How Does Statutory Redemption Affect a Buyer's Decision to Purchase at the Foreclosure Sale?

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Abstract

Statutory redemption offers the right for a mortgagor who defaulted a loan to reclaim his collateralized property within a certain period of time. This repurchase option benefits the mortgagor, but harms a buyer attending at the foreclosure sale. We derive a closed-form solution of the decision rule for the buyer, which indicates that the buyer will delay purchasing and gain less if (i) the redemption period lasts longer, (ii) the buyer is less capable of operating the property; and (iii) the buyer incurs more transaction costs in purchasing the property. However, we are not sure how uncertainty in housing price inflation affects the buyer's incentive to purchase.

Key Words: American Options, Foreclosure Sale, Real Options, Statutory Redemption

JEL Classification: G13; R52; R58

Introduction

Foreclosure takes place when a mortgagor defaults on a mortgage loan by failing to make the required payments. In the U.S. after foreclosing a mortgage, the mortgagee may or may not sell the collateralized property through a foreclosure sale because the mortgagor has the equitable right of redemption, and thus can prevent the sale by paying off the outstanding debt. Even if the foreclosure sale takes place, the mortgagor still has a statutory right of redemption.That is, during the redemption period the mortgagor can pay the selling price at the foreclosure sale to redeem the property. About half of states in the U.S. provide for statutory redemption, with the period ranging from one month to one and half years (Clauretie and Herzog, 1990).

Conceivably, the repurchase option specified by statutory redemption benefits the mortgagor, but harms the buyer at the foreclosure sale. In this article, we investigate how this option affects a buyer's decision to purchase at the foreclosure sale. The previous literature, however, has not touched this issue.[1]

There are two common types of foreclosure sale, including judicial sale supervised by a court and power-of-sale supervised usually by banks or attorney of the mortgagee. Both of them usually sell a property through auctions, with the buyer who bids the highest price wins. In this article, we focus on interactions between a buyer at a foreclosure sale and the mortgagor rather than between the buyer and the auctioneer at a foreclosure sale. To this end, will build a very simplified model in which there is only one buyer who has the privileged right to buy a property at the foreclosure sale. We model the game played by the mortgagor and the buyer as a hierarchy game and solve it backward.[2] After the foreclosure sale, the mortgagor has the option to choose any date during the redemption period to reclaim the

[1] Clauretie (1987) and Clauretie and Herzog (1990) have identified the benefits and costs associated with this option, but relate them only to default rates and mortgage loan losses.
[2] This article thus constructs a simplified version of the hierarchical game as addressed in the literature on environmental management such as Jou (2004) and Krawczyk and Zaccour (1999).

property at the price paid by the buyer. When deciding whether to purchase a property at the foreclosure sale, the buyer will rationally anticipate the path of the property value that triggers the mortgagor to reclaim the property over the redemption period. The buyer can then calculate the potential loss resulting from the option exercised by the mortgagor, which certainly discourages the buyer to purchase, as compared to the case in the absence of any statutory right of redemption.

This paper is related to Miceli and Sirmans (2005) and Baker, Miceli and Sirmans (2008), both of which build a static model to investigate how an increase in the length of equitable redemption period affects a mortgagor's incentive to devote efforts to avoid default. Given that the mortgagor enjoys a higher American-type call option value as the redemption period lasts longer, the mortgagor will thus devote less effort. By contrast, we focus on how the potential loss from the exercise of the American-type option by the mortgagor affects the decision to purchase for a buyer at the foreclosure sale.

This paper is also related to the literature on the pricing of American options. The mortgagor in our framework decides the optimal timing to exercise an American-type call option with a finite maturity, where the pricing formula has been provided by Barone-Adesi and Whalley (1987), Carr (1995), and Lee and Paxson (2003).[3] In addition, Jou and Lee (2009) have applied this pricing formula to investigate how a development moratorium affects a landowner's incentive to develop the vacant land.

