

# Heterogeneous Beliefs among Retail and Institutional Investors

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## **Abstract**

In this paper, we extend the heterogeneous agent modelling framework by making use of a data set that allows us to uniquely classify trades as either retail or institutional. These data allow us to investigate whether it is indeed the switching in beliefs of traders that causes the unique dynamics documented by heterogeneous agent models, or whether agents have fixed beliefs, but it is the changing proportion of trader types (retail versus institutional) that causes this dynamics. Over our sample period from March 2000 to December 2011, we find strong evidence of heterogeneous beliefs among both trader groups, which actually is stronger than what we observe in the model that does not account for the different trader types. We find that retail traders behave as momentum or positive feedback traders, whereas institutional traders act as contrarian or negative feedback traders.

JEL Codes: C22; G12.

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# 1 Introduction

This paper empirically evaluates a heterogeneous agent model in a market with different trader types. One of the key assumptions in a heterogeneous agent model is that agents have different beliefs and expectations, and can switch between these different expectations based on which belief has performed well in the recent past (Brock and Hommes 1997, 1998, De Grauwe and Grimaldi 2005, Boswijk et al. 2007). The outcome of these empirical heterogeneous agent models is that agents switch between beliefs, and it is this switching that introduces dynamics in the market that cannot be generated under the assumption of a representative agent model.

In this paper, we extend the heterogeneous agent modelling framework by making use of a data set that allows us to uniquely classify trades as either retail or institutional. These data allow us to investigate whether it is indeed the switching in beliefs of traders that causes the unique dynamics documented by heterogeneous agent models, or whether agents have fixed beliefs, but it is the changing proportion of trader types (retail versus institutional) that causes this dynamics. We employ a unique data set of the Finnish market that classifies each trade into retail or institutional, which allows us to compute the daily share holdings of retail and institutional traders for the Finnish market over the period 1 March 2000 to 30 December 2011. We document strong time variation in the holdings of retail/institutional traders. Estimating a heterogeneous agent model along the lines of Boswijk et al. (2007) without recognizing the different trader types shows that there is evidence of heterogeneous beliefs in the Finnish market with traders acting as negative feedback traders. When we introduce the two different trader groups, retail and institutional traders, we find strong evidence of heterogeneous beliefs among both trader groups, which actually is stronger than what we observe in the model that does not account for the different trader types. We find that retail traders behave as momentum or positive feedback traders, whereas institutional traders act as contrarian or negative feedback traders.

Our paper makes important contributions to the literature on adaptive rational expectations. First, we document that the switching behavior across different beliefs observed in previous research is not a consequence of fixed beliefs across different trader types who trade in different proportions, but we find strong evidence for heterogeneous beliefs across different trader types. Second, the result that retail traders are momentum traders and institutional traders are contrarian traders implies that the trade of the two groups offset each other. Indeed, we find much

stronger evidence of switching behavior within the two trader types, then what we observe in the overall market. This suggests that the evidence on switching behavior documented in previous studies probably presents a lower bound on the actual degree of switching that occurs in the market.

The remainder of this paper is structured as follows. In section 2, we cover the literature on heterogeneous agent models and literature that focuses on the stylized facts that we observe among retail and institutional traders. Section 3 derives a heterogeneous agent model that allows for different types of traders with different beliefs. In section 4, we present the data and section 5 reports the results from our model. We conclude in section 6.

## 2 Related Literature

One of the seminal papers in the field of heterogeneous agent models (HAMs) is Brock and Hommes (1997). In this paper, Brock and Hommes (1997) introduce the concept of Adaptive Rational Equilibrium Dynamics, where rational agents can switch between various predictors or beliefs depending on the past realized profitability of such a belief. Brock and Hommes (1997) show that the presence of small costs associated with superior predictors and a high sensitivity towards past profits can lead to cyclical and chaotic market dynamics in a typical cobweb model. Brock and Hommes (1998) apply the notion of heterogeneous beliefs to a simple asset pricing model, where investors are of different but specified belief types. Specifically, they consider the dynamics that can be generated in markets with fundamentalist traders (Brock and Hommes (1998) use this term to refer to traders that have perfect foresight about the fundamental value of the asset), trend-chasers and contrarian traders, and traders with biased expectations. They show that the presence of traders with different beliefs and switching between these beliefs can lead to interesting market dynamics, including markets with bubbles and busts.

Following this initial work by Brock and Hommes (1997, 1998), several studies have extended the adaptive belief systems for asset pricing models. For instance, Chiarella and He (2002) relax the common assumptions of homogeneity in risk aversion and homogeneity in expectations on the expected volatility of the stock market, and demonstrate that this can lead to additional dynamics in the market.

De Grauwe and Grimaldi (2005) use a slightly different model setup compared to e.g. Brock and Hommes (1998), and are probably one of the first to develop an empirically estimable version

of a HAM. They develop a two-type model where investors can either have fundamentalist beliefs or chartist beliefs (i.e. they trade based on feedback rules), and apply this model to explain the dynamics of exchange rates. Boswijk et al. (2007) develop an empirically estimable version of the original Brock and Hommes (1998) framework and use this model to explain the dynamics of the S&P 500 index over an extensive period of time (from 1871-2003), and document the existence of two groups of traders: fundamentalists versus chartists, with substantial switching between the two trader types. They further document that switching between beliefs can explain the price deviations observed between the actual price and the fundamental value.

After the development of empirically estimable HAMs, several studies have applied these types of models in various empirical settings. For instance, Frijns et al. (2010) implement a HAM in the option market, where option traders have heterogeneous expectations about the future evolution of the volatility process and therefore price options differently. The paper shows that allowing for heterogeneity in beliefs significantly improves the pricing of options across a wide range of strike prices and maturities. Frijns et al. (2011) implement a HAM in a GARCH specification, where investors have different beliefs about the evolution of the volatility process. Their paper demonstrates that allowing for heterogeneity in beliefs leads to a very parsimonious GARCH specification with time-varying parameters. Their model is relative easy to implement compared with e.g. a model that introduces time-variation through Markov switching, and performs very well, when compared with these more complicated and less parsimonious specifications.

### 3 Model

To examine the role of heterogenous beliefs among traders in the stock market, we develop a model similar to Boswijk et al. (2007). In this model, agents have heterogeneous expectations with respect to future prices, and form these expectations by extrapolating past price patterns. We extend the model of Boswijk et al. (2007) by allowing for different trader types to be present in the market. Hence we introduce a model with *different trader groups* and within these groups traders can have *heterogeneous beliefs*.

