Tail Risk in Technical Trading Rule Returns*

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November 2014

Abstract

We investigate the effects of simple moving average, time series technical trading rules and a mix of the two on the tail risk of stock index returns. A large body of the current literature is focused on the first moment performance of the trading strategies relative to a benchmark. This paper focuses on the effect on the risk profile with special attention to the tail risk of the technical trading rule payoffs. We find for various specifications of the trading strategies that the downside risk measures are substantially lower without a loss of expected returns.

*This version is for submission to the New Zealand Finance Colloquium

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1 Introduction

Active traders often use heuristic rules to make trading decisions, i.e. technical trading rules (TTR). By applying such rules they attempt to outperform the passive Buy & Hold investment strategy. In analyzing the performance of these TTR the academic literature tends to focus on the mean and standard deviation of the returns. The extreme tail events usually are not explicitly analyzed, even though they will carry the biggest punch. Our contribution is to focus on the down-side risk, specifically the tail risk, of the returns generated by simple TTR. We use the returns on the Dow Jones Index (DJI) over the period 1897-2013 to analyze the tail risk of several TTR. The results show that applying these rules will not necessarily increase the expected or risk adjusted return, but tail risk is reduced substantially.

The academic literature is divided on the ability of TTR to outperform the Buy & Hold strategy. The weak form of the efficient market hypothesis (Fama, Fisher, Jensen and Roll, 1969) stipulates that future prices cannot be predicted by analyzing past prices and therefore TTR should not have any predictive power. In spite of the EMH the stock predictability literature is voluminous, e.g. Ang and Bekaert (2007) and Pesaran and Timmermann (1995). This predictability is attributed to time varying risk premia rather than a risk adjusted out performance of the market benchmark Bollerslev and Hodrick (1992). Related to this issue is the out performance of trading strategies relative to the Buy & Hold strategy of the well diversified portfolio. The theory stipulates that given that a TTR outperforms the market by obtaining a higher expected return these higher expected returns are a compensation for exposure to additional risk factors (Fama and French, 1992).

Brock, Lakonishok, and LeBaron (1992) is the seminal paper on TTR and their relative performance to the Buy & Hold strategy. They find that during periods of the buy signals the annual positive returns are 12% as opposed to the -7% during sell signals periods. Neftci (1991) analyzes TTR in the light of the Wiener-Kolmogorov prediction theory. Under the assumption of stationary linear stochastic processes the theory states that Vector Auto Regressive should produce the best stochastic forecasts using the mean-squared-error as the penalty function. He finds that the DJI might be governed by non-linear processes and therefore the tested simple moving average prediction rule utilizes information the Wiener-Kolmogorov prediction theory ignores. Trading strategies are not only employed in equity markets trades. Bajgrowicz and Scaillet (2012) test for the increased efficiency in financial markets and the profitability of TTR. They find that trading strategies stop out performing after 1986. Sweeney (1986) uses the CAPM to test for excess profits in the exchange market for simple filter rules. He finds that there are indeed excess profits which are not explained by the CAPM.

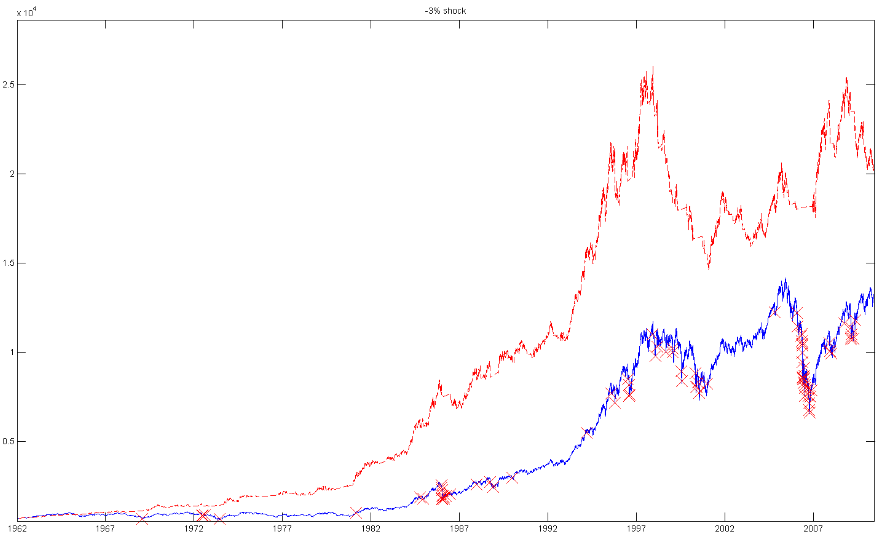
The paper starts with the introduction of the simple TTR and introduces the risk measures for evaluating the performance of the TTR. This is followed by the empirical result section. Here the trading strategies are analyzed for their risk-return trade-off with a special focus on tail risk. We also look at the trading strategies through the economic cycles and the auto-correlation structure of the index returns as suggested by Neftci (1991).

2 Technical trading rule returns

The TTR in this paper are based on simple moving averages of stock index levels, time series models for the return series and a mix of the two. The Moving average M_j, with $j = 1$ or 2, is defined as $M_j = \sum_{i=1}^{S_j} \frac{p_{t+1-i}}{S_j}$, where S_j is the number of observations used, and p_t is the index level at time t. Return at time t is defined as $R_t = \frac{p_t - p_{t-1}}{p_t}$. Here R_t is used to estimate time series models to predict R_{t+1}. We use $AR_{S_2}(1)$ with homeskedastic disturbance terms. Where S_2 are the number of observations used to estimate the time series model. Alternatively, we use the $GARCH_{S_2}(1,1)$ and $EGARCH_{S_2}(1,1)$ model the disturbance terms in the $AR_{S_2}(1)$ model. The TTR entail that all funds are invested in the DJI when the short-term moving average M_1 exceeds the long-term moving average M_2 by $p_t d$. For the time series trading strategy the funds are invested in the DJI when $R_{t+1} > d$. Otherwise the funds are invested in a risk-free investment. In the mixed strategy we combine the two signals from the simple moving average and the time series strategy. When both strategies give off a buy signal the funds are invested in the DJI, otherwise it is invested in the risk-free asset.

