Expected Stock Returns and Forward Variance*

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Abstract

In this paper, we construct model-free 3-, 6-, and 9-month forward variances and study their predictive power for stock market excess returns. We find that (i) the combination of the 6- and 9-month forward variances predict next month stock market returns with an adjusted R^2 of 2.26%, (ii) a similar tent-shaped pattern of the slope coefficients is observed in forecasting returns at 1-, 3- and 6-month horizons, (iii) a single forward variance factor, a linear combination of the forward variances, predicts stock market returns at the three horizons with slope coefficients 0.37, 1.08 and 1.55 and adjusted R^2s of 2.38%, 6.55% and 5.33%, respectively, (iv) out-of-sample analysis confirms predictive power of the single forward variance factor relative to the historical average and other traditional economic variables.

Keywords: Stock returns; Forward variance; Predictability
JEL Classification Code: C5; G12; G13

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Abstract

In this paper, we construct model-free 3-, 6-, and 9-month forward variances and study their predictive power for stock market excess returns. We find that (i) the combination of the 6- and 9-month forward variances predict next month stock market returns with an adjusted R^2 of 2.26%, (ii) a similar tent-shaped pattern of the slope coefficients is observed in forecasting returns at 1-, 3- and 6-month horizons, (iii) a single forward variance factor, a linear combination of the forward variances, predicts stock market returns at the three horizons with slope coefficients 0.37, 1.08 and 1.55 and adjusted R^2s of 2.38%, 6.55% and 5.33%, respectively, (iv) out-of-sample analysis confirms predictive power of the single forward variance factor relative to the historical average and other traditional economic variables.

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1 Introduction

It has been a long history in studying risk-return relation, however, puzzling findings are obtained when different models are employed to calculate conditional variance.[1] While previous literature often focuses on the relation between next period excess return and the conditional variance in that period, we investigate the relation augmented with variances beyond next period ($[t, t+1]$), e.g., 6- and 9-month forward variances over the intervals $[t+3, t+6]$ and $[t+6, t+9]$, respectively.[2] Using monthly data, we find that the combination of 6- and 9-month forward variances predicts next month stock market returns with adjusted R^2s of 2.26% and 10.82%, without or with traditional predictive variables. More importantly, the medium-term beta (the slope coefficient of 6-month forward variance) is positive while the long-term beta is negative. In this sense, the positive (negative) risk-return relation between expected market returns and the 6-month (9-month) forward variance may help to explain mixed findings in the risk-return tradeoff literature.

More specifically, using the unique data provided by the Chicago Board Options Exchange (CBOE), we construct monthly model-free forward variances with maturities 3, 6, and 9 months from January 1992 to August 2008 and study their predictive power for stock market excess returns. We show that the 6- and 9-month forward variances improve predictive power of traditional predictors substantially, with adjusted R^2 from 3.21% to 10.82% at 1-month horizon. Second, both 6- and 9-month forward variances remain significant in predicting returns at 3- and 6-month horizons, without or with traditional predictive variables. Third, a similar tent-shaped pattern of the slope coefficient of the three forward

[1]For example, a positive relation is found in French, Schwert, and Stambaugh (1987) using the rolling window model, Scruggs (1998) using the GARCH model, Ghysel, Santa-Clara, and Valkanov (2005) with a novel mixed data sampling approach, and Guo and Whitelaw (2006) by instrumental variables method; a negative relation in Nelson (1991) using a EGARCH-M specification, and Brandt and Kang (2004) using a latent VAR approach; a time-varying relation in Glosten, Jagannathan, and Runkle (1993) using a modified GARCH-M model.

[2]In this paper, we use 6-month (9-month) forward variance to denote 3-month (6-month) ahead variances with 3-month period.

variances is observed in forecasting returns at 1-, 3- and 6-month horizons. Recall the similar pattern obtained in Cochrane and Piazzesi (2005) by predicting excess returns on one- to five-year maturity bonds using forward rates, we see that the investor's risk aversion is consistent in both bond and stock markets, which are revealed by forward rates or forward variances, respectively. Essentially, these empirical findings may indicate that risks embedded in longer-term (6- and 9-month) forward variances are markedly different from that in short-term (3-month) forward variance in terms of forecasting future stock market returns. In other words, the aggregate investor's risk attitude depend on time horizons.

Furthermore, we construct a single forward variance factor, a linear combination of the three forward variances, and investigate its forecasting ability. Interestingly, as the powerful single forward rate factor in predicting bond returns documented in Cochrane and Piazzesi (2005), we find that the single forward variance factor predicts excess stock market returns at 1-, 3- and 6-month horizons with adjusted R^2s of 2.38%, 6.55% and 5.33%, respectively. Moreover, the single forward variance factor is statistically and economically significant at 5% level at the three horizons. For example, the slope coefficients of the single forward variance factor are 0.37, 1.08, and 1.55 at the three return horizons, which means that we indeed find a positive risk-return relation. In addition, the single factor remains significant when the slope of yield curve, the earnings-price ratio and the credit spread are included at all forecasting horizons.

Out-of-sample analysis further confirms predictive power of the single forward variance factor relative to the historical average and other economic variables. Recently, Welch and Goyal (2008) raise concerns related to predictive ability of traditional economic variables in terms of poor out-of-sample performance. Following Campbell and Thompson (2008) and Rapach, Strauss, and Zhou (2010), we use an out-of-sample R^2 statistics to measure the out-of-sample performance. Particularly, we consider the out-of-sample periods 01/2007-12/2008 and 01/2000-12/2008 to investigate effects of the "technology bubble" and the

2007-2008 financial crisis. We find that the single forward variance factor outperforms the historical average and many economic variables for the two sample periods. For example, the out-of-sample R^2 statistics of the single forward variance factor, the dividend-price ratio, the dividend yield, the default return spread, the inflation, the term spread, the Treasury bill rate, the long-term return and the long-term yield are 4.63%, 0.03%, -0.35%, -0.16%, -2.29%, -1.98%, -1.11%, -0.85% and -0.16%, respectively, for the 01/2000-12/2008 period, where positive out-of-sample R^2 indicates that predictive regression forecast is better than the historical average. It is interesting to note that corresponding out-of-sample R^2 values are 8.88%, -5.68%, -5.52%, 3.91%, -1.34%, 0.14%, 5.49%, -1.52% and 4.67% for the 01/2007-12/2008 period, which means that the single forward variance factor has consistent predictive power for different out-of-sample periods while some economic variables exhibit noteworthy varying performance (e.g., the default return spread, the Treasury bill rate and the long-term yield). In this sense, our results are complement to Bakshi, Panayotov and Skoulakis (2011), which focuses on in-sample analysis, in demonstrating predictive power of forward variances.

Recently, Bakshi, Panayotov and Skoulakis (2011) show that forward variances can improve predictability of real economic growth, Treasury bill returns and stock market returns.[3] However, with the model-free and longer maturity data, we do observe several important differences. First, the combination of 3- and 9-month forward variances predict next month returns with an adjusted R^2 of 1.27% as well while the combination of 3- and 6-month forward variances has no predictive power. Further, the best performance comes from the combination of 6- and 9-month forward variances, which is different from the information content of short-term (with maturities correspond to 19 and 49 days) forward

[3] In fact, Bakshi, Panayotov and Skoulakis (2011) construct forward variances from exponential claims on integrated variance by using theoretical results in Carr and Lee (2008). Our forward variances are calculated by using a simple relation between forward variance and spot variance, as will be seen in a later section.

variances detected in Bakshi, Panayotov and Skoulakis (2011). Second, the coefficients of both 6- and 9-month forward variances are statistically significant whether or not the traditional predictor variables are included, while only 49-day forward variance is significant in Bakshi, Panayotov and Skoulakis (2011). Third, a similar tent-shaped pattern of regression coefficients for the three forward variances is observed in predicting different horizon returns. Note that the coefficients of 49-day forward variance are larger than that of 19-day forward variance in Bakshi, Panayotov and Skoulakis (2011), although the coefficient of 19-day forward variance is negative in 1-month regression and positive in 3- and 6-month regressions.