In our market, there are two types of investors, retail and institutional (i.e.  $i = \{R, I\}$ ). Within each group of investors, traders can have different belief formation and we refer to these different beliefs as  $i_h$ , with  $h = \{1, \dots, H\}$ . Investors trade in a market with two assets: a

risk-free asset that is in infinite supply and has a gross rate of return of  $R^f$ , and a risky asset that is in fixed supply and has a gross excess rate of return of  $ER$ . The risky asset has a current price (ex-dividend) of  $P_t$ , and a cash flow (e.g. dividend or earnings), which is stochastic, of  $Y_t$ . We can thus define the excess rate of return as  $ER_t = \frac{(P_{t+1} + Y_{t+1} - R^f P_t)}{P_t}$ . Since investors have heterogeneous beliefs about the future value of the asset, the expected wealth of an investor of type  $i_h$  is given as,

$$E_{i_h,t}[W_{i,t+1}] = R^f W_{i,t} + z_{i_h,t} E_{i_h,t}[ER_{t+1}], \quad (1)$$

where  $E_{i_h,t}$  is the expectation at time  $t$  based on belief  $i_h$ ,  $W_{i,t}$  is wealth at time  $t$  of investor group  $i$ , and  $z_{i_h,t}$  is the dollar demand of the risky asset for investors of type  $i$  with belief  $h$ . Likewise, the variance of wealth is given as

$$Var_{i_h,t}[W_{i,t+1}] = z_{i_h,t}^2 Var_{i_h,t}[R_{t+1}], \quad (2)$$

where  $R_{t+1}$  is the return on the risky asset. We assume that investors are mean-variance optimizers, i.e. they maximize expected utility

$$E_{i_h,t}[U(W_{i,t+1})] = E_{i_h,t}[W_{i,t+1}] - \frac{1}{2} a_i Var_{i_h,t}[W_{i,t+1}], \quad (3)$$

where  $a_i$  is the coefficient of risk aversion of investor type  $i$ . As in Brock and Hommes (1998) we assume that agents have homogeneous expectations with regards to the variance of future wealth and that agents have the same risk aversion across the different beliefs. However, we allow investors of different types to have different degrees of risk aversion.

Maximizing expected utility in Equation (3) provides us with the demand for the risky assets coming from the different beliefs within the different investor groups, i.e.,

$$z_{i_h,t} = \frac{E_{i_h,t}[ER_{t+1}]}{a_i Var_{i_h,t}[R_{t+1}]}, \quad (4)$$

where  $z_{i_h,t}$  is the demand for the risky asset for investor type  $i$  with belief  $h$ . Setting supply equal to demand and normalizing supply to 1, we obtain the pricing equation similar to Boswijk et al. (2007), i.e.,

$$\sum_{i_h} w_{i_h,t} z_{i_h,t} = 1, \quad (5)$$

where  $w_{i_h,t}$  is the proportion of investors of type  $i$  that follow belief  $h$ . Given that we have

two trader types, we can define  $a_i = \{a_R, a_I\}$  as  $a_R = ka$  and  $a_I = k^{-1}a$ . We can thus rewrite Equation (5) as,

$$\sum_h w_{R_h,t} \frac{E_{R_h,t}[ER_{t+1}]}{a_R \sigma^2} + \sum_h w_{I_h,t} \frac{E_{I_h,t}[ER_{t+1}]}{a_I \sigma^2} = 1. \quad (6)$$

Substituting in the risk aversion of the different trader types yields

$$k^{-1} \sum_h w_{R_h,t} E_{R_h,t}[ER_{t+1}] + k \sum_h w_{I_h,t} E_{I_h,t}[ER_{t+1}] = a\sigma^2. \quad (7)$$

We can now enter the expression for expected returns, providing us with the expression

$$k^{-1} \sum_h w_{R_h,t} E_{R_h,t}[P_{t+1} + Y_{t+1} - R^f P_t] + k \sum_h w_{I_h,t} E_{I_h,t}[P_{t+1} + Y_{t+1} - R^f P_t] = P_t a \sigma^2, \quad (8)$$

which we can rearrange to

$$k^{-1} \sum_h w_{R_h,t} R^f P_t + k \sum_h w_{I_h,t} R^f P_t + P_t a \sigma^2 = k^{-1} \sum_h w_{R_h,t} E_{R_h,t}[P_{t+1} + Y_{t+1}] + k \sum_h w_{I_h,t} E_{I_h,t}[P_{t+1} + Y_{t+1}], \quad (9)$$

or

$$k_t R^f P_t + P_t a \sigma^2 = k^{-1} \sum_h w_{R_h,t} E_{R_h,t}[P_{t+1} + Y_{t+1}] + k \sum_h w_{I_h,t} E_{I_h,t}[P_{t+1} + Y_{t+1}], \quad (10)$$

where  $k_t = k^{-1} \sum_h w_{R_h,t} + k \sum_h w_{I_h,t}$ . Multiplying everything by  $k_t^{-1}$ , yields

$$P_t R^f + P_t k_t^{-1} a \sigma^2 = k_t^{-1} \{k^{-1} \sum_h w_{R_h,t} E_{R_h,t}[P_{t+1} + Y_{t+1}] + k \sum_h w_{I_h,t} E_{I_h,t}[P_{t+1} + Y_{t+1}]\}. \quad (11)$$

Let  $R_t = R^f + k_t^{-1} a \sigma^2$ , then we can rewrite this expression as

$$P_t = R_t^{-1} k_t^{-1} \{k^{-1} \sum_h w_{R_h,t} E_{R_h,t}[P_{t+1} + Y_{t+1}] + k \sum_h w_{I_h,t} E_{I_h,t}[P_{t+1} + Y_{t+1}]\}. \quad (12)$$

The present value expression in Equation (12) nests several models. It captures the representative agent model, where  $k = 1$ ,  $H = 1$  and both investor groups have homogeneous expectations. In that case the model reduces to

$$P_t = \frac{E_t[P_{t+1} + Y_{t+1}]}{R}. \quad (13)$$

Equation (12) also nests the asset pricing model with heterogeneous beliefs of Boswijk et al. (2007), where  $k = 1$ ,  $H = 2$  and investors across types form the same beliefs, i.e.,

$$P_t = R^{-1} \sum_h w_{h,t} E_{h,t} [P_{t+1} + Y_{t+1}]. \quad (14)$$

Finally, the model nests a specification where there are two different groups of investors, with homogeneous beliefs within the investor group and different levels of risk aversion, i.e.

$$P_t = R_t^{-1} k_t^{-1} \{k^{-1} w_{R,t} E_{R,t} [P_{t+1} + Y_{t+1}] + k w_{I,t} E_{I,t} [P_{t+1} + Y_{t+1}]\}. \quad (15)$$

If, as in Boswijk et al. (2007), we further assume that the heterogeneity in expectations is only with regards to prices and not to cash flow, and assume that cash flows follow a geometric Brownian motion,<sup>1</sup> we can express Equation (12) in terms of price multiples, i.e.,

$$\delta_t = R_t^{-1} k_t^{-1} (1 + g) \{k^{-1} \sum_h w_{R_h,t} (E_{R_h,t} [\delta_{t+1}] + 1) + k \sum_h w_{I_h,t} (E_{I_h,t} [\delta_{t+1}] + 1)\}, \quad (16)$$

where  $\delta_t = \frac{P_t}{Y_t}$  and  $g$  is the growth rate in cash flows.