Daily closing values of the DJI are obtained from the MeasuringWorth webpage for the period October 7, 1896 until January 1, 2013. The risk-free asset data, measured by the one-month US Treasury bill rate, is obtained from the data library of Kenneth R. French. We follow Brock, Lakonishok, and LeBaron (1992), and use the following combinations to calculate moving averages (S_1, S_2): (1,50), (1,150), (5,150), (1,200), (2,200).

Figure 1: Buy and Hold vs Trading Strategy.



This figure depicts the level of the daily Dow Jones industrial average and the level of the trading strategy. The marks in the graph are the −3% shocks in the daily Dow Jones industrial average. The implemented trading strategy utilizes the difference between the $S_1 = 1$ and $S_2 = 150$ moving average on the level. When the difference becomes negative the investor moves out of the investment object and into a risk free investment. The time series of the daily Dow Jones industrial average are from 1963 till 2013 obtained from the measuring worth webpage.

In Figure 1 the Buy & Hold strategy of the Dow Jones Industrial index is contrasted to the simple moving average trading strategy. The trading strategy clearly outperforms the Buy & Hold strategy. In the recent financial crisis the trading strategy performed extremely well. During this time period a considerable share of the large negative shocks are evaded by investing in the risk-free asset. This type of analysis is very sensitive to the starting point of the measurement period. We therefore analyze the returns structure from various perspectives in the remainder of the paper.

2.1 Risk Measures

We use an Extreme Value Theory (EVT) framework to study the tails of the return distributions. This framework is well-suited to investigate extremely large falls in asset prices. It allows us to determine the Value at Risk (VaR) and Expected Shortfall (ES) semi-parametrically. See Danielsson, Jorgensen, Sarma, and de Vries (2006) for a detailed description of the EVT-methodology. Assuming the tail is regularly varying and under the assumption of self-similarity we are able to use the Pareto distribution to model the tail as the scaled Pareto distribution. We therefore can derive the semi-parametric Value-at-Risk estimator.

$$1 - F(x) = P(X > x) = Ax^{-\alpha} \rightarrow \left(\frac{A}{P(X > x)} \right)^{\frac{1}{\alpha}} = x = VaR$$

The scaling coefficient A is estimated from the empirical distribution and is dependent on the number of observations used for the tail exponent estimate.

$$P(X > x_k) = Ax_k^{-\alpha_k} \rightarrow A = P(X > x_k)\, x_k^{\alpha_k}$$

where k is the number of observations used for the Hill estimator and P is the empirical CDF, which is substituted with k/n.

The Expected Shortfall (ES) is a measure of the expected return, given that a certain threshold is crossed. This threshold is often set at the VaR-level. The ES can alternatively be described by the conditional expectation of the random variable, leading to:

$$E\left(X|X>VaR\right)=\int_{VaR}^{\infty}\frac{xf\left(x\right)}{1-F\left(VaR\right)}dx=\frac{\alpha}{\alpha-1}VaR$$

The above formula shows that once the value of VaR is calculated, the ES can be obtained relatively easily.

We use the CAPM model, the Up and Downside beta framework and the EVT based tail risk factor to measure the tail sensitivity of the TTR strategies to state variables. The linear regression representation of the CAPM is,

$$R_t^{strat}-R^f=\alpha+\beta\left(R_t^m-R^f\right)+\epsilon_t$$

where R_t^f, R_t^{strat} and R_t^m are the risk-free, TTR, market return at time t. This is estimated with data on a monthly frequency. The Up and Downside beta model by Ang, Chen, and Xing (2006) use a non-linear beta model to distinguish between up and downside exposure. The up and downside beta are defined as,

$$\beta^-=\frac{cov\left(R_t^{strat},R_t^m|R_t^m<\mu^m\right)}{var\left(R_t^m|R_t^m<\mu^m\right)}$$

$$\beta^+=\frac{cov\left(R_t^{strat},R_t^m|R_t^m\geq\mu^m\right)}{var\left(R_t^m|R_t^m\geq\mu^m\right)}$$

where μ^m is the expected return on the market index. The μ^m functions as the boundary for the upside and downside risk in the market. We also employ a more extreme version of the up and downside risk exposure. This measure derived from EVT evaluates the dependency in the tail of the distribution between two variables. These measures are stated as a count measure,

$$EVT^{up} = \frac{\sum_{i=1}^{n} I_{\left\{R_t^{strat}, R_t^m > C\right\}}}{\sum_{i=1}^{n} I_{\{R_t^m > C\}}}$$

$$EVT^{down} = \frac{\sum_{i=1}^{n} I_{\left\{R_t^{strat}, R_t^m < -C\right\}}}{\sum_{i=1}^{n} I_{\{R_t^m < -C\}}}$$

here I is the Indicator function and C is the high threshold, which indicates when an observation is extreme. To measure the extreme dependence for the daily observations are used. These three measures of co-movement allow us to look at different perceptive of the risk-return relationship of the trading strategy.

3 Empirical analysis

The payoff structure is viewed from different angles to shed more light sensitivity of the trading strategy to alterations and different economic circumstances. We therefore include analysis over different time periods. We also use the NBER business cycle data to further explore the payoff structure during different economic environment. An additional autocorrelation analysis of the DJI time series investigates the possible source of linear predictability. The analysis is carried out for various parameter choices of the trading strategy for robustness reasons.

3.1 Univariate Risk Measures

The first step towards understanding the tail risk is by analyzing the payoff distribution of the trading strategies. A first glance at the different univariate risk measures presented above will an give initial feel for the tail risk profile.