To further examine the single forward variance factor's ability to predict excess returns, we provide robustness analysis by considering measurement error and run regressions while controlling for the variance risk premium. As noted by Cochrane and Piazzesi (2005), the measurement error issue can be addressed with the help of lagged variables. We conduct predictive regressions as before by using lagged forward variances and the single forward variance factor to predict returns at 1-, 3- and 6-month horizons, and obtain same conclusion. The importance of variance risk premium, defined as the difference between the squared VIX index and realized variance, in predicting stock return has been well documented in the recent studies (see, for example, Bollerslev, Tauchen, and Zhou (2009), Drechsler and Yaron (2011) and Zhou and Zhu (2009)). Specifically, we consider both the pre-crisis sample (01/1992-06/2007) and the full sample (01/1992-08/2009) periods to examine the effect of huge volatility spikes during the 2007-2008 financial crisis. It turns out that the single forward variance factor has more predictive power than the variance risk premium throughout both the pre-crisis and the full sample periods at 1-, 3- and 6-month horizons. While the variance risk premium is significant in univariate regressions for the pre-crisis sample period, it is not significant for the full sample period. However, the single factor is significant in both the univariate and the multivariate regressions for the two sample periods. In addition, we find similar results as before when both lagged single forward variance factor and lagged variance risk premium are considered.

The rest of the paper is organized as follows. Section 2 describes the data and presents in-sample results. Section 3 reports out-of-sample analysis. Section 4 provides robustness check by considering measurement error and variance risk premium. Section 5 concludes the paper.

2 Data and empirical results

In this section, we introduce the VIX term structure data, construct the forward variances used in the paper, and describe S&P 500 (SPX) returns and traditional predictors.

2.1 Data

The model-free VIX term structure is calculated by applying the CBOE's methodology of VIX formula to a single strip of options having the same expiration dates, and model-free forward variances can be interpolated from the VIX term structure.[4] However, unlike the VIX index, VIX term structure data does not reflect constant-maturity volatility. Generally, the CBOE lists SPX option series in three near-term contract months plus at least three additional contracts expiring on the March quarterly cycle; that is, on the third Friday of March, June, September and December. Therefore, for each day, there are different numbers of expiration dates and corresponding VIXs. Note that the CBOE use "business day" convention to measure time to expiration in computing VIX term structure data.[5]

In order to construct a VIX with particular maturity, we use interpolation as in the CBOE 30-day VIX calculation procedure. For example, on October 31, 2008, we use

[4]The CBOE revised the methodology of VIX formula by using results of Carr and Madan (1998) and Demeterfi et al (1999). The concept of forward variance is initially proposed by Dupire (1993). Theoretical foundation of computing model-free implied variance by current option prices is developed by Britten-Jones and Neuberger (2000) for diffusion model and extended to jump-diffusion setting by Jiang and Tian (2005).

[5]Both VIX and VIX term structure white papers can be obtained from the CBOE websites.

implied volatility values of two SPX options with expiration dates March 21, 2009 (95 business days) and June 20, 2009 (158 business days) to compute the VIX with 126 trading days to expiration. That is,

$$VIX^2_{t,126d} = \left[T_1\sigma_1^2 \left(\frac{N_{T_2} - N_{126}}{N_{T_2} - N_{T_1}} \right) + T_2\sigma_2^2 \left(\frac{N_{126} - N_{T_1}}{N_{T_2} - N_{T_1}} \right) \right] \times \frac{N_{252}}{N_{126}}, \tag{1}$$

where T_1 and T_2 are 95 and 158 business days to expiration of the two SPX options, respectively, and σ_1 and σ_2 are corresponding volatilities. N_T denotes business day measure. Same methodology can be employed to calculate the forward VIX term structure. Particularly, at time t, the forward squared VIX between time interval $[T_1, T_2]$, $VIX^2_{t,(T_1,T_2)}$, is given by

$$VIX^2_{t,(T_1,T_2)} = VIX^2_{t,T_2-t} \left(\frac{N_{T_2} - t}{N_{T_2} - N_{T_1}} \right) - VIX^2_{t,T_1-t} \left(\frac{N_{T_1} - t}{N_{T_2} - N_{T_1}} \right), \ t < T_1 < T_2. \tag{2}$$

We know that, at time t, τ_1-ahead forward variance in the period $[t+\tau_1, t+\tau_1+\tau_2]$, $FV_{t,(t+\tau_1,t+\tau_1+\tau_2)}$, is equivalent to the forward squared VIX in the same period. That is[6],

$$FV_{t,(t+\tau_1,t+\tau_1+\tau_2)} = E_t^Q[VIX^2_{t+\tau_1,\tau_2}], \tag{3}$$

$$= VIX^2_{t,(t+\tau_1,t+\tau_1+\tau_2)}. \tag{4}$$

Therefore, the three forward variances over intervals $[t, t+3m]$, $[t+3m, t+6m]$ and $[t+6m, t+9m]$ can be calculated using the formula (2) and the VIX term structure data with maturities that bracket 3, 6 and 9 months, respectively. The monthly values are the last observations in the month. Note that the CBOE calculates three separate volatility values based on SPX option bid, offer and midpoint prices at each point. We will focus on midpoint data in the following sections.

Table 1 provides descriptive statistics for the monthly returns, the forward variances, and other predictor variables from 01/1992 to 08/2008. The excess return on the S&P 500

[6]It means that the forward implied variance is a linear combination of the spot implied variances, which is also used in Carr and Wu (2009). For example, $FV_{t,(t+\tau,t+2\tau)} = 2VIX^2_{t,2\tau} - VIX^2_{t,\tau}$.

in the next month, xr_{t+1}, is the total return in excess of the risk-free rate, which is given by the three-month T-bill rate. $VIX^2_{t,3m}$, $FV_t^{(2)}$, and $FV_t^{(3)}$ are the three forward variances corresponding to the previous three time intervals, i.e.

$$VIX^2_{t,3m} \equiv FV_{t,(t,t+3m)}, \tag{5}$$

$$FV_t^{(2)} \equiv FV_{t,(t+3m,t+6m)}, \tag{6}$$

$$FV_t^{(3)} \equiv FV_{t,(t+6m,t+9m)}. \tag{7}$$

Since the VIX term structure values reported by the CBOE are annualized volatility in percent, we divide by 12 to obtain monthly quantity, which follows Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2011). The $(e/p)_t$ denotes the earnings-price ratio $\log(E/P)$, the $yslope_t$ denotes the slope of yield curve, defined as the difference between the ten-year and three-month Treasury yields, and the $credit_t$ denotes credit spread, defined as Moody's BAA bond yield minus AAA bond yield.[7] In the following regressions, $(e/p)_t$ and $credit_t$ are scaled by 12 to obtain monthly values as in Bollerslev, Tauchen, and Zhou (2009).

Some observations are in order. First, for the S&P 500 excess return, the mean over the sample equals 0.457% monthly; the skewness is negative and the kurtosis is larger than 3, which are consistent with well documented pattern. Second, the average forward variances term structure is concave, rises from 33.111 to 34.302 and then decreases to 31.754; the standard deviations are large, the skewness is positive and much larger than 0, and the kurtosis is extremely high, which indicate that forward variances are significantly different from normality. Note that the variations of forward variances term structure are downward sloping as maturity increases. More importantly, all the AR(1) coefficients related to forward variances are relatively small compared with traditional predictors. Third, the

[7] We acknowledge Amit Goyal for making these data available at his personal website: http://www.bus.emory.edu/agoyal/Research.html

excess return is positively correlated with 3- and 6-month forward variances and negatively correlated with 9-month forward variance. Further, the augmented Dickey-Fuller (ADF) unit root test shows that ADF t-statistic values (lag 6) of the three forward variances are -3.68, -3.77, and -6.02, respectively. Note that the critical value for significance level 1% is -3.46, which means the three forward variances are stationary.