As in Boswijk et al. (2007), we wish to define our model in terms of deviations from the fundamental value. Define  $\delta^*$  as the long-run fundamental multiplier observed in the market. This fundamental multiplier can be obtained from the Gordon Growth model, i.e.

$$P_t^* = \delta^* Y_t, \text{ where } \delta^* = \left( \frac{1 + g}{r - g} \right). \quad (17)$$

If we define  $x_t = \delta_t - \delta^*$  as the difference between the observed price multiplier and fundamental price multiplier, then we can express Equation (16) in term of deviations from fundamentals, i.e.,

$$x_t = R_t^{-1} k_t^{-1} (1 + g) \{k^{-1} \sum_h w_{R_h,t} E_{R_h,t} [x_{t+1}] + k \sum_h w_{I_h,t} E_{I_h,t} [x_{t+1}]\}. \quad (18)$$

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<sup>1</sup>As in Boswijk et al. (2007) we assume that the the log of cash flow follows a Gaussian random walk with drift

$$\log Y_{t+1} = \mu + \log Y_t + v_{t+1}.$$

This implies that  $E_t[Y_{t+1}] = (1 + g)Y_t$ , where  $g = e^{\mu + \frac{1}{2}\sigma^2} - 1$ .



### 3.1 Heterogeneous Expectations across Investor Types

At this stage, it is convenient to discuss the different investors types and evaluate the dynamics that can be introduced if different types of traders are present in the market. Expectations of the different trader types can take on different forms. First, traders can have non-zero expectations, i.e.  $E_{i_h,t}[x_{t+1}] = c^i$ , where depending on the sign of  $c^i$  traders of a specific group either consistently over- or underestimate the fundamental value of the risky asset. Such over- or undervaluation reflects heterogeneity with regards to risk aversion of the different trader types. For instance, if we assume that retail investors are more risk averse than institutional investors, we would expect these investors to have different fundamental values (or fundamental price multiples) than what is observed in the market. If we define this fundamental price multiple as  $\delta^{*,R}$ , then these investors would compare the actual multiple to their fundamental price multiple, and if they would act as pure fundamentalists (i.e. expect immediate mean-reversion of any deviation of the price multiple to the fundamental price multiple) their expectation would be  $E_{R_h,t}[x_{t+1}^R] = 0$ , where  $x_{t+1}^R = \delta_t - \delta^{*,R}$ . If we define  $\delta^{*,R} = \delta^* + c^R$ , then  $x_{t+1}^R = \delta_t - \delta^* - c^R$ , and  $E_{R_h,t}[x_{t+1}^R] - E_{R_h,t}[x_{t+1}] = c^R$ .

It is further clear from the definition of the fundamental multipliers that  $c^R$  and  $c^I$  are function of  $k$  and  $a$ , i.e.,

$$c^i = \delta^{*,i} - \delta^* = \left( \frac{1+g}{r^i - g} \right) - \left( \frac{1+g}{r - g} \right), \quad (19)$$

where  $r^i = r^f + ka\sigma^2$ . We can thus use the constant deviations of the two groups from the fundamental value to identify the differences in the degree of risk aversion of the two investor groups.

In addition to having non-zero expectations, traders can have extrapolative expectations, i.e.  $E_{i_h,t}[x_{t+1}] = \phi^{i,h}x_{t-1}$ ,<sup>2</sup> where depending on the magnitude of  $\phi^{i,h}$  traders either expect prices to drift further away from fundamentals (traders with such expectations are often referred to as chartists), when  $|\phi^{i,h}| > 1$  or expect prices to mean-revert to fundamentals (traders with such expectations are often referred to as fundamentalists) when  $|\phi^{i,h}| < 1$ .

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<sup>2</sup>We assume first-order extrapolative dynamics. However, the model can easily be extended to multiple lags.

### 3.2 Adaptive Beliefs

Besides investors belonging to specific investor groups, investors can also change their beliefs about their expectations of future prices. We assume that investors do this on the basis of past profitability of a specific trading strategy and compare performance of the different strategies.

Define the profit of a particular trading strategy,  $\pi_{i_h,t-1}$ , as follows:

$$\pi_{i_h,t-1} = DR_{t-1} z_{i_h,t-2} = DR_{t-1} \frac{E_{i_h,t-2}[DR_{t-1}]}{a_i \text{Var}_t[DR_{t+1}]} \quad (20)$$

In this equation, we define profit as the dollar return in excess of the average return, i.e.

$$DR_{t-1} = P_{t-1} + Y_{t-1} - R^f P_t, \quad (21)$$

which we can rewrite in terms of price multiples, i.e.,

$$DR_{t-1} = (1+g)\{\delta_{t-1} + 1 - m\delta_{t-2}\}Y_{t-2}, \quad (22)$$

where  $m = R^f(1+g)^{-1}$ .

Likewise we have an expression for  $E_{i_h,t-2}[DR_{t-1}]$ ,

$$E_{i_h,t-2}[DR_{t-1}] = (1+g)\{E_{i_h,t-2}[\delta_{t-1}] + 1 - m\delta_{t-2}\}Y_{t-2}. \quad (23)$$

Finally, we can express the variance as

$$\begin{aligned} \sigma_{t-1}^2(DR_{t-1}) &= \sigma_{t-1}^2(P_{t-1} + Y_{t-1} - (1+r)P_t) \\ &= \sigma_{t-1}^2(\delta_{t-1}^*(1+g)Y_{t-2}) \\ &= \eta^2 Y_{t-2}^2. \end{aligned} \quad (24)$$

Substituting all these parts into Equation (20), we obtain,

$$\pi_{i_h,t-1} = \left( \frac{(1+g)^2}{a_i \eta^2} \right) (\delta_{t-1} + 1 - \delta_{t-2})(E_{i_h,t-2}[\delta_{t-1}] + 1 - m\delta_{t-2}), \quad (25)$$

which can be written in terms of deviations from fundamentals as

$$\pi_{i_h,t-1} = \left( \frac{(1+g)^2}{a_i \eta^2} \right) (x_{t-1} - mx_{t-2})(E_{i_h,t-2}[x_{t-1}] - mx_{t-2}). \quad (26)$$

The profit function in Equation (26) is essentially the same as in Boswijk et al. (2007), apart from the fact that retail and institutional investors would have different degrees of risk aversion. However, since the degree of risk aversion is assumed to be constant across the different beliefs, the different degree of risk aversion does not affect the switching between the different beliefs. Given that investors can follow two beliefs, i.e. fundamentalist or chartist, we can also define profitability in terms of differences between the fundamentalist and chartist belief, i.e.,

$$\Delta\pi_{i,t-1} = \left( \frac{(1+g)^2}{a_i\eta^2} \right) (x_{t-1} - mx_{t-2})(\phi_1^i - \phi_2^i)x_{t-3}. \quad (27)$$