Table 1: Descriptive statistics of simple moving TTR

	B & H	(1,50)		(1,150)		(5,150)		(2,200)	
d		0	0.1	0	0.1	0	0.1	0	0.1
HP Return	5.16	8.03	8.23	7.53	7.54	6.97	7.10	7.26	7.20
$E(R^m)$	4.35	6.14	6.32	5.51	5.52	4.94	5.06	5.32	5.26
σ^m	18.36	12.28	12.12	11.90	11.82	11.83	11.72	11.86	11.81
Sharp Ratio	0.24	0.50	0.52	0.46	0.47	0.42	0.43	0.45	0.45
Skewness	-0.19	-0.04	0.00	-0.23	-0.24	-0.21	-0.20	-0.38	-0.38
Kurtosis	20.84	20.65	21.04	18.69	18.86	18.46	18.65	16.92	17.09
Max Drawdown	53.78	29.79	26.68	43.84	45.25	33.15	35.28	45.34	45.03
Best year	80.02	42.83	43.36	66.83	66.83	66.16	66.16	66.16	66.16
Worst year	-52.70	-20.30	-20.30	-21.74	-21.99	-18.68	-20.33	-29.79	-29.79
VaR 2.5%	2.04	1.38	1.37	1.39	1.39	1.40	1.40	1.41	1.40
VaR 1%	2.92	1.94	1.93	1.93	1.93	1.96	1.95	1.96	1.95
VaR 0.5%	3.83	2.50	2.50	2.48	2.48	2.52	2.51	2.51	2.50
E S 2.5%	3.35	2.19	2.19	2.17	2.17	2.20	2.20	2.20	2.19
E S 1%	4.80	3.07	3.08	3.01	3.01	3.08	3.07	3.05	3.04
E S 0.5%	6.29	3.97	4.00	3.85	3.87	3.96	3.95	3.91	3.89

This table reports different statistics on performance and risk for the trading strategies. In the first row the different trading strategies are reported. B & H indicates that it is the Buy and Hold strategy. In the other columns the moving average strategies are given. The first and second numbers between brackets are the number of trading days for the short and long moving average, respectively. The second row indicates by which percentage from the price level the moving averages should differ to produce a strategy signal. The data for the trading strategies are the daily Dow Jones industrial average levels from October 7 1896 till January 1 2013 obtained from the measuring worth webpage. From the third row on the various performance measures for the trading strategy are reported.

The results in Table 1 show that over the full sample period, the TTR yield average returns that somewhat exceed the average Buy & Hold return, but with lower standard deviations. As a result the Sharpe Ratios are higher. Skewness and kurtosis levels are comparable across the various rules. The main finding in Table 1 is that the TTR consistently reduce investment risk. All technical trading rules generate substantially lower maximum drawdowns, VaR- and ES-levels than the Buy & Hold rule. Moreover, the VaR- and ES-levels hardly change across the different moving average parameter combinations. Hence, the risk-mitigation is robust to changes in the moving average rules. Extreme tail risk levels are reduced by around one third, e.g. the 1%-VaR for (S_1,S_2) = (1,150) equals 1.9% while for Buy & Hold it is 2.9%, and thus around 34% lower. Following Brock, Lakonishok, and LeBaron (1992), we also use a band-rule to limit transaction costs. Under this rule, a certain percentage difference is needed between the short and long term moving averages M_1 and M_2 to generate a trading signal. These rules show similar reductions in levels of risk, while average returns are affected very little.

Table 6 presents the results for the time series trading strategy. The results show a similar picture in the risk profiles as the moving average TTR. The expected return on the time series strategy is higher than the Buy & Hold and simple moving average strategy. Especially the trading strategy based on the EGARCH model is able to provide a return of 8.89% on average. This comes at the cost of a higher variance, but overall a higher Sharp Ratio. The univariate tail risk measures are also slightly elevated compared to the Simple moving average strategy, but below the B&H strategy. Mixing the two types of trading strategies results in a risk profile similar to that of the moving average trading strategy. This is an indication that the simple moving average TTR is dominant in producing the trading signals.

3.2 Avoiding largest losses

In this section we analyze why simple TTR strongly lowers the tail risk measures. Table 2 reports the percentage of negative returns in the DJI below a certain threshold level that are avoided because of a selling signal (% Out). The moving average (1, 150) is chosen as the benchmark TTR, following Brock, Lakonishok, and LeBaron (1992). Table 2 also reports total numbers of negative returns in excess of a threshold return level (# Shocks).

Table 2: Downward shocks avoided

	1896-1927		1927-1963		1963-1990		1990-2013	
	Perc out	total #	Perc out	total #	Perc out	total #	Perc out	total #
-0.5%	0.528	2112	0.481	2356	0.502	1672	0.424	1481
-1%	0.589	1007	0.574	1215	0.542	685	0.528	750
-1.5%	0.640	470	0.634	666	0.585	265	0.601	386
-2%	0.636	247	0.681	420	0.663	98	0.673	205
-2.5%	0.669	127	0.719	278	0.750	40	0.771	105
-3%	0.634	71	0.731	193	0.700	20	0.797	64
-3.5%	0.682	44	0.736	140	0.692	13	0.907	43
-4%	0.630	27	0.732	97	0.750	8	0.935	31

This table reports the percentage of negative shocks which are avoided due to the trading strategy. The implemented trading strategy utilizes the difference between the $S_1 = 1$ and $S_2 = 150$ moving average on the level. When the difference becomes negative the investor moves out of the investment object and into a risk free investment. The time series of the daily Dow Jones industrial average are from October 7 1896 till January 1 2013 obtained from the measuring worth webpage. The different rows indicate the threshold of the negative shock. The Perc out column indicates the percentage of shocks which are avoided. The Total # column reports the total number of shock observed. The headings of the columns report the non-overlapping time interval the models are estimated over.