2.2 Predicting stock market excess returns

It is documented in the literature that implied volatility is a powerful predictive variable for realized volatility.[8] In this section, we use predictive regressions to investigate the predictive power of the forward variances for stock market excess returns. First, we consider the following regressions:

$$xr_{t+1} = \alpha + \beta_1 VIX^2_{t,3m} + \beta_2 FV_t^{(2)} + \beta_3 FV_t^{(3)} + \theta_1 \, yslope_t + \theta_2 \, (e/p)_t + \epsilon_{t+1}. \tag{8}$$

Table 2 reports results from the regressions of equation (8) when the forward variances are combined with or without traditional predictors. Among forward variances regressions, the adjusted R^2 (2.26%) from the combination of 6- and 9-month forward variances greatly exceeds that from other combinations, and both slope coefficients for 6-month (β_2) and 9-month (β_3) forward variances are statistically significant at 5% level. Combined with the slope of yield curve and the earnings-price ratio, forward variances substantially increases the adjusted R^2 from 3.21% up to 10.82% when the combination of 6- and 9-month forward variances are considered. More importantly, the magnitudes of 6-month beta (β_2), 9-month beta (β_3) and the adjusted R^2s are comparable with those reported in Bakshi, Panayotov and Skoulakis (2011) whether or not the traditional predictors are included. Note that joint p-value for the regression with combination of 6-month and 9-month forward variances is 0.12, which is similar to the joint p-value 0.18 for the regression with combination of 19-day

[8] See, for example, Christensen and Prabhala (1998) and Jiang and Tian (2005).

and 49-day forward variances in Bakshi, Panayotov and Skoulakis (2011). Nevertheless, the two joint p-values are less than 1% when traditional predictors are included.

There are also some important observations that are different from Bakshi, Panayotov and Skoulakis (2011). First, we find positive slope coefficient for 6-month (β_2) while negative slope coefficient for 9-month (β_3) forward variances. When 3-month forward variance is combined with 9-month forward variance, the short-term (3-month) beta (β_1) is positive while the long-term (9-month) beta (β_3) is negative and both betas are significant at 10% level. However, Bakshi, Panayotov and Skoulakis (2011) find that longer-term (49-day) beta is positive while shorter-term (19-day) beta is negative. This difference can also be seen when traditional predictors are included. Second, while both medium-term beta (β_2) and long-term beta (β_3) are significant here, the shorter-term (19-day) beta is not significant in Bakshi, Panayotov and Skoulakis (2011). Third, these differences can also be seen at both 3- and 6-month horizons forecasting as demonstrated in the following.

It is of interest to investigate whether the predictive ability of the forward variance term structure for 1-month returns remains in predicting longer-horizon returns. Specifically, we consider the following long-horizon regressions:

$$xr_{t+h} = \alpha + \beta_1 VIX^2_{t,3m} + \beta_2 FV^{(2)}_t + \beta_3 FV^{(3)}_t + \theta_1\ yslope_t + \theta_2\ (e/p)_t + \epsilon_{t+h}, \qquad (9)$$

where $h = 3$ and 6 months. Table 3 reports results on 3- and 6-month horizon regressions in Panels A and B, respectively. When focus on the combination of 6- and 9-month forward variances, we find that (i) the forward variances increase R^2s from 8.53% to 20.42% and from 11.56% to 22.26% for 3- and 6-month horizons, respectively, (ii) both slope coefficients for 6-month (β_2) and 9-month (β_3) forward variances are statistically significant at 1% level at both 3- and 6-month horizons even when combined with the slope of yield curve and the earnings-price ratio, (iii) the magnitudes of 6-month beta (β_2), 9-month beta (β_3) and the adjusted R^2s are comparable with those reported in Bakshi, Panayotov and Skoulakis

(2011) at both 3- and 6-month horizons.

An important observation is that a similar tent-shaped pattern of the three forward variances slope coefficients is presented whether or not they are combined with traditional predictors in predicting the three horizon returns in Tables 2 and 3. Note that, a similar tent-shaped pattern is obtained when predict excess returns on different maturity bonds by one- to five-year forward rates in Cochrane and Piazzesi (2005). It may indicate that investors' risk attitude depends on duration and it is consistent in both bond and stock markets which are revealed by forward rates or forward variances, respectively. The beautiful pattern encourages us to predict the three horizon returns by constructing a single forward variance factor. The results are presented in the next section.

2.3 The single forward variance factor

We conduct the following two-step regression:

`Step 1`: Average (across horizons) regressions:

$$\frac{1}{3}\sum_{h} xr_{t+h} = \alpha + \beta_1 VIX^2_{t,3m} + \beta_2 FV^{(2)}_t + \beta_3 FV^{(3)}_t + \bar{\epsilon},\ h = 1, 3, 6. \tag{10}$$

`Step 2`: Three individual-horizon regressions:

$$Restricted : xr_{t+h} = b_h FVF_t + \epsilon_{t+h},\ h = 1, 3, 6, \tag{11}$$

where $FVF_t \equiv \hat{\beta}' FV_t$ is the single forward variance factor, $\hat{\beta} \equiv [\hat{\alpha}\ \ \hat{\beta}_1\ \ \hat{\beta}_2\ \ \hat{\beta}_3]'$ are obtained in the first step and $FV_t \equiv [1\ VIX^2_{t,3m}\ FV^{(2)}_t\ FV^{(3)}_t]'$.[9] The two-step regressions is similar to the procedure to construct the single forward rate factor considered in Cochrane and Piazzesi (2005), but with average across three forecasting horizons. Note that, while the single forward rate factor predicts cross section returns on one- to five-year maturity bonds, the single forward variance factor forecasts stock market returns at different horizons. The

[9]Our results are robust when the single forward variance factor is constructed without constant term.

single factor, FVF_t, is a state variable for time-varying expected stock market returns at all three different horizons.

Table 4 presents the results for the two-step regression. For robustness check, we report both the Newey-West (1987) t-statistics and the Hodrick (1992) t-statistics. The number in brackets, "[ ]", below the parameter estimates are the Newey-West (1987) t-statistics with 18 lags. The number in parentheses, "( )", are the Hodrick (1992) t-statistics. Adjusted R^2 is shown as $\bar{R}^2$. The Durbin-Watson statistic is reported as DW. The p-values for all the slope coefficients are jointly zero are joint p-val. The χ^2 statistics are in the last column. Some observations are in order. First, the β estimates in Panel A are similar to the results reported in previous tables. A tent-shaped pattern of the three forward variances slope coefficients is also observed. Particularly, the slope coefficients of the three forward variances are 3.64, 12.17 and -20.04, respectively. In the average regression, the adjusted R^2 is 5.84% and the joint p-values are less than 5%. Second, the loadings b_h (h =1,3,6) are 0.37, 1.08, and 1.55 for corresponding return horizons, which are significant at 5% level for both the Newey-West and the Hodrick corrections. Note that the loadings increase with forecasting horizons. Third, in the three individual-horizon restricted regressions, the adjusted R^2s are 2.38%, 6.55%, and 5.33% for 1-, 3- and 6-month return horizons, respectively. The adjusted R^2s as a function of the return horizons exhibit an interesting hump shape, which is similar to the findings reported in Bollerslev, Tauchen, and Zhou (2009) in which they forecast expected returns by using variance risk premium. We compare relative performance between the single forward variance factor and the variance risk premium in later section as a robustness check.

Table 5 reports further results on the predictive power of the single forward variance factor by including traditional forecast variables. In particular we consider the regressions:

$$xr_{t+h} = \alpha + \beta\, FVF_t + \theta_1\, yslope_t + \theta_2\, (e/p)_t + \theta_3\, credit_t + \epsilon_{t+h},\ h = 1, 3, 6, \qquad (12)$$

where FVF_t denotes the single forward variance factor, obtained in the Panel A of Table 4. Additional predict variables are the slope of yield curve, $yslope_t$, the earnings-price ratio, $(e/p)_t$, and the credit spread, $credit_t$. From Table 5, we note that the single forward variance factor is statistically and economically significant at 10% level in all cases. All the joint p-values are less than 5%. The R^2s are increasing with forecasting horizons. For example, combined with the three other predictors, the R^2s are 4.82%, 18.94% and 29.80% for 1-, 3- and 6-month return horizons, respectively.