Finally, we assume that investors in each group  $i$  compare the past profitability of the different investment beliefs and reallocate their investment based on past relative performance. We assume that this reallocation is based on a discrete choice model of Manski and McFadden (1981) The weights of a specific investor type are then given by

$$\mu_t^i = \frac{1}{1 + \exp\{-\gamma^i(\phi_1^i - \phi_2^i)x_{t-3}(x_{t-1} - kx_{t-2})\}}, \quad (28)$$

where  $\mu_t^i$  is the weight on the fundamentalist strategy for investor type  $i$ , and  $\gamma^i = \gamma^{i,*} \left( \frac{(1+g)^2}{a_i\eta^2} \right)$  is the so-called intensity of choice parameter. This parameter defines the sensitivity of investors to differences in past profitability of the different investment strategies and control the aggressiveness with which investors switch between the different beliefs. For instance, if  $\gamma^i = 0$  then investors are not sensitive to past profitability and do not switch between the different beliefs. if  $\gamma^i = +\infty$ , then investors switch immediately, and invest all their capital according to the belief that had the highest recent relative performance. Finally, the sign of  $\gamma^i$  informs us about how investors interpret past signals. If  $\gamma^i > 0$ , then investor chase the belief that has offered the highest past returns. We refer to such a strategy as a momentum or a positive feedback trading strategy. If  $\gamma^i < 0$ , then investor chase the belief that has performed worst in the recent past. In this case, we refer to such a strategy as a contrarian or a negative feedback trading strategy.

We can then extend Equation (18) into the following

$$x_t = R_t^{-1}k_t^{-1}(1+g)\{k^{-1}n_t^R(c^R + \mu_t^R\phi_1^R x_{t-1} + (1 - \mu_t^R)\phi_2^R x_{t-1}) + k(1 - n_t^R)(c^I + \mu_t^I\phi_1^I x_{t-1} + (1 - \mu_t^I)\phi_2^I x_{t-1})\} + \varepsilon_t, \quad (29)$$

where  $n_t^R$  is the proportion of retail traders in the market.

## 4 Data

In this study, we make use of a unique intraday dataset from Euroclear, which is the clearing house for all stocks traded on the Helsinki Stock Exchange.<sup>3</sup> To trade on this exchange, investors must register with Euroclear and are given a unique account number, even when they trade through multiple brokers. Euroclear provides us with the unique trader's identifier and an indicator identifying the type of trader. The database classifies each trader into one of 37 categories and two main ownership types (either nominee account for foreign traders or individual account for traders domiciled in Finland). We separate investors into two main groups using the information from Euroclear: Institutions (I) consisting of domestic and foreign institutions; and Retail (R) consisting of trades by individuals. We obtain data on trader types for the period 1 March 2000 to 30 December 2011.

From the record of trades, we can compute the holdings (and trading activity) of institutions and individuals on a daily basis, and we do so for all stocks that are traded on the Helsinki Stock Exchange. On average, we have about 130 firms in the sample although this ranges from a minimum of 91 companies to a maximum of 147. We then compute the proportion of the stocks held by institutions and retail traders, and compute an equally-weighted average to represent the weighted average holdings of institutional and individual investors.<sup>4</sup>

INSERT FIGURE 1 HERE

In Figure 1, we plot the daily proportion of stocks owned by institutional traders ( $n_t^I$ ).<sup>5</sup> As can be seen the percentage of stocks held by institutions has decreased substantially following the burst of the dot com bubble, where institutional ownership of Finnish companies initially was about 70%. By 2005, the institutional ownership was about 30%, and from 2005 onwards, the percentage ownership of institutions has been between 30-35%. This decline is related to the decline value of the Nokia stock, which resulted in a lower market capitalization for the whole Finnish stock market and therefore a decline in institutional investment in Finland.

Similar to the proportion of institutional investors, we also construct a market index as the equally-weighted total returns index of the stocks in our sample, and based on the dividends paid by the stocks in the index we construct a price dividend ratio. In Figure 2, we plot the

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<sup>3</sup>This database is formerly known as the Finnish Central Share Depository (FCSD). Grinblatt and Keloharju (2000) provide a detailed description of the database.

<sup>4</sup>We compute an equally-weighted average, as a value-weighted average would almost completely be dominated by Nokia.

<sup>5</sup>The proportion of shares held by retail investors is simply  $n_t^R = 1 - n_t^I$ .

Price-Dividend ratio together with the fundamental ratio.<sup>6</sup> We observe that over time there are large fluctuations, where the ratio deviates from the fundamental ratio. Specifically, during the initial part of the sample, we note that the observed PD-ratio is below the fundamental ratio, suggesting a relative undervaluation of stocks relative to fundamentals. Similarly, we observe substantial deviations from the fundamental PD ratio following the Global Financial Crisis (where the observed PD ratio is lower than the fundamental ratio, again showing a relative undervaluation of the market relative to fundamentals), while we observe a larger PD ratio than the fundamental ratio during the European Debt Crisis (explaining an overvaluation in the market relative to fundamental).

INSERT FIGURE 2 HERE

## 5 Results

In this section, we present the result for the model derived in Section 3. We start our analysis by estimating a simple AR(1) specification, i.e.

$$R(1 + g)^{-1}x_t = c + \phi x_{t-1} + \varepsilon_t. \quad (30)$$

In this specification, we do not allow for switching between trading rules, and do not recognize the existence of different trader types. We estimate this equation: 1. to set a benchmark for the switching model that we estimate subsequently; and 2. to see whether, unconditionally, we observe mean-reversion towards fundamentals in the long-run.

We report the results for this regression in Panel A of Table 1. From these results, we observe that the intercept  $c$  is insignificant, which suggests that overall the market has an unbiased view of the fundamental PD ratio. We also observe that  $\phi$ , the first order autoregressive coefficient, is equal to 0.9887. This is close to the AR(1) coefficient observed by Boswijk et al. (2007), who find a coefficient of 0.97. This coefficient suggests that deviations from fundamental values are mean-reverting, but very persistent.

In Panel B, we report the results where we allow for switching between the fundamentalist

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<sup>6</sup>We estimate this fundamental ratio as per Equation (17). We find that  $\delta^* = 26.89$ , with  $r = 20.45\%$  p.a. and  $g = 16.13\%$  p.a.

and chartist rule. Specifically, we estimate the following specification

$$R(1 + g)^{-1}x_t = c + \mu_t\phi_1x_{t-1} + (1 - \mu_t)\phi_2x_{t-1} + \varepsilon_t, \quad (31)$$

where  $\mu_t$  is defined as

$$\mu_t = \frac{1}{1 + \exp\{-\gamma(\phi_1 - \phi_2)x_{t-3}(x_{t-1} - mx_{t-2})\}}.$$

The results in Panel B show that indeed, we find evidence of two regimes, a fundamentalist regime ( $\phi_1 = 0.9337$ ), where agents believe that the price of the stock market will mean-revert to the fundamental value, and a chartist regime ( $\phi_2 = 1.0464$ ), where agents extrapolate past price trends and believe that the PD ratio's deviation from its fundamental value will continue to grow. Both coefficients are highly significant and further testing reveals that we can reject the null hypothesis  $H_0 : \phi_1 = \phi_2$ . The switching parameter,  $\gamma$ , is negative and significant. Although it is pleasing to see that this coefficient is significant, the statistical significance of this parameter as observed by a t-test is not conclusive. The reason for this is that the switching parameter enters the model nonlinearly. As Teräsvirta (1994) points out, as long as there is sufficient heterogeneity in the regimes, significance of the t-statistic should not be of concern. As a Likelihood Ratio (LR) test confirms, the addition of a second regime significantly improves the fit of the model (we obtain an LR statistic of 179.10, well exceeding the critical value at the 1% level of 9.21). The negative sign of the switching parameter suggests that traders act as negative feedback traders, i.e. they switch away from the rule that has been more profitable in the recent past.

