K(K-2)}D^2 \\
&= \Delta m_i\left(p^R - a + \frac{b}{K(K-2)}QC - \frac{b\omega\left((K-1)QC_i + \overline{QC}_j\right)}{K(K-1)(K-2)}\right)D \\
&- \frac{b\Delta m_i}{K(K-2)}\left(K-1-M-\frac{\Delta m_i}{2}\right)D^2 \\
&+ \frac{\omega b\Delta m_i}{K(K-1)(K-2)}\left(1-(K-1)m_i - \overline{m}_j + \frac{\omega\Delta m_i}{2(K-1)}\right)D^2 \\
&= \Delta m_i\left(p^R - a + \frac{b}{K(K-2)}QC - \frac{b\omega\,(QC + K(K-2)QC_i))}{K(K-1)^2(K-2)}\right)D \\
&- \frac{b\Delta m_i}{K(K-2)}\left(K-1-M-\frac{\Delta m_i}{2}\right)D^2 \\
&+ \frac{\omega b\Delta m_i}{K(K-1)^2(K-2)}\left(K-1-M-K(K-2)m_i + \frac{\omega\Delta m_i}{2}\right)D^2 \\
&= \Delta m_i\left(p^R - a + \frac{b}{K(K-2)}\left(1-\frac{\omega}{(K-1)^2}\right)QC - \frac{b\omega}{(K-1)^2}QC_i\right)D
\end{aligned}$$

$$
\begin{aligned}
& -\frac{b\Delta m_i}{K(K-2)}\left((K-1-M)\left(1-\frac{\omega}{(K-1)^2}\right)\right)D^2 \\
& -\frac{b\Delta m_i}{K(K-2)}\left(\frac{\omega K(K-2)}{(K-1)^2}m_i-\left(1+\frac{\omega}{(K-1)^2}\right)\frac{\Delta m_i}{2}\right)D^2 \\
= \; & \hat{A}D-\hat{B}D^2.
\end{aligned}
$$

E Proof of lemma 4

Consider that the demand follows a geometric brownian motion,

$$
dD_t = D_t\mu dt + D_t\sigma dW_t. \tag{35}
$$

The present value of the inflows V_t is equal to:

$$
\begin{aligned}
V_t &= \hat{A}\int_t^T E_t\left[D_\tau\right]e^{-r(\tau-t)}d\tau - \hat{B}\int_t^T E_t\left[D_\tau^2\right]e^{-r(\tau-t)}d\tau \\
&= \hat{A}\int_t^T e^{\mu(\tau-t)}e^{-r(\tau-t)}d\tau D_t - \hat{B}\int_t^T e^{(2\mu+\sigma^2)(\tau-t)}e^{-r(\tau-t)}d\tau D_t^2 \\
&= \hat{A}\frac{\left(1-e^{-(r-\mu)(T-t)}\right)}{r-\mu}D_t - \hat{B}\frac{\left(1-e^{-(r-2\mu-\sigma^2)(T-t)}\right)}{r-2\mu-\sigma^2}D_t^2 \\
&= \tilde{A}D_t - \tilde{B}D_t^2.
\end{aligned} \tag{36}
$$

Therefore, we have:

$$
V_t = \tilde{A}D_t - \tilde{B}D_t^2 - I \tag{37}
$$

$\tilde{A}$ is clearly greater than zero if $p^R > p^c$. We can easily show that $\tilde{B} \geq 0$ as well. By contradiction, suppose that $\tilde{B} < 0$. Then the following holds:

$$\begin{aligned}
\left(1+\frac{\omega}{(K-1)^2}\right)\frac{\Delta m_i}{2} &\geq (K-1-M)\left(1-\frac{\omega}{(K-1)^2}\right)+\frac{\omega K(K-2)}{(K-1)^2}m_i \\
\left(1+\frac{\omega}{(K-1)^2}\right)\frac{\Delta m_i}{2} &\geq (K-1-M)\left(1-\frac{\omega}{(K-1)^2}\right) \\
\Delta m_i &\geq 2\frac{1-\frac{\omega}{(K-1)^2}}{1+\frac{\omega}{(K-1)^2}}(K-1-M).
\end{aligned}$$

The second term of the inequality achieves his minimum value if $\omega = 1$, $K = 3$ and $M = 1$. Therefore in this case,

$$\begin{aligned}
\Delta m_i &\geq 2\frac{1-\frac{1}{(3-1)^2}}{1+\frac{1}{(3-1)^2}}(3-1-1) \\
\Delta m_i &\geq \frac{6}{5}.
\end{aligned}$$

However, this is a contradiction because $\Delta m_i \leq 1$. Therefore, $\tilde{B} \geq 0$.