3 Out-of-sample forecast

In this section, we investigate out-of-sample performance of the single forward variance factor by following out-of-sample forecasts adopted by Welch and Goyal (2008) and Rapach, Strauss, and Zhou (2010).

3.1 Forecast procedure

As in Welch and Goyal (2008) and Rapach, Strauss, and Zhou (2010), we divide the total sample of T observations into an in-sample part of the first m observations and an out-of-sample part of the last q observations, and employ a recursive estimation window. In particular, the initial out-of-sample forecast, $\hat{xr}_{m+1}$, is obtained by

$$\hat{xr}_{m+1} = \hat{\alpha}_m + \hat{\beta}_m \, FVF_m, \tag{13}$$

where FVF_m denotes single forward variance factor, which is determined by using slope coefficients at one-month horizon regression[10], and $\hat{\alpha}_m$ and $\hat{\beta}_m$ are the estimates of α and β, respectively, from a standard predictive regression:

$$xr_{t+1} = \alpha + \beta \, FVF_t + \epsilon_{t+1}, \tag{14}$$

[10]Because the single forward variance factor obtained in Table 4 is given by the average regression across 1-, 3- and 6-month, it is not suitable to compare its forecasts with historical average.

with first $m-1$ observations. The next out-of-sample forecast, $\hat{xr}_{m+2}$, is obtained by

$$\hat{xr}_{m+2} = \hat{\alpha}_{m+1} + \hat{\beta}_{m+1}\, FVF_{m+1}, \tag{15}$$

where $\hat{\alpha}_{m+1}$ and $\hat{\beta}_{m+1}$ are generated by using the predictive regression and a expanding estimation window or first m observations. We continue the procedure to obtain q out-of-sample forecasts, $\hat{xr}_{m+k}$, $k = 1, ..., q$.

3.2 Forecast evaluation

A natural benchmark used in Welch and Goyal (2008) and Rapach, Strauss, and Zhou (2010) is the historical average, $\bar{r}_{t+1} = t^{-1}\sum_{j=1}^{t} r_j$. The performance of the single forward variance factor relative to the historical average is measured by the out-of-sample R^2 statistic, R^2_{OS}, proposed by Campbell and Thompson (2008). Specifically, the R^2_{OS} is given by

$$R^2_{OS} = 1 - \frac{\sum_{k=1}^{q}(r_{m+k} - \hat{r}_{m+k})^2}{\sum_{k=1}^{q}(r_{m+k} - \bar{r}_{m+k})^2}, \tag{16}$$

where $\hat{r}_{m+k}$ are forecasts obtained by the single forward variance factor using previous forecast procedure. Because R^2_{OS} statistics measure reduction in mean square prediction error (MSPE) for the predictive regression model compared to the historical average forecasts, positive R^2_{OS} indicates that $\hat{r}_{m+k}$ forecasts outperform $\bar{r}_{m+k}$ forecasts.

To further demonstrate the performance of the single forward variance factor, we also consider eight economic variables from Welch and Goyal (2008). The eight economic variables are the dividend-price ratio (log), (d/p), the dividend yield (log), (d/y), the default return spread, dfr, the inflation, $infl$, the term spread, tms, the Treasury bill rate, tbl, the long-term return, ltr, and the long-term yield, lty. The detailed descriptions of the eight economic variables and monthly data from 01/1992 to 12/2008 are available from Amit Goyal's website. Welch and Goyal (2008) note that many economic variables lose their predictive ability due to significant shocks, such as the Oil Shock. We take this into account and consider two different out-of-sample forecast evaluation periods: a short out-of-sample period over 01/2007-12/2008, which covers recent financial crisis, and a long out-of-sample period over 01/2000-12/2008, which covers the "technology bubble" as well. As stated by Welch and Goyal (2008) and Rapach, Strauss, and Zhou (2010), the multiple out-of-sample periods analysis provides a kind of robustness check to the out-of-sample forecasting results.

Table 6 presents out-of-sample forecasting results. From Panels A and B, we find that the single forward variance factor generates positive R^2_{OS} values, that are 8.88% and 4.63%, for both the short and long out-of-sample periods. Notice that R^2_{OS} statistics for predictive regression models are generally small. However, as argued in Campbell and Thompson (2008), very small positive R^2_{OS}, such as 0.5% for monthly data, may signal an economically significant degree of return predictability. In particular, we report R^2_{OS} in Panel C for the out-of-sample period 01/2000-12/2005, which is also considered in Rapach, Strauss, and Zhou (2010), in order to compare with previous findings. For example, the R^2_{OS} values for the previous eight economic variables are 3.08%, 2.38%, -2.43%, -3.01%, -3.17%, -4.74%, -0.46% and -2.72%, compared with 10.32%, 10.40%, -2.10%, -1.42%, -4.98%, -2.50%, -1.72% and -0.32% that are obtained in Rapach, Strauss, and Zhou (2010). It is important to note that the R^2_{OS} statistics depend on the historical average calculated by using different samples. This explains different magnitude of R^2_{OS} values observed in Rapach, Strauss, and Zhou (2010) and current paper for the identical out-of-sample period. However, it is interesting to find that the relative performance of the eight economic variables to the historical average are the same in the two papers. Note that the R^2_{OS} value for our single forward variance factor is 3.14%. In Panel D, we provide average of the R^2_{OS} values for the out-of-sample periods starting from 01/2000,...,12/2007. It turns out that the single forward variance factor and the inflation have consistent performance over different out-of-sample periods. More importantly, while the inflation underperforms the historical average (the R^2_{OS} value is -1.44%), the single forward variance factor outperforms the historical average (the R^2_{OS} value is 7.17%). Therefore, we provide out-of-sample evidence on predictive ability of forward variances, which complements to in-sample evidence by Bakshi, Panayotov and Skoulakis (2011).

4 Robustness check

In this section, we provide robustness check by considering measurement error and by controlling for variance risk premium.

4.1 Measurement error

As suggested by Cochrane and Piazzesi (2005), we use lagged variables to consider measurement error issue. Particularly, we run the following regressions:

$$xr_{t+h} = \alpha + \beta_1 VIX^2_{t-1,3m} + \beta_2 FV^{(2)}_{t-1} + \beta_3 FV^{(3)}_{t-1} + \theta_1\, yslope_{t-1} + \theta_2\, (e/p)_{t-1} + \epsilon_{t+h}, \quad (17)$$

where $h = 1, 3, 6$. We also investigate lagged single forward variance factor by the regressions:

$$xr_{t+h} = \alpha + \beta\, \mathrm{FVF}_{t-1} + \theta_1\, yslope_{t-1} + \theta_2\, (e/p)_{t-1} + \epsilon_{t+h},\ h = 1, 3, 6, \quad (18)$$

where FVF_{t-1} denotes the lagged single forward variance factor, obtained in the Panel A of Table 4.

Tables 7-9 report the results for predicting excess returns of SPX at 1-, 3- and 6-month horizons by using individual forward variances and the single forward variance factor. We get similar results as before, which indicates that measurement errors do not account for the observed predictability.

4.2 The variance risk premium

In this subsection, we further investigate the performance of the single forward variance factor in predicting expected excess returns by controlling for the variance risk premium, which is documented in Bollerslev, Tauchen, and Zhou (2009), Carr and Wu (2009), Drechsler and Yaron (2011) and Zhou and Zhu (2009).

Tables 10-12 report results for the following regressions at the three forecasting horizons:

$$xr_{t+h} = \alpha + \beta\, FVF_t + \theta\, VRP_t + \epsilon_{t+h},\ \ h = 1, 3, 6, \tag{19}$$

where FVF_t denotes the single forward variance factor, obtained in the Panel A of Table 4, and VRP_t denotes the variance risk premium used in Bollerslev, Tauchen, and Zhou (2009).[11] Specifically, we consider both the pre-crisis sample period (01/1992 to 06/2007) and the full sample period (01/1992 to 08/2009) at the three horizons to examine the effect of huge volatility spikes during the 2007-2008 financial crisis.