#### INSERT TABLE 1

In Panel C of Table 1, we report the results for an AR(1) specification, where different trader types (R - Retail; I - Institutions) are present in the market. Specifically, we estimate the following equation

$$(1 + g)^{-1}x_t = R_t^{-1}k_t^{-1}\{k^{-1}n_t^R(c^R + \phi^R x_{t-1}) + kn_t^I(c^I + \phi^I x_{t-1})\} + \varepsilon_t, \quad (32)$$

where  $n_t^R$  and  $n_t^I$  are the proportions owned by retail and institutional traders, respectively. The results shown in Panel C show some interesting results. First, when we consider the intercepts

of the model, we observe that neither  $c^R$  nor  $c^I$  are significant. This suggests that, on average, retail and institutional investors have similar expectation of the fundamental PD ratio and therefore also the fundamental value of the stock market. Additional tests, reveal that the difference between  $c^R$  and  $c^I$  is insignificant. As we documented before different views on the fundamentals are due to investors having different degrees of risk aversion. However, tests show that  $k = 1.0004$  is not significantly different from 1.

When we consider the AR(1) coefficients for both trader groups, we note that the coefficient for retail investors is greater than unity, about 1.03, and less than unity for institutional investors at 0.92. The difference in the coefficients of the two groups is statistically significant, indicating that the two groups, on average, have different beliefs on the persistence, or mean reversion, in the PD ratio, where retail traders, on average, tend to be chartists and institutional investors tend to be fundamentalists. Finally, we observe that allowing for two different trader types in our model leads to a significant improvement with respect to the AR(1) specification without switching, the LR statistic of 14.64 easily exceeds the 1% critical value of 9.21.

Finally, in Panel D, we report the results where we allow for switching between beliefs for each type of trader. We estimate the equation

$$x_t = R_t^{-1} k_t^{-1} (1 + g) \{ k^{-1} n_t^R (c^R + \mu_t^R \phi_1^R x_{t-1} + (1 - \mu_t^R) \phi_2^R x_{t-1}) + k(1 - n_t^R) (c^I + \mu_t^I \phi_1^I x_{t-1} + (1 - \mu_t^I) \phi_2^I x_{t-1}) \} + \varepsilon_t, \quad (33)$$

where

$$\mu_t^i = \frac{1}{1 + \exp\{-\gamma^i (\phi_1^i - \phi_2^i) x_{t-3} (x_{t-1} - m x_{t-2})\}}$$

with  $i = \{R, I\}$ .

The results in Panel D show that in terms of the expectations of the different trader group, we still observe the same as in Panel C, i.e. retail and institutional investors, on average, hold the same beliefs about the fundamental values, as explained by a similar degree of risk aversion. If we focus on the group of retail investors, we find evidence of both fundamentalist and chartist beliefs. The difference between these coefficients is highly significant, and more extreme than the difference we observed in Panel B. This shows that within the group of retail traders, we find significant evidence of both chartist and fundamentalist behavior. The switching parameter,  $\gamma^R$  is positive but insignificant according to the t-test. However, as we discussed earlier, t-tests are not suitable, as the switching parameter enters the model non-linearly. An additional test,

where we restrict  $\gamma^R = 0$  shows that the likelihood of that specification drops significantly to -3,232.22. This implies that retail traders act as positive feedback traders or momentum traders, i.e. they switch towards the trading rule that has been most profitable in the recent past.

When we consider the group of institutional traders, we also find strong evidence of both fundamentalist and chartist beliefs with  $\phi_1^I < 1 < \phi_2^I$ . The difference between the coefficients for the two beliefs is again highly significant. Hence, similar to retail investors, institutional investors also engage in switching behavior. The intensity of choice parameter for this group of investors,  $\gamma^I$  is negative. Testing for the significance of  $\gamma^I$  by restricting the parameter to zero leads to a significant decrease in the likelihood of the model (it decreases to -3,266.10). This suggests that institutional traders act as negative feedback traders or contrarian traders.

Finally, when we consider the likelihood function, we observe a huge improvement in the log-likelihood of the model when including trader types and allowing for switching within both groups. The LR-statistic that compares the model with the static AR(1) model is 241.58 and is highly statistically significant. The improvement in the likelihood is also very large when compared with the switching model in Panel B and the trader types model in Panel C (which are both nested in our model), and suggests that allowing for dynamics within each trader group contributes substantially to the fit of the model.

The findings documented so far have several important implications. First, our results demonstrate that switching behavior between different beliefs is not driven by groups of traders with fixed beliefs but trading in different proportions. On the contrary, our results provide strong support for the existence of heterogeneous beliefs among traders and switching between those beliefs. In fact, our findings show that after controlling for different trader groups, the evidence on different beliefs and switching behavior becomes stronger. Second, our results show that while retail traders act as positive feedback traders, institutional investors act as negative feedback traders. Thus, when e.g. institutional traders are increasing their beliefs in a chartist trading strategy, retail traders are decreasing their beliefs in the chartist strategy. This offsetting behavior of the different trader groups causes the overall degree of switching that occurs in the market to decline. This is what explains the large improvement in the likelihood between the switching model with no trader types and the switching model that allows for different types of traders. It also suggests that when studies find little evidence of switching behavior, it does not imply that there are no heterogeneous beliefs among investors, but that the beliefs of different trader types may be offsetting. Finally, our results have some implications



for existing empirical studies on heterogeneous beliefs and switching behavior that are not able to distinguish between different trader groups. Literature to date has documented evidence of both positive and negative feedback trading in markets. We offer an additional explanation to those mixed findings, arguing that the differences could come from which group of traders dominates the market.

In Figure 3, we plot the time series of the market weights. As we can observe, there is substantial switching behavior among the two different trader types in the market. We observe more aggressive switching among institutional investors (middle graph) compared with retail investors (top graph). This is in line with the magnitudes of the switching parameters  $\gamma^R$  and  $\gamma^I$  observed in Panel D of Table 1, which is larger for institutional investors than for retail investors. However, when we consider the overall weight on the fundamental belief (computed as  $\mu_t = n_t^R \mu_t^R + n_t^I \mu_t^I$ ), we observe much less fluctuation. This is due to the fact that retail and institutional traders have different signs on the switching parameters, retail traders being positive feedback traders and institution being negative feedback traders. Thus, and in line with the observation in Panel B of Table 1, we observe less switching behavior in the model where we cannot distinguish between different trader types as their switching behavior offsets each other.