Apparently, as a result of applying the TTR, a high percentage of downward shocks in the DJI are avoided. The effectiveness of the rule tends to increase with the absolute size of the negative return. Moreover, the effectiveness does not decrease over time.

3.3 Business cycle analysis

To further investigate the payoff structure of the trading strategy we look at the behavior during economic recessions. We use the NBER business cycle dates to divide the time series in economic recessions and booms.

Table 3: Downward shocks avoided in NBER recessions

	1896-1927		1927-1963		1963-1990		1990-2013	
	Perc out	total #	Perc out	total #	Perc out	total #	Perc out	total #
-0.5%	0.720	743	0.757	820	0.810	336	0.818	253
-1%	0.813	358	0.797	541	0.830	176	0.859	184
-1.5%	0.859	170	0.819	353	0.870	92	0.893	121
-2%	0.888	89	0.839	255	0.902	41	0.943	70
-2.5%	0.909	44	0.836	189	0.889	18	0.978	46
-3%	0.857	21	0.837	141	0.833	6	0.971	34
-3.5%	0.909	11	0.854	103	0.500	2	1.000	25
-4%	1.000	5	0.838	74	NaN	0	1.000	21

This table reports the percentage of negative shocks which are avoided due to the trading strategy in NBER recession periods. The implemented trading strategy utilizes the difference between the $S_1 = 1$ and $S_2 = 150$ moving average on the level. When the difference becomes negative the investor moves out of the investment object and into a risk free investment. The time series of the daily Dow Jones industrial average are from October 7 1896 till January 1 2013 obtained from the measuring worth webpage. The different rows indicate the threshold of the negative shock. The Perc out column indicates the percentage of shocks which are avoided. The Total # column reports the total number of shock observed. The headings of the columns report the non-overlapping time interval the models are estimated over.

From Table 3 it is clear that during economic recession on average a larger percentage of large negative shocks are mitigated. This is uniform over all time periods and shock sizes. When the shocks are extremely negative, e.g. past the -2.5% mark, the trading strategy predicts them more effective. The 1963 till 1990 period is an exception, where the extreme shocks become spares. When contrasting Table 2 and Table 3 it shows that a large portion of the negative shock do not take place in economic recession. This difference is partially attributed to the lead-lag relationship of the stock market and real economy, but there are overlapping periods.[1]

These results point in the direction of using the trading strategy as a hedge. In the light of seeing the trading strategy as a hedge the co-movement with state variables is the way to go to describe the risk profile. The timing of the NBER recessions and the downturn in the stock market do not run parallel. Economic recessions normally lag the stock market recessions and therefore a further investigation is necessary.

3.4 Multivariate return analysis

To further analyze the TTR returns, we estimate the CAPM model, the Upside and Downside beta framework, and the EVT-based tail dependence measure. These measures allow us to study different perspectives of the risk-return relationship of the TTR.

[1]NBER recession periods are determined as a function of real economic activity as opposed to decline of the stock market index

Table 4: Modelling TTR investment returns

	(1, 50)		(1, 150)		(5, 150)		(2, 200)	
d	0	0.1	0	0.1	0	0.1	0	0.1
$CAPM\alpha$	0.042	0.044	0.036	0.036	0.030	0.032	0.035	0.034
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$CAPM\beta$	0.474	0.465	0.452	0.443	0.445	0.438	0.434	0.432
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β_{down}	0.063	0.054	0.144	0.136	0.172	0.153	0.193	0.188
β_{up}	0.822	0.822	0.588	0.574	0.528	0.527	0.454	0.458
EVT β_{down}	0.272	0.256	0.288	0.288	0.304	0.297	0.319	0.319
EVT β_{up}	0.192	0.188	0.214	0.211	0.236	0.236	0.230	0.227

This table reports different risk loadings for different risk factors. The models are further specified in the main text. In the first row the different trading strategies are reported. B & H indicates that it is the Buy and Hold strategy. In the other columns the moving average strategies are given. The first and second numbers between brackets are the number of trading days for the short and long moving average, respectively. The second row indicates by which percentage from the price level the moving averages should differ to produce a strategy signal. For the EVT dependence measures the threshold return level C is set at the 1% empirical quantile. The data for the trading strategies are the daily Dow Jones industrial average levels from October 7 1896 till January 1 2013 obtained from the measuring worth webpage. From the third row on the model parameters are indicated.

Table 4 shows that α^{CAPM} is significantly positive, in line with the findings in Brock, Lakonishok, and LeBaron (1992). We also estimate a Five Factor model, using the three Fama French factors, Momentum and a Liquidity factor. For the Liquidity factor, the data are available only from 1962 onwards. Table 10 show that abnormal excess return become insignificant for the five factor model in the period after 1963. The CAPM abnormal excess returns for this periods have decrease compared to the complete subsample. This might be the source of the insignificant abnormal excess returns. The crux of the findings in Table 4, however, lies in the different ranges found for β_{down} (0.06, 0.19) and β_{up} (0.45 , 0.82). Obviously, the TTR returns are significantly more sensitive to upward movements in the underlying index than to downward movements. This difference in sensitivity may explain part of the reduction in risk reported in Table 1. Most of the upward index returns transfer into equal-sized upward TTR returns. Per contrast, most of the downward index returns are avoided and the sensitivity to negative index returns thus is much lower. The difference between the Up and Downside betas is strongly positive, hence reducing risks while increasing the average return. Finally, our unreported results show that excess returns tend to fall in the last two decades in the data.

Interestingly, the sensitivities of altering TTR to extreme upward and downward movements in the DJI are of rather comparable levels. As an illustration, for $(S_1, S_2) = (1, 150)$, the β_{up}^{tail} is 0.21 while β_{down}^{tail} is 0.29, thus only modestly different. For the other moving average parameter combinations, this finding is comparable. Obviously, tail risk is mitigated not because the TTR reduce the sensitivity to the most extreme downward returns exclusively.