The univariate full sample regressions show that the single forward variance factor is statistically and economically significant at 1% level at the three horizons while the variance risk premium is not significant. However, in univariate regressions for the pre-crisis sample, both the single forward variance factor and the variance risk premium are significant at 1% level at the three horizons. Moreover, in multiple regression for the pre-crisis sample, both the single forward variance factor and the variance risk premium are significant at 5% level at 3-month horizon, while the single forward variance factor is significant at 6-month horizon as well.

It is interesting to note that Drechsler and Yaron (2011) find that the lagged variance risk premium perform better than the immediate variance risk premium in forecasting 1-month ahead returns. To compare with the lagged variance risk premium, we consider the following regressions:

$$xr_{t+h} = \alpha + \beta\, FVF_{t-1} + \theta\, VRP_{t-1} + \epsilon_{t+h},\ h = 1, 3, 6,$$

[11] The end-of-month variance risk premium data, which is the difference between the squared VIX and the sum of squared 5-minute log returns of SPX over the month, are obtained from Hao Zhou's personal website. We thank Hao Zhou for making the data publicly available. Note that Drechsler and Yaron (2011) calculate variance risk premium by subtracting from the squared VIX the realized variance forecasts based on the SPX futures.

where FVF_{t-1} denotes the lagged single forward variance factor, obtained in the Panel A of Table 4, VRP_{t-1} is lagged variance risk premium. Tables 13-15 show results for the pre-crisis period (01/1992 to 06/2007) and the full sample period (01/1992 to 08/2009). Generally, we find similar results as we use time-t variables.

5 Conclusion

In this paper, we construct the three forward variances and study their predictive power in forecasting expected stock returns. We find that the forward variances can predict excess stock returns at 1-, 3- and 6-month horizons. Taking advantage of the unique longer-maturity data provided by the CBOE, we observe some important differences from Bakshi, Panayotov and Skoulakis (2011) in which they show that front-end forward variances (with maturities correspond to 19 and 49 days) help to predict stock market returns. First, the betas of medium- (6-month) and long-term (9-month) forward variances are statistically and economically significant whether or not the traditional predictor variables are included. Moreover, the medium-term beta is positive while the long-term beta is negative. Second, the predictive ability of the forward variances remain significant when forecast stock returns at longer horizons. Third, a similar tent-shaped pattern of regression coefficients is observed in forecasting different horizon returns, which may indicate that the aggregate investor's risk aversion depends on time horizons.

Further, we construct a single forward variance factor, a linear combination of the forward variances, and investigate its forecasting ability. It is interesting to note that the single forward variance factor predicts excess stock market returns at 1-, 3- and 6-month horizons, similar to the powerful single forward rate factor in forecasting bond returns with different maturities identified in Cochrane and Piazzesi (2005). To further examine the single factor's ability to forecast excess returns, we also run regressions while controlling for the slope of yield curve, the earnings-price ratio, the credit spread, and the variance risk premium. The single forward variance factor remains significant.

Out-of-sample analysis demonstrates that the single forward variance factor outperforms the historical average and other traditional economic variables for both 01/2007-12/2008 and 01/2000-12/2008 out-of-sample periods. In particular, while economic variables, for example, the default return spread, the Treasury bill rate and the long-term yield, display different performance for the two out-of-sample periods, the single forward variance factor maintains consistent performance. This also adds evidence to predictive power of forward variances established in Bakshi, Panayotov and Skoulakis (2011).

In the Merton (1973) intertemporal capital asset pricing model (ICAPM), state variables that capture the dynamics of changes in future investment opportunity set are able to explain expected returns. The significance of both positive slope coefficient of the 6-month forward variance and negative slope coefficient of the 9-month forward variance indicates that the 6- and 9-month forward variances are potential proxies for future investment opportunities. The tent-shaped pattern of the slope coefficients also has implication for intertemporal asset pricing models.

Recently, Adrian and Rosenberg (2008) document that the expected market returns depends negatively on short-run volatility but positively on long-run volatility, and prices of risk are negative for both volatility components when market volatility is decomposed into short- and long-run components. Moreover, they find that growth and value stocks have positive and negative loadings on short-run volatility, respectively. The model-free short-, medium- and long-term forward variances in this paper are different from the filtered short- and long-run volatilities in Adrian and Rosenberg (2008). However, it will be also interesting to investigate role of the forward variances in forecasting portfolio returns across both the size and book-to-market dimensions in the future.

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Table 1: **Summary Statistics**

This table provides descriptive statistics for the monthly data from 01/1992 to 08/2008 (200 observations). xr_{t+1} denotes the next period excess return on the S&P 500 in excess of the risk free rate. $VIX^2_{t,3m}$, $FV^{(2)}_t$, and $FV^{(3)}_t$ are forward variances within the intervals $[t, t+3m], [t+3m, t+6m]$, and $[t+6m, t+9m]$, respectively. The $(e/p)_t$ denotes the earnings-price ratio $\log(E/P)$, the $yslope_t$ denotes the slope of yield curve, defined as the difference between the ten-year and three-month Treasury yields, and the $credit_t$ denotes credit spread, defined as Moody's BAA bond yield minus AAA bond yield.

	xr_{t+1}	$VIX^2_{t,3m}$	$FV^{(2)}_t$	$FV^{(3)}_t$	$(e/p)_t$	$yslope_t$	$credit_t$
Panel A: Summary statistics							
Mean	0.457	33.111	34.302	31.754	-3.143	1.691	0.838
Std.dev.	3.909	20.964	18.749	17.085	0.256	1.207	0.223
Skewness	-0.612	1.446	1.137	1.486	-0.533	0.080	1.103
Kurtosis	3.935	5.687	3.924	6.041	2.746	1.767	3.564
AR(1)	-0.010	0.755	0.900	0.884	0.984	0.976	0.935
Panel B: Correlation matrix							
xr_{t+1}	1.000						
$VIX^2_{t,3m}$	0.070	1.000					
$FV^{(2)}_t$	0.055	0.917	1.000				
$FV^{(3)}_t$	-0.036	0.749	0.873	1.000			
$(e/p)_t$	0.204	-0.625	-0.632	-0.616	1.000		
$yslope_t$	-0.037	-0.122	-0.173	-0.128	-0.226	1.000	
$credit_t$	-0.149	0.321	0.271	0.303	-0.442	0.231	1.000

Table 2: **Predicting excess returns of S&P 500 index**

This table presents the results for the following regressions:

$$xr_{t+1} = \alpha + \beta_1 VIX^2_{t,3m} + \beta_2 FV^{(2)}_t + \beta_3 FV^{(3)}_t + \theta_1 \, yslope_t + \theta_2 \, (e/p)_t + \epsilon_{t+1}.$$

The sample period is 01/1992 to 08/2008 (200 observations). The numbers in parentheses below the parameter estimates are the Hodrick (1992) t-statistics. Adjusted R^2 is shown as $\bar{R}^2$. The p-values for all the slope coefficients are jointly zero are in the last column.

	α	β_1	β_2	β_3	θ_1	θ_2	$\bar{R}^2(\%)$	Joint p-val
Panel A								
	0.00	2.32	-1.23				-0.46	0.75
	(0.23)	(0.47)	(-0.25)					
	0.01	4.11		-4.59			1.27	0.28
	(1.09)	(1.76)		(-1.67)				
	0.00		7.54	-8.04			2.26	0.12
	(0.84)		(2.67)	(-2.19)				
	0.10				0.03	0.38	3.21	0.02
	(3.13)				(0.14)	(2.78)		
	0.00	0.22	7.27	-7.98			1.76	0.24
	(0.85)	(0.04)	(1.10)	(-2.01)				
Panel B								
	0.21	4.36	3.60		0.43	0.90	10.30	0.00
	(5.12)	(0.95)	(0.73)		(1.83)	(4.97)		
	0.19	7.36		-0.91	0.35	0.83	9.92	0.00
	(4.80)	(3.32)		(-0.36)	(1.43)	(4.50)		
	0.19		11.92	-5.58	0.42	0.82	10.82	0.00
	(4.54)		(4.80)	(-1.72)	(1.79)	(4.36)		
	0.19	2.99	8.34	-4.68	0.42	0.84	10.74	0.01
	(4.81)	(0.62)	(1.31)	(-1.32)	(1.78)	(4.57)		

Table 3: **Predicting excess returns of S&P 500 index: Long-horizon regressions**

This table presents the results for the following long-horizon regressions:

$$xr_{t+h} = \alpha + \beta_1 VIX^2_{t,3m} + \beta_2 FV^{(2)}_t + \beta_3 FV^{(3)}_t + \theta_1\ yslope_t + \theta_2\ (e/p)_t + \epsilon_{t+h},$$

where $h = 3, 6$, and all variables are defined as in Table 1. The sample period is 01/1992 to 08/2008 (200 observations). The number in parentheses below the parameter estimates are the Hodrick (1992) t-statistics. Adjusted R^2 is shown as $\bar{R}^2$. The p-values for all the slope coefficients are jointly zero are in the last column.