INSERT FIGURE 3 HERE

In Figure 4, we plot the weights on the fundamentalist belief as a function of profit differences (fundamentalist minus chartist), where the top (bottom) graph shows the plot for the retail (institutional) investors. The graph clearly shows the sensitivity of the weights on the fundamentalist strategy as a function of profit differences. Retail investors are less sensitive to profit differences and switch less aggressively. As the positive switching parameter indicates, they switch towards the most profitable strategy acting as negative feedback traders. Institutions act as negative feedback traders, and their switching is much more sensitive to profit differences compared with individual investors.

INSERT FIGURE 4 HERE

The weights that investors put on the chartist/fundamentalist strategy can cause the market to either display destabilizing or bubble/crash behavior or stabilizing behavior. To assess this behavior over time Boswijk et al. (2007) construct a measure which they label market sentiment,

which essentially is the time-varying AR(1) coefficient, where the time variation is driven by the weights investors put on the different strategies. In Figure 5, we plot the market sentiment for retail (red line -  $\phi_t^R = \mu_t^R \phi_1^R + (1 - \mu_t^R) \phi_2^R$ ) and institutional (blue line -  $\phi_t^I = \mu_t^I \phi_1^I + (1 - \mu_t^I) \phi_2^I$ ) investors, and for the market as a whole (green line -  $\phi_t = n_t^R \phi_t^R + n_t^I \phi_t^I$ ). The graph shows several interesting results. First, we observe that the different belief about the degree of mean-reversion towards the fundamental value (the intercepts  $\phi^R$  and  $\phi^I$  reported in Panel C of Table 1) show up as different estimates for market sentiment. Institutional sentiment shows that, on average, there is more mean-reversion of prices to fundamentals compared with the sentiment of retail investors. We also observe the differences in the variation in market sentiment between the two groups. Institutional investors have sentiments that are more explosive than retail investors. When we consider the overall sentiment of the market over the sample period, we observe that this sentiment is less extreme than the sentiment of the two groups separately. This again highlights that the two trader groups offset each other by switching in opposite directions.

INSERT FIGURE 5 HERE

## 6 Conclusion

In this paper, we extend the heterogeneous agent modelling framework by making use of a data set that allows us to uniquely classify trades as either retail or institutional. These data allow us to investigate whether it is indeed the switching in beliefs of traders that causes the unique dynamics documented by heterogeneous agent models, or whether agents have fixed beliefs, but it is the changing proportion of trader types (retail versus institutional) that causes this dynamics. Over our sample period from March 2000 to December 2011, we find strong evidence of heterogeneous beliefs among both trader groups, which actually is stronger than what we observe in the model that does not account for the different trader types. We find that retail traders behave as momentum or positive feedback traders, whereas institutional traders act as contrarian or negative feedback traders. In addition, we observe that both trader groups have different expectations with regards to the fundamental price, with retail traders having lower expectations than institutional traders. We attribute these different expectations to different levels of risk aversion of the two trader types.

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Figure 1: Proportion of Stocks held by Institutional Investors