Table 8 shows similar results for the time series TTR. The trading strategy with a threshold for trading costs shows a reduced exposure to the market factor across all risk factors. Except for β_{up} all of the risk measures are higher for the time series strategies. The reduced β_{up} exposure tells us that for market returns above μ^m the trading strategy more often miss predicts a upswing compared to the simple moving average strategy. There is a significant high positive abnormal return when for the Five factor model. This is in contrast to the simple moving average TTR results. The mixed strategy shows again similar results compared to the simple moving average strategy.[2]

[2]The Fama-French factors and the liquidity factors have only been included in the analysis after 1963. The results are reported in the Appendix.

3.5 Autocorrelations

Moskowitz, Ooi, and Pedersen (2012) convincingly show that most asset prices tend to exhibit time series momentum, or trending, over a one-year period. The moving average TTR may be capitalizing on such return dynamics. Hence, intuitively, one would expect autocorrelations to reflect the tendency of the DJI returns to trend. However, our unreported results show that autoregression coefficients are unstable over time, in line with the results in Amini, Hudson, and Keasey (2010). Moreover, we find that autocorrelations strongly and asymmetrically depend on historic returns. Table 5 reports estimates of a conditional first-order autocorrelation model of DJI-returns over the period 1986-2013. It is conditioned on observing a negative index return over a predetermined thres , ranging from -0.5 percent to -4.0 percent. To measure the duration of the autoregressive effects, the dependent variable is measured as the average return across various period lengths.

Table 5: Auto correlation table with sub-sample conditional shock

z	1	2	3	4	5	10	20	30	100
-0.5%	-0.118	-0.112	-0.071	-0.037	-0.019	-0.015	-0.011	-0.009	-0.004
-1%	-0.152	-0.148	-0.098	-0.054	-0.031	-0.022	-0.017	-0.012	-0.006
-1.5%	-0.202	-0.201	-0.134	-0.071	-0.037	-0.027	-0.022	-0.014	-0.004
-2%	-0.194	-0.236	-0.152	-0.072	-0.027	-0.022	-0.021	-0.013	-0.007
-2.5%	-0.208	-0.265	-0.176	-0.091	-0.034	-0.032	-0.029	-0.015	-0.011
-3%	-0.221	-0.336	-0.203	-0.103	-0.038	-0.035	-0.042	-0.024	-0.014
-3.5%	-0.304	-0.411	-0.244	-0.119	-0.042	-0.033	-0.038	-0.017	-0.008
-4%	-0.359	-0.446	-0.248	-0.119	-0.032	-0.034	-0.047	-0.025	-0.013

This table reports various coefficient estimates of AR(1) models where the dependent variable is the mean return after a shock. The different rows indicate the threshold of the negative shock and reports the autoregressive coefficient of an AR(1) model on a subsample conditional on the negative return shock. The headings of the columns report the time interval of the estimated model. The time series of the daily Dow Jones industrial average are from 1896 till 2013 obtained from the measuring worth webpage.

Table 5 shows that conditional autocorrelations are strongly negative, especially for the shortest durations and after the most negative shocks, over the period 1896-2013. Unreported results for the other time periods in the sample are qualitatively similar. We find that the index tends to rebound immediately after a large negative return. This price behavior deviates distinctly from the moving average TTR notion that markets tend to trend. Obviously, the tendency to rebound contributes to a reduction in investment risks. It is possible that the TTR capture some of such hidden patterns in the asymmetric non-linear index return dynamics.

To test the robustness of the outcomes, we run the estimations and tests with the S&P500 index. This index is often invested in by technical traders. Our unreported results are qualitatively similar but somewhat stronger than the ones that are documented above for the DJI series index data. These findings strengthen our earlier conclusions.

4 Conclusions

We document that extreme tail risk in stock returns is strongly reduced through the use of moving average TTR. This mitigation of tail risk does not diminish in recent periods. Part of the tail risk reduction is caused by the finding that many of the largest index-losses are avoided when applying the TTR. In the process, the trading strategy also misses out on the extreme positive shocks.

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A Appendix

A.1 Descriptive statistics

The table below describes how the different descriptive statistics are calculated. The subscript used for the different time notations is related to the frequency of the observations. The d, m, and y indicates the use of daily, monthly and annual observations, respectively.

HP Return $= \left(1 + \frac{p_{1_d} - p_{T_d}}{p_{1_d}}\right)^{\frac{1}{T_y}} - 1$

$E\left(R_m\right) = E\left(\frac{p_{t_m} - p_{t-1_m}}{p_{t-1_m}} - R^f_{t_m} + 1\right)^{12} - 1$

$\sigma_m = \sigma\left(R_{t_m}\right) * \sqrt{12}$

Sharp Ratio $= E\left(R_{t_m}\right) / \sigma_m$

Skewness $= \frac{E\left(R_{t_d} - \mu\right)^3}{\sigma_d^3}$

Kurtosis $= \frac{E\left(R_{t_d} - \mu\right)^4}{\sigma_d^4}$

Max Drawdown$= \frac{\max\limits_t \left[\max\limits_t (p_{1,t}) - p_t\right]}{\max\limits_t (p_{1,t})}$