	α	β_1	β_2	β_3	θ_1	θ_2	$\bar{R}^2(\%)$	Joint p-val
Panel A: 3m								
	0.02		18.06	-21.94			5.44	0.02
	(2.21)		(3.27)	(-3.39)				
	0.29				0.09	1.05	8.53	0.00
	(5.08)				(0.23)	(4.62)		
	0.52	19.10	-2.00		0.91	2.22	21.93	0.00
	(7.83)	(3.42)	(-0.27)		(2.45)	(7.43)		
	0.49	20.74		-7.01	0.81	2.07	22.98	0.00
	(7.43)	(6.90)		(-1.46)	(2.22)	(6.95)		
	0.46		28.33	-16.01	0.91	1.94	20.42	0.00
	(6.32)		(6.15)	(-2.89)	(2.48)	(5.92)		
	0.50	15.79	9.44	-11.29	0.89	2.09	23.06	0.00
	(7.43)	(3.01)	(1.22)	(-2.24)	(2.43)	(6.94)		
Panel B: 6m								
	0.04		24.39	-33.04			4.98	0.01
	(3.14)		(3.09)	(-3.18)				
	0.52				0.51	1.93	11.56	0.00
	(6.50)				(0.96)	(6.01)		
	0.85	18.77	6.90		1.77	3.63	21.32	0.00
	(9.19)	(2.11)	(0.65)		(3.17)	(8.63)		
	0.81	26.98		-7.29	1.52	3.35	21.60	0.00
	(8.22)	(5.20)		(-0.99)	(2.75)	(7.47)		
	0.78		42.11	-23.17	1.76	3.28	22.26	0.00
	(8.10)		(6.74)	(-2.68)	(3.21)	(7.41)		
	0.81	13.13	26.40	-19.25	1.74	3.41	22.71	0.00
	(8.44)	(1.53)	(2.24)	(-2.24)	(3.16)	(7.76)		

Table 4: **Predicting excess returns of S&P 500 index with the single forward variance factor**

This table presents the results for the following two-step regressions:

`Step 1`: Average (across forecasting horizons) regressions:

$$\bar{xr} = \beta_0 + \beta_1 VIX^2_{t,3m} + \beta_2 FV^{(2)}_t + \beta_3 FV^{(3)}_t + \bar{\epsilon}.$$

where $\bar{xr} = \frac{1}{3}\sum_h xr_{t+h}$, $h = 1, 3, 6$ months, and

`Step 2`: Restricted individual horizon regressions:

$$Restricted: \quad xr_{t+h} = b_h \; FVF_t + \epsilon_{t+h}, \; h = 1, 3, 6,$$

where $FVF_t = \hat{\beta}' FV_t$ is the single forward variance factor, with $\hat{\beta} \equiv [\hat{\beta}_0 \;\; \hat{\beta}_1 \;\; \hat{\beta}_2 \;\; \hat{\beta}_3]'$ obtained in the Step 1 and $FV_t \equiv [1 \; VIX^2_{t,3m} \; FV^{(2)}_t \; FV^{(3)}_t]'$ is forward variance vector. The sample period is 01/1992 to 08/2008 (200 observations). The number in brackets, "[ ]", below the parameter estimates are the Newey-West (1987) t-statistics with 18 lags. The number in parentheses, "( )", are the Hodrick (1992) t-statistics. Adjusted R^2 is shown as $\bar{R}^2$. The Durbin-Watson statistic is reported as DW. The p-values for all the slope coefficients are jointly zero are p-val. The χ^2 statistics are in the last column.

Panel A: Average

	β_0	β_1	β_2	β_3	$\bar{R}^2$(%)	DW	Joint p-val	$\chi^2(3)$
	0.02	3.64	12.17	-20.04	5.84	0.51		
	[1.16]	[0.68]	[1.79]	[-2.63]			[0.01]	[10.61]
	(2.79)	(0.73)	(1.63)	(-3.23)			(0.02)	(12.45)

Panel B: Restricted

h	b_h	$\bar{R}^2$(%)	DW	p-val	$\chi^2(1)$
1	0.37	2.38	2.01		
	[2.40]			[0.02]	[5.74]
	(2.09)			(0.05)	(4.36)
3	1.08	6.55	0.63		
	[2.97]			[0.00]	[8.83]
	(3.44)			(0.01)	(11.81)
6	1.55	5.33	0.28		
	[2.06]			[0.04]	[4.26]
	(3.31)			(0.01)	(10.96)

Table 5: **Predicting excess returns of S&P 500 index with the single forward variance factor and other predictors**

This table presents the results for the following regressions:

$$xr_{t+h} = \alpha + \beta\, FVF_t + \theta_1\, yslope_t + \theta_2\, (e/p)_t + \theta_3\, credit_t + \epsilon_{t+h}.$$

where FVF_t denotes the single forward variance factor, obtained in the Panel A of Table 4. The sample period is 01/1992 to 08/2008 (200 observations). The number in parentheses below the parameter estimates are the Hodrick (1992) t-statistics. Adjusted R^2 is shown as $\bar{R}^2$. The Durbin-Watson statistic is reported as DW. The p-values for all the slope coefficients are jointly zero are reported as p-val. The χ^2 statistics are in the last column.

h	α	β	θ_1	θ_2	θ_3	$\bar{R}^2$(%)	DW	Joint p-val	χ^2
1	0.09	0.33	0.03	0.35		4.95	2.03	0.02	11.60
	(2.81)	(1.73)	(0.15)	(2.60)					
	0.09	0.32	0.06	0.30	-1.17	4.82	2.05	0.03	10.94
	(2.95)	(1.72)	(0.30)	(1.95)	(-0.72)				
3	0.25	0.96	0.09	0.97		13.59	0.63	0.00	33.54
	(4.51)	(2.68)	(0.26)	(4.30)					
	0.23	0.90	0.32	0.60	-8.73	18.94	0.70	0.00	26.07
	(4.10)	(2.70)	(0.91)	(2.37)	(-2.74)				
6	0.47	1.32	0.52	1.81		15.37	0.27	0.00	34.37
	(5.93)	(2.42)	(1.00)	(5.62)					
	0.40	1.18	1.08	0.88	-22.00	29.80	0.35	0.00	52.25
	(5.58)	(2.63)	(2.16)	(2.60)	(-4.25)				

Table 6: **Out-of-sample forecasts**

This table presents the out-of-sample forecasting results, the R^2_{OS} statistics, defined by

$$R^2_{OS} = 1 - \frac{\sum_{k=1}^{q}(r_{m+k} - \hat{r}_{m+k})^2}{\sum_{k=1}^{q}(r_{m+k} - \bar{r}_{m+k})^2},$$

where $\hat{r}_{m+k}$ are forecasts obtained by the single forward variance factor, which is determined by using slope coefficients of forward variances at one-month regression in Table 2, and $\bar{r}_{m+k}$ are forecasts based on the historical average. The results for the short period (01/2007 to 12/2008) and the long sample period (01/2000 to 12/2008) are reported in Panels A and B, respectively. Panel C presents R^2_{OS} for the out-of-sample period 01/2000-12/2005. Panel D provides average R^2_{OS} for the out-of-sample periods starting from 01/2000,..., 01/2007. Additional eight economic variables are the dividend-price ratio (log), (d/p), the dividend yield (log), (d/y), the default return spread, dfr, the inflation, $infl$, the term spread, tms, the Treasury bill rate, tbl, the long-term return, ltr, and the long-term yield, lty, described in Welch and Goyal (2008). The results are presented in percent form for monthly data.