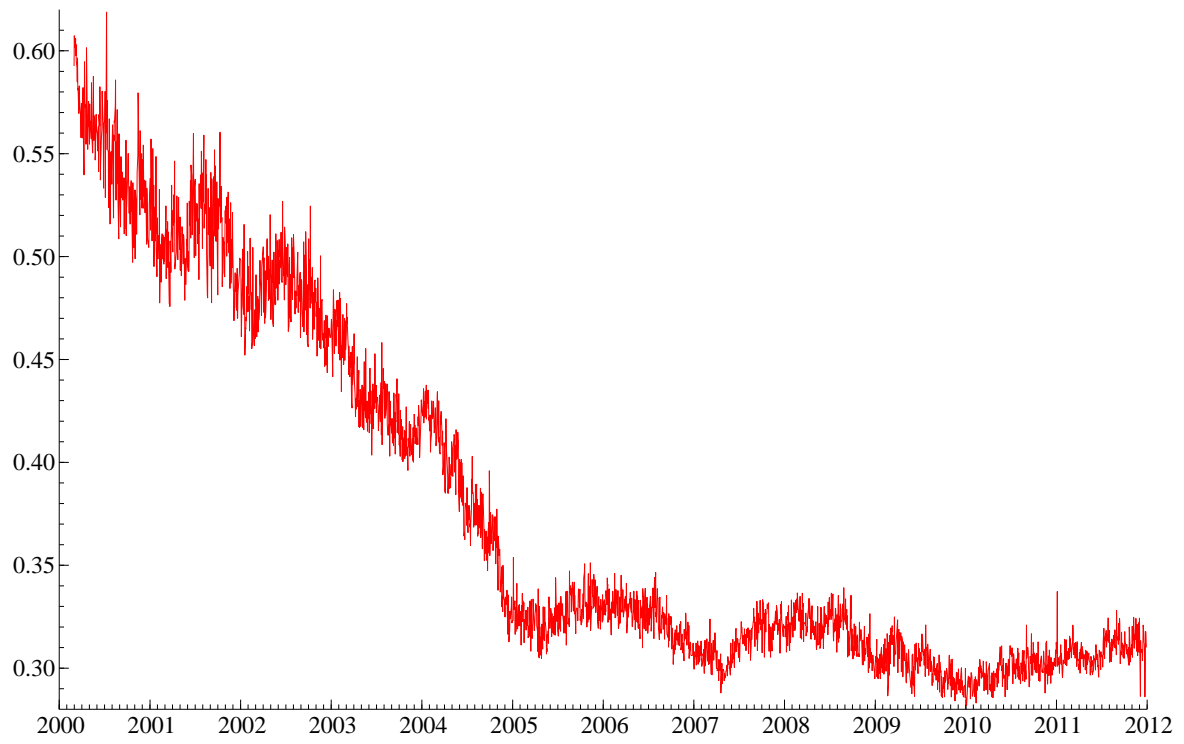


Figure 2: Price-Dividend Ratio versus Fundamental Ratio

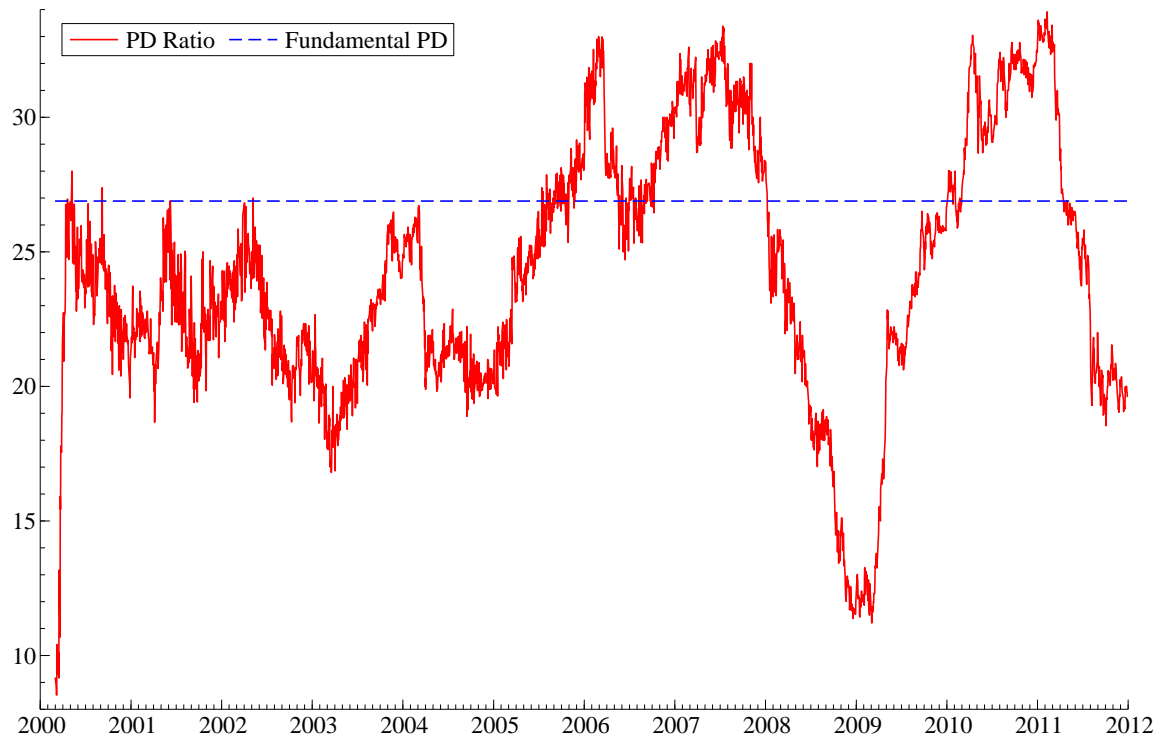


Figure 3: Weights on Fundamentalist Beliefs for the Different Trading Groups

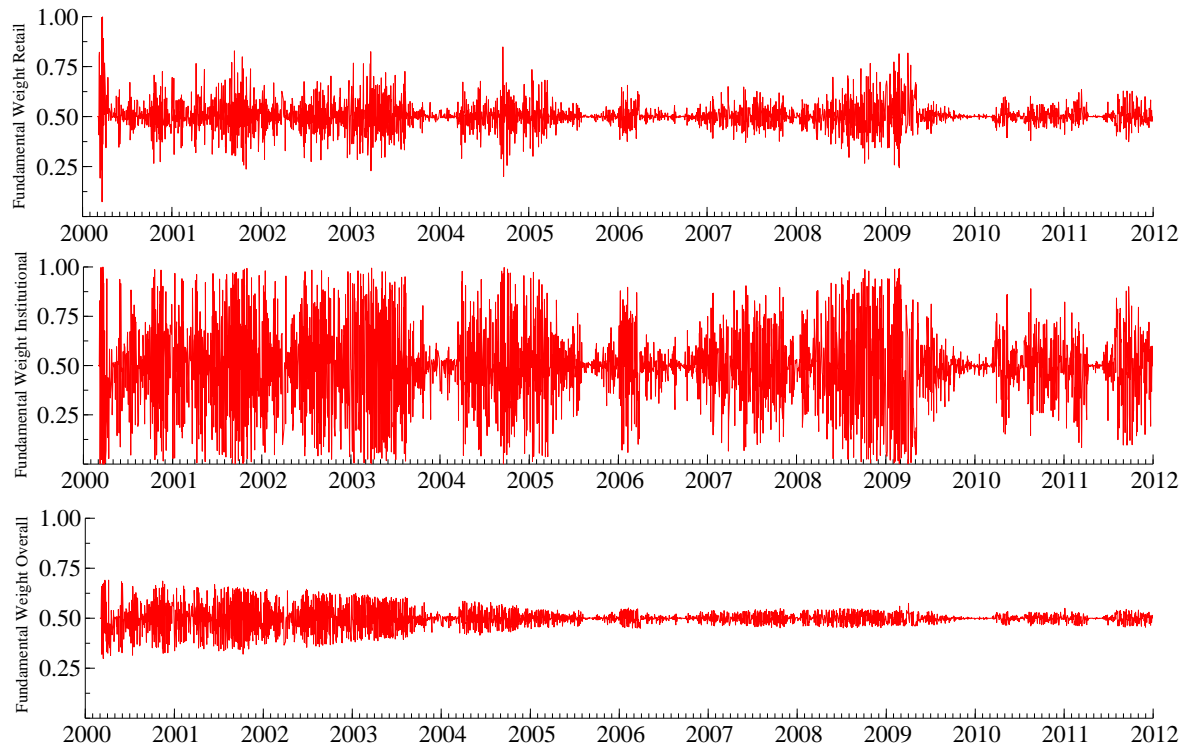


Figure 4: Fundamentalists Weights versus Profit Difference

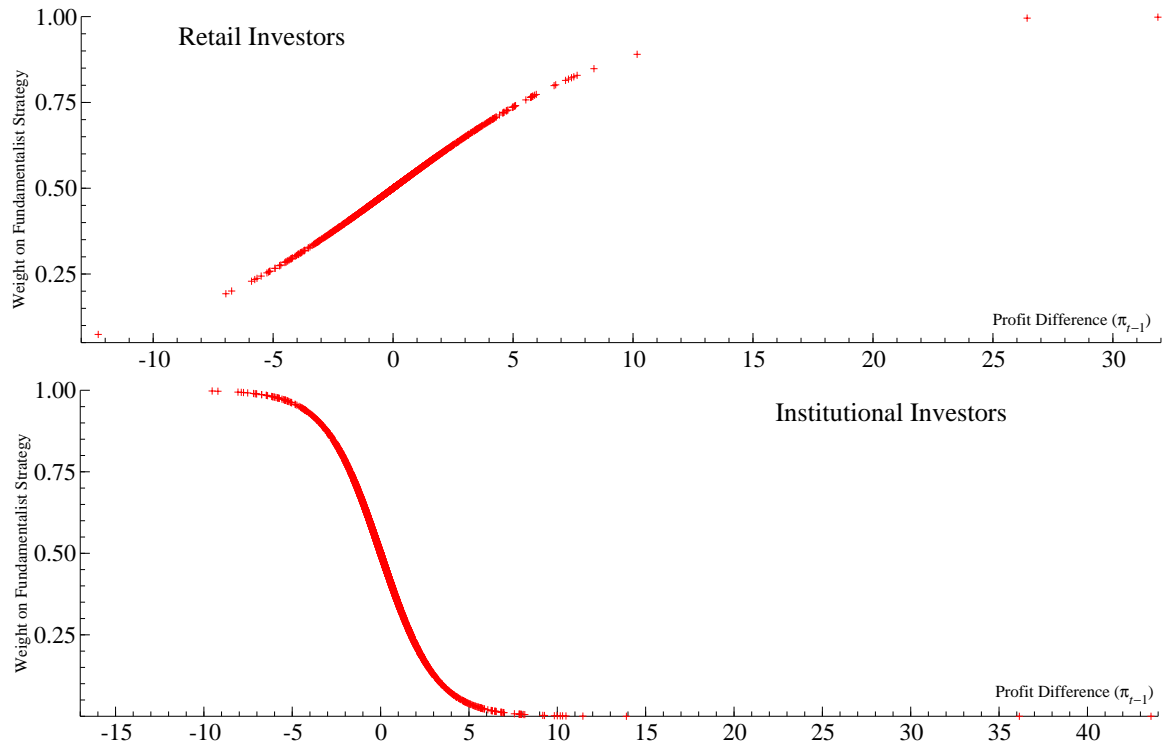


Figure 5: Market Sentiment

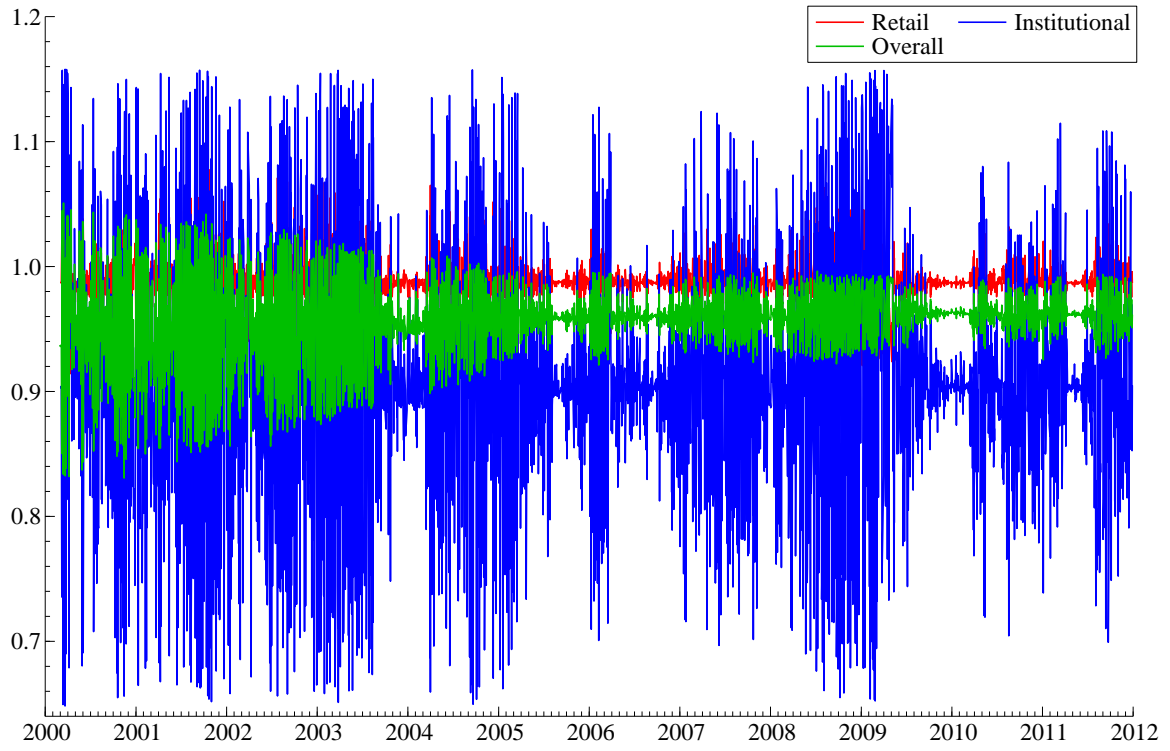




Table 1: Estimation Results based on Price-Dividend Multiples

Panel A: AR(1) specification											
$c$	$\phi$									LL	
-0.0711	0.9887 (365.20)									-3,345.84	
Panel B: Switching - No Trader Types											
$c$	$\phi_1$	$\phi_2$	$\gamma$							LL	LR
-0.0597	0.9337 (76.76)	1.0464 (102.58)	-3.4379 (-2.17)							-3,256.29	179.10
Panel C: Trader Types - No Switching											
$c^R$	$\phi^R$	$c^I$	$\phi^I$	$k$						LL	LR
-0.0426	1.0296 (75.70)	0.0427	0.9200 (34.73)	1.0004 (1.18)						-3,338.52	14.64
Panel D: Switching and Trader Types											
$c^R$	$\phi_1^R$	$\phi_2^R$	$\gamma^R$	$c^I$	$\phi_1^I$	$\phi_2^I$	$\gamma^I$	$k$	LL	LR	
-0.0390	0.8307 (4.95)	1.217 (7.42)	0.2057 (0.62)	0.03912	0.6724 (8.18)	1.2008 (14.49)	-0.6414 (-1.65)	1.0004 (1.03)	-3,225.05	241.58	

Note: This table reports the estimation results for the heterogeneous agent model developed in Section 3. Specifically, we report the results for the following equation:

$$x_t = R_t^{-1} k_t^{-1} (1 + g) \{ k^{-1} n_t^R (c^R + \mu_t^R \phi_1^R x_{t-1} + (1 - \mu_t^R) \phi_2^R x_{t-1}) + k(1 - n_t^R) (c^I + \mu_t^I \phi_1^I x_{t-1} + (1 - \mu_t^I) \phi_2^I x_{t-1}) \} + \varepsilon_t,$$

where the switching is defined by:

$$\mu_t^i = \frac{1}{1 + \exp\{-\gamma^i (\phi_1^i - \phi_2^i) x_{t-3} (x_{t-1} - k x_{t-2})\}}.$$

In Panel A we report the results for the model that does not allow for heterogeneity in beliefs ( $H = 1$ ), and we do not distinguish between trader types. Panel B reports the results for heterogeneity in beliefs ( $H = 2$ ) but does not distinguish between trader types. Panel C does not allow for heterogeneity in beliefs ( $H = 1$ ), but recognizes the existence of retail and institutional traders. Finally, Panel D allows for heterogeneity in beliefs ( $H = 2$ ) and different trader types. The last penultimate column reports the Log-Likelihood of the model ( $LL$ ), while the last column reports the Likelihood ratio ( $LR$ ) statistic of the model relative to the base model in Panel A. For Panels B and C, we add 2 parameters to the base model and thus the LR statistic will be  $\chi^2(2)$ -distributed (the critical value at the 5% and 1% levels are 5.99 and 9.21, respectively). For Panel D, we add 5 parameters to the base model and thus the LR statistic will be  $\chi^2(5)$ -distributed (the critical value at the 5% and 1% levels are 11.07 and 15.09, respectively).