Best return = max $\left(R_{t_y}\right)$

Worst return = min $\left(R_{t_y}\right)$

A.2 Tables

Table 6: Descriptive statistics of time series TTR 1896-2013

	B & H	$AR(50)$		$AR(150)$		$GARCH(200)$		$EGARCH(200)$	
d		0	0.1	0	0.1	0	0.1	0	0.1
HP Return	5.16	8.97	7.96	9.63	8.03	9.91	8.21	11.14	9.07
$E(R^m)$	4.35	7.03	5.54	7.41	5.48	7.84	5.81	8.98	6.75
σ^m	18.36	12.58	8.91	11.85	8.64	13.12	8.58	13.75	9.81
Sharp Ratio	0.24	0.56	0.62	0.62	0.63	0.60	0.68	0.65	0.69
Skewness	-0.19	-0.23	0.09	-0.29	0.67	-0.44	0.49	-0.05	0.09
Kurtosis	20.84	19.10	41.09	30.94	65.01	25.31	71.52	25.28	65.61
Max Drawdown	53.78	43.67	10.26	24.35	25.40	21.34	19.49	16.12	21.92
Best year	80.02	58.08	52.18	49.55	44.40	62.69	48.72	63.58	65.56
Worst year	-52.70	-27.94	-31.35	-23.49	-25.62	-24.00	-10.93	-33.63	-35.01
VaR 2.5%	2.04	1.47	1.13	1.46	1.08	1.50	1.04	1.63	1.20
VaR 1%	2.92	2.08	1.77	2.09	1.75	2.13	1.70	2.33	1.87
VaR 0.5%	3.83	2.71	2.49	2.75	2.52	2.76	2.47	3.05	2.60
E S 2.5%	3.35	2.37	2.22	2.40	2.28	2.42	2.25	2.67	2.31
E S 1%	4.80	3.35	3.47	3.44	3.68	3.42	3.69	3.81	3.59
E S 0.5%	6.29	4.36	4.86	4.51	5.30	4.44	5.35	4.99	5.01

This table reports different statistics on performance and risk for the trading strategies. In the first row the different trading strategies are reported. B & H indicates that it is the Buy and Hold strategy. In the other columns the time series model for the strategies are given. The number of days to estimated the time series model is stated between brackets. The second row indicates by which percentage from the price level the moving averages should differ to produce a strategy signal. The data for the trading strategies are the daily Dow Jones industrial average levels from October 7 1896 till January 1 2013 obtained from the measuring worth webpage. From the third row on the various performance measures for the trading strategy are reported.

Table 7: Descriptive statistics of Mixed TTR 1896-2013

	B & H	$(1,50)$		$(1,150)$		$(5,150)$		$(2,200)$	
d		0	0.1	0	0.1	0	0.1	0	0.1
HP Return	5.16	7.56	6.98	7.43	7.47	7.17	6.84	7.74	6.63
$E(R^m)$	4.35	5.57	4.94	5.40	5.46	5.14	4.89	5.77	4.79
σ^m	18.36	11.61	11.62	11.72	11.56	11.64	11.66	11.97	12.87
Sharp Ratio	0.24	0.48	0.43	0.46	0.47	0.44	0.42	0.48	0.37
Skewness	-0.19	-0.36	-0.28	-0.33	-0.39	-0.28	-0.36	-0.03	-0.45
Kurtosis	20.84	18.65	18.75	17.40	17.17	17.47	17.01	22.97	29.14
Max Drawdown	53.78	29.30	30.45	44.04	15.62	39.61	16.35	38.97	28.62
Best year	80.02	42.49	42.10	65.74	61.43	65.74	61.43	65.74	63.74
Worst year	-52.70	-21.76	-24.01	-29.02	-23.96	-25.43	-23.96	-20.78	-25.25
VaR 2.5%	2.04	1.37	1.40	1.41	1.40	1.41	1.41	1.42	1.41
VaR 1%	2.92	1.91	1.96	1.96	1.95	1.98	1.96	1.98	1.97
VaR 0.5%	3.83	2.46	2.53	2.52	2.50	2.54	2.52	2.55	2.54
E S 2.5%	3.35	2.15	2.21	2.21	2.19	2.23	2.21	2.23	2.22
E S 1%	4.80	3.00	3.10	3.07	3.04	3.11	3.08	3.12	3.10
E S 0.5%	6.29	3.86	4.00	3.95	3.90	4.00	3.95	4.02	3.99

This table reports different statistics on performance and risk for the trading strategies. In the first row the different trading strategies are reported. B & H indicates that it is the Buy and Hold strategy. The mixed strategies are the combination of the moving average and time series strategies. The AR model is applied for the time series strategy. In the brackets the length of the short-term and long-term moving average is given, respectively. The investor invests in the risk free rate unless there is a buy signal from both strategies. The second row indicates by which percentage from the price level the moving averages should differ to produce a strategy signal. The data for the trading strategies are the daily Dow Jones industrial average levels from October 7 1896 till January 1 2013 obtained from the measuring worth webpage. From the third row on the various performance measures for the trading strategy are reported.

Table 8: Modelling risk of time series TTR 1896-2013

	$AR(50)$		$AR(150)$		$GARCH(200)$		$EGARCH(200)$	
d	0	0.1	0	0.1	0	0.1	0	0.1
$CAPM\alpha$	0.05	0.05	0.06	0.05	0.06	0.05	0.07	0.06
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$CAPM\beta$	0.48	0.25	0.42	0.22	0.53	0.23	0.59	0.28
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
β_{down}	0.31	0.14	0.30	0.16	0.42	0.20	0.45	0.20
β_{up}	0.53	0.35	0.38	0.26	0.53	0.30	0.61	0.35
EVT β_{down}	0.40	0.23	0.41	0.25	0.42	0.22	0.52	0.29
EVT β_{up}	0.31	0.21	0.35	0.26	0.34	0.21	0.48	0.28

This table reports different risk loadings for different risk factors. The models are further specified in the main text. In the first row the different trading strategies are reported. B & H indicates that it is the Buy and Hold strategy. In the other columns the time series model for the strategies are given. The number of days to estimated the time series model is stated between brackets. The second row indicates by which percentage from the price level the moving averages should differ to produce a strategy signal. For the EVT dependence measures the threshold return level C is set at the 1% empirical quantile. The data for the trading strategies are the daily Dow Jones industrial average levels from October 7 1896 till January 1 2013 obtained from the measuring worth webpage. From the third row on the model parameters are indicated.