	FVF	(d/p)	(d/y)	dfr	$infl$	tms	tbl	ltr	lty
Panel A: 01/2007-12/2008									
	8.88	-5.68	-5.52	3.91	-1.34	0.14	5.49	-1.52	4.67
Panel B: 01/2000-12/2008									
	4.63	0.03	-0.35	-0.16	-2.29	-1.98	-1.11	-0.85	-0.16
Panel C: 01/2000-12/2005									
	3.14	3.08	2.38	-2.43	-3.01	-3.17	-4.74	-0.46	-2.72
Panel D: Average									
	7.17	-3.43	-3.35	2.42	-1.44	-0.36	2.39	-1.29	2.19

Table 7: **Predicting excess returns of S&P 500 index with lagged forward variances**

This table presents the results for the following regressions:

$$xr_{t+1} = \alpha + \beta_1 VIX^2_{t-1,3m} + \beta_2 FV^{(2)}_{t-1} + \beta_3 FV^{(3)}_{t-1} + \theta_1\, yslope_{t-1} + \theta_2\, (e/p)_{t-1} + \epsilon_{t+1}.$$

The sample period is 01/1992 to 08/2008 (200 observations). The numbers in parentheses below the parameter estimates are the Hodrick (1992) t-statistics. Adjusted R^2 is shown as $\bar{R}^2$. The p-values for all the slope coefficients are jointly zero are in the last column.

	α	β_1	β_2	β_3	θ_1	θ_2	$\bar{R}^2$(%)	Joint p-val
Panel A								
	0.01	4.90	-4.89				0.09	0.61
	(0.85)	(1.03)	(-0.88)					
	0.01	4.53		-5.98			2.22	0.22
	(1.47)	(2.04)		(-1.85)				
	0.01		6.56	-8.10			1.98	0.14
	(1.28)		(2.64)	(-2.52)				
	0.09				0.03	0.34	2.35	0.05
	(2.79)				(0.15)	(2.48)		
	0.01	2.99	2.86	-7.31			1.87	0.27
	(1.31)	(0.63)	(0.48)	(-2.40)				
Panel B								
	0.17	6.58	-1.20		0.29	0.71	6.53	0.02
	(4.18)	(1.47)	(-0.20)		(1.26)	(3.88)		
	0.16	7.07		-3.10	0.25	0.65	7.20	0.02
	(3.87)	(3.36)		(-0.87)	(1.10)	(3.53)		
	0.14		9.77	-6.24	0.29	0.61	6.29	0.02
	(3.31)		(3.73)	(-1.83)	(1.26)	(3.01)		
	0.16	5.21	3.54	-4.70	0.28	0.66	6.95	0.03
	(3.85)	(1.17)	(0.60)	(-1.46)	(1.23)	(3.50)		

Table 8: **Predicting excess returns of S&P 500 index with lagged forward variances: Long-horizon regressions**

This table presents the results for the following long-horizon regressions:

$$xr_{t+h} = \alpha + \beta_2 FV_{t-1}^{(2)} + \beta_3 FV_{t-1}^{(3)} + \theta_1\, yslope_{t-1} + \theta_2\, (e/p)_{t-1} + \epsilon_{t+h},$$

where $h = 3, 6$, and all variables are defined as in Table 1. The sample period is 01/1992 to 08/2008 (200 observations). The number in parentheses below the parameter estimates are the Hodrick (1992) t-statistics. Adjusted R^2 is shown as $\bar{R}^2$. The p-values for all the slope coefficients are jointly zero are in the last column.

	α	β_2	β_3	θ_1	θ_2	$\bar{R}^2$(%)	Joint p-val
Panel A: 3m							
	0.02	14.07	-18.57			3.60	0.05
	(2.78)	(2.86)	(-3.37)				
	0.26			0.18	0.95	6.58	0.00
	(4.42)			(0.46)	(4.05)		
	0.39	22.67	-13.64	0.80	1.62	13.58	0.00
	(4.97)	(4.81)	(-2.76)	(1.92)	(4.56)		
Panel B: 6m							
	0.04	15.35	-23.93			2.43	0.03
	(3.20)	(1.94)	(-2.35)				
	0.47			0.70	1.77	9.48	0.00
	(5.76)			(1.30)	(5.39)		
	0.68	30.97	-15.39	1.68	2.85	15.17	0.00
	(5.92)	(4.55)	(-1.76)	(2.65)	(5.32)		

Table 9: **Predicting excess returns of S&P 500 index with lagged single forward variance factor**

This table presents the results for the following regressions:

$$xr_{t+h} = \alpha + \beta\, FVF_{t-1} + \theta_1\, yslope_{t-1} + \theta_2\, (e/p)_{t-1} + \epsilon_{t+h},\ \ h = 1, 3, 6,$$

where FVF_{t-1} denotes the lagged single forward variance factor, obtained in the Panel A of Table 4. The sample period is 01/1992 to 08/2008 (200 observations). The numbers in parentheses below the parameter estimates are the Hodrick (1992) t-statistics. Adjusted R^2 is shown as $\bar{R}^2$. The p-values for all the slope coefficients are jointly zero are in the last column.

h	α	β	θ_1	θ_2	$\bar{R}^2$(%)	Joint p-val	χ^2
1	-0.00	0.40			2.76	0.05	5.98
	(-0.10)	(2.45)					
	0.08	0.36	0.04	0.31	4.52	0.04	10.28
	(2.46)	(2.13)	(0.16)	(2.30)			
3	0.00	0.91			4.47	0.02	12.05
	(0.07)	(3.47)					
	0.23	0.80	0.18	0.88	9.96	0.00	21.16
	(3.94)	(2.68)	(0.49)	(3.77)			
6	0.01	1.13			2.59	0.02	6.43
	(0.59)	(2.54)					
	0.44	0.93	0.70	1.69	11.11	0.00	27.30
	(5.33)	(1.79)	(1.33)	(5.08)			

Table 10: **Predicting excess returns of S&P 500 index at 1-month horizon with the single forward variance factor and variance risk premium**

This table presents the results for the following regressions:

$$xr_{t+1} = \alpha + \beta \, FVF_t + \theta \, VRP_t + \epsilon_{t+1}.$$

where FVF_t denotes the single forward variance factor, obtained in the Panel A of Table 4, VRP_t is variance risk premium obtained from Hao Zhou's personal website. The number in parentheses below the parameter estimates are the Hodrick (1992) t-statistics. Adjusted R^2 is shown as $\bar{R}^2$. The Durbin-Watson statistic is reported as DW. The p-values for all the slope coefficients are jointly zero are reported as p-val. The χ^2 statistics are in the last column. The results for the pre-crisis period (01/1992 to 06/2007, 186 observations) and the full sample period (01/1992 to 08/2009, 212 observations) are reported in Panels A and B, respectively.

	α	β	θ	$\bar{R}^2$(%)	DW	Joint p-val	χ^2
Panel A: Pre-crisis							
	0.00	0.48		3.24	2.07	0.03	7.28
	(-0.82)	(2.70)					
	0.00		3.94	3.09	1.92	0.06	9.48
	(-0.53)		(3.08)				
	-0.01	0.32	2.49	3.75	1.97	0.08	16.15
	(-1.36)	(1.45)	(1.41)				
Panel B: Full sample							
	0.00	0.48		4.42	1.84	0.01	14.043
	(-0.69)	(3.18)					
	0.00		0.32	0.00	1.77	0.88	0.02
	(0.80)		(0.15)				
	0.00	0.52	-1.12	4.33	1.90	0.04	10.05
	(-0.24)	(2.66)	(-0.54)				

Table 11: **Predicting excess returns of S&P 500 index at 3-month horizon with the single forward variance factor and variance risk premium**

This table presents the results for the following regressions:

$$xr_{t+3} = \alpha + \beta\, FVF_t + \theta\, VRP_t + \epsilon_{t+3}.$$

where FVF_t denotes the single forward variance factor, obtained in the Panel A of Table 4, VRP_t is variance risk premium obtained from Hao Zhou's personal website. The number in parentheses below the parameter estimates are the Hodrick (1992) t-statistics. Adjusted R^2 is shown as $\bar{R}^2$. The Durbin-Watson statistic is reported as DW. The p-values for all the slope coefficients are jointly zero are reported as p-val. The χ^2 statistics are in the last column. The results for the pre-crisis period (01/1992 to 06/2007, 186 observations) and the full sample period (01/1992 to 08/2009, 212 observations) are reported in Panels A and B, respectively.