F Proof of proposition 2

Over a short period of time, dt, the total return from holding the investment opportunity, $F(D)$, is simply its expected change in value. With a rate of return r, this is:

$$rFdt = E(dF). \tag{38}$$

Expanding dF using Ito's Lemma, we get the quadratic differential equation:

$$\frac{1}{2}\sigma^2 D^2 F''(D) + \mu D F'(D) - rF = 0. \tag{39}$$

The general solutions for F in the domains $[0, D_1^*[$ and $]D_2^*, \infty[$ are equal to $F = \alpha_{11} D^{\beta_1} + \alpha_{21} D^{\beta_2}$ and $F = \alpha_{21} D^{\beta_1} + \alpha_{22} D^{\beta_2}$, respectively. The parameters β_1 and β_2 are the characteristic roots of the partial differential equation given by equation (12). We know that β_1 is greater than 1 and β_2 is negative.[14] Therefore, the conditions given by equations (13) and (18) imply that $\alpha_{21} = 0$ and $\alpha_{12} = 0$. We obtain α_{11} and α_{22} from the remaining boundary conditions.

Assume that, for $D \in [0, D_1^*]$, $F(D)$ satisfies the following boundary conditions:

$$F(0) = 0 \tag{40}$$

$$F(D_1^*) = \tilde{A} D_1^* - \tilde{B} D_1^{*2} - I \tag{41}$$

$$F'(D_1^*) = \tilde{A} - 2\tilde{B} D_1^*. \tag{42}$$

Using the first boundary condition we have $F(D) = \alpha_{11} D^{\beta_1}$ where:

$$\beta_1 = \frac{1}{2} - \frac{(r-\mu)}{\sigma^2} + \sqrt{\left(\frac{(r-\mu)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}. \tag{43}$$

Applying the boundary conditions we have:

$$\alpha_{11} \beta_1 D_1^{*\beta_1 - 1} = \tilde{A} - 2\tilde{B} D_1^* \tag{44}$$

[14]See Dixit et al. (1994) pages 142-144.

$$\alpha_{11} D_1^{*\beta_1} \;=\; \tilde{A} D_1^* - \tilde{B} D_1^{*2} - I. \tag{45}$$

Dividing 44 by 45,

$$\begin{aligned}
\frac{1}{\beta_1} D_1^* &= \frac{\tilde{A} D_1^* - \tilde{B} D_1^{*2} - I}{\tilde{A} - 2\tilde{B} D_1^*} \\
&\Rightarrow \frac{\tilde{A}}{\beta} D_1^* - 2\frac{\tilde{B}}{\beta} D_1^{*2} = \tilde{A} D_1^* - \tilde{B} D_1^{*2} - I \\
&\Rightarrow -\frac{2-\beta_1}{\beta_1} \tilde{B} D_1^{*2} + \frac{1-\beta_1}{\beta_1} \tilde{A} D_1^* + I = 0 \\
&\Rightarrow -(2-\beta_1)\tilde{B} D_1^{*2} + (1-\beta_1)\tilde{A} D_1^* + \beta_1 I = 0. \qquad (46)
\end{aligned}$$

Lastly, we have:

$$D_1^* \;=\; \frac{(1-\beta_1)\tilde{A} + \sqrt{(1-\beta_1)^2 \tilde{A}^2 + 4(2-\beta_1)\beta_1 \tilde{B} I}}{2(2-\beta_1)\tilde{B}} \tag{47}$$

$$\alpha_{11} \;=\; \frac{\tilde{A}}{\beta_1} D_1^{*1-\beta_1} - 2\frac{\tilde{B}}{\beta_1} D_1^{*2-\beta_1}. \tag{48}$$

It is reasonable to assume that if the project has a negative value at a the critical demand, the option to exercise it has no value.

Therefore, α_{11} is, in fact, equal to:

$$\alpha_{11} \;=\; \max\left(\frac{\tilde{A}}{\beta} D^{*1-\beta} - 2\frac{\tilde{B}}{\beta} D^{*2-\beta}, 0\right). \tag{49}$$

For $D \in [D_2^*, \infty]$ we have the following boundary conditions:

$$F(\infty) \;=\; 0 \tag{50}$$

$$F(D_2^*) = \tilde{A}D_2^* - \tilde{B}D_2^{*2} - I \tag{51}$$

$$F'(D_2^*) = \tilde{A} - 2\tilde{B}D_2^*. \tag{52}$$

Using the transversality condition, the solution in this region is $F(D) = \alpha_{22}D^{\beta_2}$, where:

$$\beta_2 = \frac{1}{2} - \frac{(r-\mu)}{\sigma^2} - \sqrt{\left(\frac{(r-\mu)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}. \tag{53}$$

Analogously, we have the following results:

$$D_2^* = \frac{(1-\beta_2)\tilde{A} + \sqrt{(1-\beta_2)^2\tilde{A}^2 + 4(2-\beta_2)\beta_2\tilde{B}I}}{2(2-\beta_2)\tilde{B}} \tag{54}$$

$$\alpha_{22} = \frac{\tilde{A}}{\beta_2}D_2^{*1-\beta_2} - 2\frac{\tilde{B}}{\beta_2}D_2^{*2-\beta_2}. \tag{55}$$