Table 9: Modelling risk of mixed strategy TTR 1896-2013

	(1, 50)		(1, 150)		(5, 150)		(2, 200)	
d	0	0.1	0	0.1	0	0.1	0	0.1
$CAPM\alpha$	0.04	0.03	0.04	0.04	0.03	0.03	0.04	0.03
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$CAPM\beta$	0.43	0.43	0.44	0.42	0.43	0.42	0.46	0.49
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
β_{down}	0.10	0.15	0.16	0.21	0.18	0.24	0.20	0.36
β_{up}	0.60	0.53	0.48	0.39	0.45	0.37	0.52	0.47
EVT β_{down}	0.28	0.30	0.31	0.33	0.31	0.34	0.33	0.36
EVT β_{up}	0.18	0.24	0.23	0.23	0.24	0.24	0.24	0.24

This table reports different risk loadings for different risk factors. The models are further specified in the main text. In the first row the different trading strategies are reported. B & H indicates that it is the Buy and Hold strategy. The mixed strategies are the combination of the moving average and time series strategies. The AR model is applied for the time series strategy. In the brackets the length of the short-term and long-term moving average is given, respectively. The investor invests in the risk free rate unless there is a buy signal from both strategies. The second row indicates by which percentage from the price level the moving averages should differ to produce a strategy signal. For the EVT dependence measures the threshold return level C is set at the 1% empirical quantile. The data for the trading strategies are the daily Dow Jones industrial average levels from October 7 1896 till January 1 2013 obtained from the measuring worth webpage. From the third row on the model parameters are indicated.

Table 10: Modelling risk of the simple moving average TTR 1963-2013

	(1, 50)		(1, 150)		(5, 150)		(2, 200)	
d	0	0.1	0	0.1	0	0.1	0	0.1
$CAPM\alpha$	0.014	0.016	0.013	0.015	0.006	0.008	0.013	0.013
p-value	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
$CAPM\beta$	0.476	0.464	0.485	0.483	0.492	0.476	0.515	0.506
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$FF\alpha$	0.011	0.013	0.008	0.008	0.002	0.002	0.004	0.007
p-value	0.420	0.331	0.572	0.579	0.900	0.863	0.755	0.625
$FF\beta$	0.403	0.391	0.427	0.428	0.429	0.416	0.460	0.450
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$FFSMB$	-0.073	-0.062	-0.113	-0.111	-0.092	-0.100	-0.113	-0.124
p-value	0.022	0.048	0.000	0.001	0.004	0.002	0.000	0.000
$FFHML$	0.022	0.022	0.042	0.047	0.036	0.028	0.042	0.039
p-value	0.528	0.527	0.240	0.183	0.304	0.434	0.230	0.267
FF MoM	-0.045	-0.039	0.059	0.062	0.068	0.062	0.082	0.074
p-value	0.047	0.078	0.009	0.007	0.003	0.006	0.000	0.001
FF Liq	0.010	0.012	0.027	0.025	0.032	0.025	0.032	0.033
p-value	0.518	0.426	0.089	0.104	0.043	0.116	0.041	0.035
β_{down}	0.095	0.075	0.201	0.202	0.265	0.218	0.318	0.301
β_{up}	0.808	0.792	0.577	0.579	0.512	0.518	0.525	0.519
EVT β_{down}	0.262	0.238	0.238	0.238	0.254	0.246	0.254	0.254
EVT β_{up}	0.167	0.167	0.127	0.127	0.167	0.159	0.135	0.127

This table reports different risk loadings for different risk factors. The models are further specified in the main text. In the first row the different trading strategies are reported. B & H indicates that it is the Buy and Hold strategy. In the other columns the moving average strategies are given. The first and second numbers between brackets are the number of trading days for the short and long moving average, respectively. The second row indicates by which percentage from the price level the moving averages should differ to produce a strategy signal. For the EVT dependence measures the threshold return level C is set at the 1% empirical quantile. The data for the trading strategies are the daily Dow Jones industrial average levels from October 7 1896 till January 1 2013 obtained from the measuring worth webpage. From the third row on the model parameters are indicated.

Table 11: Modelling risk of time series TTR 1963-2013

	$AR(50)$		$AR(150)$		$GARCH(200)$		$EGARCH(200)$	
d	0	0.1	0	0.1	0	0.1	0	0.1
$CAPM\alpha$	0.05	0.04	0.05	0.05	0.06	0.05	0.09	0.06
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$CAPM\beta$	0.52	0.25	0.45	0.20	0.58	0.22	0.58	0.24
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$FF\alpha$	0.05	0.03	0.04	0.03	0.05	0.04	0.06	0.05
p-value	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00
$FF\beta$	0.46	0.23	0.40	0.20	0.53	0.21	0.53	0.22
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$FFSMB$	-0.16	-0.04	-0.09	-0.06	-0.13	-0.04	-0.15	-0.06
p-value	0.00	0.10	0.01	0.03	0.00	0.14	0.00	0.04
$FFHML$	0.06	0.08	0.06	0.06	0.09	0.08	0.03	0.03
p-value	0.09	0.01	0.13	0.03	0.02	0.01	0.36	0.36
FF MoM	-0.04	-0.03	0.00	-0.01	-0.01	-0.02	-0.03	-0.06
p-value	0.14	0.10	0.87	0.77	0.74	0.28	0.19	0.01
FF Liq	0.01	-0.02	0.00	-0.02	0.00	-0.01	-0.04	-0.03
p-value	0.75	0.19	0.91	0.19	0.80	0.37	0.02	0.06
β_{down}	0.35	0.15	0.24	0.10	0.44	0.17	0.30	0.06
β_{up}	0.63	0.41	0.56	0.34	0.68	0.34	0.77	0.40
EVT β_{down}	0.33	0.18	0.37	0.17	0.40	0.21	0.50	0.28
EVT β_{up}	0.34	0.20	0.40	0.26	0.42	0.20	0.63	0.33

This table reports different risk loadings for different risk factors. The models are further specified in the main text. In the first row the different trading strategies are reported. B & H indicates that it is the Buy and Hold strategy. In the other columns the time series model for the strategies are given. The number of days to estimated the time series model is stated between brackets. The second row indicates by which percentage from the price level the moving averages should differ to produce a strategy signal. For the EVT dependence measures the threshold return level C is set at the 1% empirical quantile. The data for the trading strategies are the daily Dow Jones industrial average levels from October 7 1896 till January 1 2013 obtained from the measuring worth webpage. From the third row on the model parameters are indicated.