	α	β	θ	$\bar{R}^2$(%)	DW	Joint p-val	χ^2
Panel A: Pre-crisis							
	0.00	1.12		6.65	0.74	0.03	15.03
	(-0.61)	(3.88)					
	0.00		9.20	6.51	0.62	0.05	16.54
	(0.05)		(4.07)				
	-0.01	0.73	5.90	8.20	0.67	0.08	24.47
	(-1.21)	(2.55)	(2.28)				
Panel B: Full sample							
	0.00	1.08		7.04	0.61	0.01	18.32
	(-0.44)	(4.28)					
	0.00		4.37	1.36	0.48	0.32	0.87
	(0.36)		(0.93)				
	-0.01	1.02	1.56	6.81	0.58	0.02	19.44
	(-0.64)	(3.24)	(0.36)				

Table 12: **Predicting excess returns of S&P 500 index at 6-month horizon with the single forward variance factor and variance risk premium**

This table presents the results for the following regressions:

$$xr_{t+6} = \alpha + \beta\, FVF_t + \theta\, VRP_t + \epsilon_{t+6}.$$

where FVF_t denotes the single forward variance factor, obtained in the Panel A of Table 4, VRP_t is variance risk premium obtained from Hao Zhou's personal website. The number in parentheses below the parameter estimates are the Hodrick (1992) t-statistics. Adjusted R^2 is shown as $\bar{R}^2$. The Durbin-Watson statistic is reported as DW. The p-values for all the slope coefficients are jointly zero are reported as p-val. The χ^2 statistics are in the last column. The results for the pre-crisis period (01/1992 to 06/2007, 186 observations) and the full sample period (01/1992 to 08/2009, 212 observations) are reported in Panels A and B, respectively.

	α	β	θ	$\bar{R}^2$(%)	DW	Joint p-val	χ^2
Panel A: Pre-crisis							
	0.01	1.40		5.17	0.36	0.03	12.57
	(0.79)	(3.55)					
	0.02		10.71	4.27	0.30	0.07	11.03
	(1.84)		(3.32)				
	0.00	1.00	6.23	5.81	0.32	0.10	22.69
	(0.45)	(2.22)	(1.56)				
Panel B: Full sample							
	0.01	1.44		5.21	0.31	0.01	16.58
	(0.57)	(4.07)					
	0.02		4.79	0.47	0.25	0.36	0.79
	(1.26)		(0.89)				
	0.00	1.40	0.92	4.79	0.30	0.02	17.68
	(0.36)	(3.21)	(0.19)				

Table 13: **Predicting excess returns of S&P 500 index at 1-month horizon with the lagged single forward variance factor and variance risk premium**

This table presents the results for the following regressions:

$$xr_{t+1} = \alpha + \beta\, FVF_{t-1} + \theta\, VRP_{t-1} + \epsilon_{t+1}.$$

where FVF_{t-1} denotes the single forward variance factor, obtained in the Panel A of Table 4, VRP_{t-1} is variance risk premium obtained from Hao Zhou's personal website. The number in parentheses below the parameter estimates are the Hodrick (1992) t-statistics. Adjusted R^2 is shown as $\bar{R}^2$. The p-values for all the slope coefficients are jointly zero are reported as p-val. The χ^2 statistics are in the last column. The results for the pre-crisis period (01/1992 to 06/2007, 186 observations) and the full sample period (01/1992 to 08/2009, 212 observations) are reported in Panels A and B, respectively.

	α	β	θ	$\bar{R}^2$(%)	Joint p-val	χ^2
Panel A: Pre-crisis						
	0.00	0.45		2.69	0.04	9.22
	(-0.76)	(3.04)				
	0.00		3.80	2.84	0.08	7.99
	(-0.47)		(2.83)			
	0.00	0.28	2.54	3.22	0.09	12.23
	(-1.20)	(1.42)	(1.38)			
Panel B: Full sample						
	0.00	0.44		3.70	0.01	11.43
	(-0.60)	(3.38)				
	0.00		3.24	2.92	0.05	4.42
	(-0.66)		(2.10)			
	-0.01	0.35	2.29	4.76	0.02	15.31
	(-1.45)	(2.29)	(1.53)			

Table 14: **Predicting excess returns of S&P 500 index at 3-month horizon with the lagged single forward variance factor and variance risk premium**

This table presents the results for the following regressions:

$$xr_{t+3} = \alpha + \beta\, FVF_{t-1} + \theta\, VRP_{t-1} + \epsilon_{t+3}.$$

where FVF_{t-1} denotes the single forward variance factor, obtained in the Panel A of Table 4, VRP_{t-1} is variance risk premium obtained from Hao Zhou's personal website. The number in parentheses below the parameter estimates are the Hodrick (1992) t-statistics. Adjusted R^2 is shown as $\bar{R}^2$. The p-values for all the slope coefficients are jointly zero are reported as p-val. The χ^2 statistics are in the last column. The results for the pre-crisis period (01/1992 to 06/2007, 186 observations) and the full sample period (01/1992 to 08/2009, 212 observations) are reported in Panels A and B, respectively.

	α	β	θ	$\bar{R}^2$(%)	Joint p-val	χ^2
Panel A: Pre-crisis						
	0.00	0.88		3.88	0.07	10.57
	(0.07)	(3.25)				
	0.00		8.18	5.03	0.06	10.55
	(0.39)		(3.25)			
	0.00	0.49	5.99	5.48	0.13	16.02
	(-0.48)	(1.49)	(1.92)			
Panel B: Full sample						
	0.00	0.79		3.49	0.02	13.48
	(0.25)	(3.67)				
	0.01		2.56	0.15	0.52	0.36
	(0.83)		(0.60)			
	0.00	0.77	0.45	3.05	0.07	13.70
	(0.11)	(2.87)	(0.11)			

Table 15: **Predicting excess returns of S&P 500 index at 6-month horizon with the lagged single forward variance factor and variance risk premium**

This table presents the results for the following regressions:

$$xr_{t+6} = \alpha + \beta \, FVF_{t-1} + \theta \, VRP_{t-1} + \epsilon_{t+6}.$$

where FVF_{t-1} denotes the single forward variance factor, obtained in the Panel A of Table 4, VRP_t is variance risk premium obtained from Hao Zhou's personal website. The number in parentheses below the parameter estimates are the Hodrick (1992) t-statistics. Adjusted R^2 is shown as $\bar{R}^2$. The p-values for all the slope coefficients are jointly zero are reported as p-val. The χ^2 statistics are in the last column. The results for the pre-crisis period (01/1992 to 06/2007, 186 observations) and the full sample period (01/1992 to 08/2009, 212 observations) are reported in Panels A and B, respectively.

	α	β	θ	$\bar{R}^2(\%)$	Joint p-val	χ^2
Panel A: Pre-crisis						
	0.02	0.88		1.68	0.11	4.68
	(1.81)	(2.16)				
	0.02		7.69	1.94	0.14	5.36
	(2.58)		(2.32)			
	0.02	0.53	5.30	1.98	0.24	8.05
	(1.59)	(1.14)	(1.27)			
Panel B: Full sample						
	0.01	0.85		1.49	0.04	6.39
	(1.49)	(2.53)				
	0.01		6.13	1.06	0.10	3.32
	(1.26)		(1.82)			
	0.01	0.67	4.29	1.69	0.11	8.77
	(0.70)	(1.96)	(1.32)			