Table 12: Modelling risk of Mixed TTR 1963-2013

	$(1,50)$		$(1,150)$		$(5,150)$		$(2,200)$	
d	0	0.1	0	0.1	0	0.1	0	0.1
$CAPM\alpha$	0.02	0.02	0.02	0.03	0.01	0.02	0.02	0.02
p-value	0.04	0.10	0.04	0.01	0.29	0.04	0.11	0.03
$CAPM\beta$	0.49	0.49	0.49	0.49	0.49	0.50	0.51	0.52
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$FF\alpha$	0.01	0.00	0.01	0.02	0.00	0.01	0.01	0.00
p-value	0.28	0.89	0.35	0.14	0.72	0.39	0.64	0.73
$FF\beta$	0.41	0.43	0.43	0.44	0.43	0.45	0.46	0.48
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$FFSMB$	-0.08	-0.04	-0.11	-0.10	-0.10	-0.10	-0.11	-0.12
p-value	0.01	0.20	0.00	0.00	0.00	0.00	0.00	0.00
$FFHML$	0.01	0.08	0.02	0.09	0.04	0.09	0.04	0.08
p-value	0.72	0.02	0.52	0.01	0.23	0.01	0.28	0.02
FF MoM	-0.05	-0.00	0.05	0.06	0.08	0.09	0.10	0.11
p-value	0.02	0.97	0.03	0.01	0.00	0.00	0.00	0.00
FF Liq	-0.00	0.01	0.02	0.03	0.04	0.04	0.03	0.02
p-value	0.98	0.73	0.19	0.04	0.02	0.02	0.04	0.14
β_{down}	0.12	0.18	0.20	0.28	0.27	0.33	0.32	0.35
β_{up}	0.74	0.65	0.56	0.51	0.50	0.46	0.50	0.46
EVT β_{down}	0.21	0.28	0.25	0.25	0.25	0.27	0.28	0.28
EVT β_{up}	0.16	0.18	0.17	0.16	0.17	0.17	0.15	0.17

This table reports different risk loadings for different risk factors. The models are further specified in the main text. In the first row the different trading strategies are reported. B & H indicates that it is the Buy and Hold strategy. The mixed strategies are the combination of the moving average and time series strategies. The AR model is applied for the time series strategy. In the brackets the length of the short-term and long-term moving average is given, respectively. The investor invests in the risk free rate unless there is a buy signal from both strategies. The second row indicates by which percentage from the price level the moving averages should differ to produce a strategy signal. For the EVT dependence measures the threshold return level C is set at the 1% empirical quantile. The data for the trading strategies are the daily Dow Jones industrial average levels from October 7 1896 till January 1 2013 obtained from the measuring worth webpage. From the third row on the model parameters are indicated.

Models in table

Row 1: $y_t = \alpha + \gamma_1 y_{t-1} + e_t$

Row 2: $y_t = \alpha + \gamma_1 y_{t-1} + \gamma_2 I_{\{R_{t-1}<-1\%\}} + e_t$

Row 3: $y_t = \alpha + \gamma_1 y_{t-1} + \gamma_2 I_{\{R_{t-1}<-2\%\}} + e_t$

$y_t = \alpha + \gamma_1 y_{t-1} + \gamma_2 I_{\{R_{t-1}<-3\%\}} + e_t$

Row 4: $y_t = \alpha + \gamma_1 y_{t-1} + e_t$, if $R_{t-1} < -1\%$

Row 5: $y_t = \alpha + \gamma_1 y_{t-1} + e_t$, if $R_{t-1} < -2\%$

Row 6: $y_t = \alpha + \gamma_1 y_{t-1} + e_t$, if $R_{t-1} < -3\%$

Table 13: Auto correlation table

	1896-2013		1927-2013		1963-2013		1990-2013	
Model	Coeff	P-val	Coeff	P-val	Coeff	P-val	Coeff	P-val
AR(1)	0.023	0.000	0.027	0.000	0.020	0.026	-0.055	0.000
Dummy 0.01	0.000	0.263	-0.000	0.801	-0.001	0.002	-0.001	0.155
Dummy 0.02	0.003	0.000	0.002	0.000	0.002	0.003	0.001	0.125
Dummy 0.03	0.006	0.000	0.004	0.000	0.005	0.000	0.006	0.000
AR 0.01	-0.152	0.000	-0.115	0.000	-0.226	0.000	-0.275	0.000
AR 0.02	-0.194	0.000	-0.155	0.012	-0.326	0.000	-0.478	0.000
AR 0.03	-0.221	0.033	-0.195	0.085	-0.414	0.010	-0.625	0.009

This table reports various coefficient estimates from time series models over various time periods. The time series of the daily Dow Jones industrial average are from October 7 1896 till January 1 2013 obtained from the measuring worth webpage. The first row depicts the autoregressive coefficient estimate of an AR(1) model. Rows two to four report the coefficient estimates of a dummy for a -0.01, -0.02 and -0.03 return shock in an AR(1) model. Rows five to seven reports the autoregressive coefficient of an AR(1) model on a subsample conditional on a -0.01, -0.02 and -0.03 return shock. For every estimated coefficient the p-value is reported in the column next to it. The headings of the columns report the time interval the models are estimated over.