The remaining sections are organized as follows. We first present the assumptions of the model, and then solve for the level of the property value that triggers a buyer to purchase at the foreclosure sale. We also investigate how the following factors affect the buyer's choice of timing and the associated gain from the purchase: (1) The length of statutory redemption; (2) the discount rate employed by the buyer; (3) the irreversible transaction costs; (4) the

[3] Geske and Johnson (1984) have provided the pricing formula for the American put option with a finite maturity.

expected inflation rate in the housing price; (5) the volatility of that expected inflation rate; and (6) the buyer's managerial ability. Next, we present the simulation analysis by employing plausible parameter values. The last section concludes and offers suggestions for future research.

Basic Assumptions

Consider a mortgagor who has already defaulted a mortgage loan, and thus the mortgagor's property is subject to a judicial foreclosure sale or power-of-sale. Suppose that the value of the property, $V(t)$, evolves as:

$$\frac{dV(t)}{V(t)} = \alpha dt + \sigma dZ(t), \tag{1}$$

where α is the expected inflation rate of the housing price, σ is the instantaneous volatility of that inflation rate, and $Z(t)$ is a standard Wiener process. We assume that the mortgagor and a potential buyer at the foreclosure sale are both risk-neutral and face a constant riskless rate, r. The total return from holding the property is, therefore, equal to r, which is also equal to $\alpha + \delta$, where δ $(= r - \alpha)$ is the convenience yield, i.e., the implicit rental rate from holding the property. We can generalize our model to the case of risk aversion in the manner of Cox and Ross (1976). Our result, however, will be the same regardless of whether we consider a risk-neutral, or a risk-averse, environment. Finally, the transaction costs other than the price paid to the mortgagee, denoted by K, are assumed to be fully irreversible.[4]

The Repurchase and Purchase Trigger Values

As stated earlier, we focus on interactions between a buyer at the foreclosure sale and

[4] As Brueggeman and Fisher (2006, Chapter 4) suggest, the transaction costs consist of statutory costs and third-party charges. The former includes certain charges for legal requirements that pertain to the title transfer, recording of the deed, and other fees required by state and local law. The latter includes charges for services, such as legal fees, appraisals, surveys, past inspection, and title insurance. All of these charges, however, are unrecoverable after the property is purchased.

the mortgagor rather than between the buyer and the auctioneer at the foreclosure sale. Thus, we assume that there is a single buyer at the foreclosure sale, and the buyer can either purchase immediately or delay purchasing. As shown later, the buyer will not purchase the property until the property value reaches a threshold level. Thus, the buyer is not willing to purchase the property at the foreclosure sale if the realized property value falls short of this threshold level. We may use this threshold level to proxy the reservation price set by the auctioneer. Therefore, unless this reservation price is reached, the auctioneer will not sell the property. The auctioneer will then set a new date for the foreclosure date. This sequence will end until the property value evolves to the level of the reservation price.

Suppose that an auctioneer announces a foreclosure sale for a property at $t = 0$. After the buyer purchases the property, the statutory redemption period then lasts for T years.[5] Let us first consider a hypothetical case where $T = \infty$, i.e., the mortgagor who defaulted a loan has the right to reclaim the collateralized property at any time as he wishes. We will then use the solution for this case to derive the exercise rule for the mortgagor who has the right to reclaim the property only during a finite period of time.

Let us assume that a buyer has already paid $V(0)$ to an auctioneer to purchase a property at time $t = 0$. After then, the mortgagor can reclaim the property by paying $V(0)$ at any future date, given that the statutory redemption period lasts forever. The mortgagor thus holds a perpetual American call option, denoted as $F_2(V(t))$. Given that this call option value is independent of the calendar date, we can use Ito's lemma, thus yielding $F_2(V(t))$ satisfies the following differential equation:

[5] We may consider the case in which many buyers compete for a property. The existing literature, however, is inconclusive regarding how competition affects the investment timing of these buyers. For example, Grenadier (2002) finds that competition encourages a firm to invest earlier if the firm undertakes a continuous project. By contrast, Jou and Lee (2008) find that competition encourages a firm to delay investment if the firm can choose the scale and timing of a discrete investment project.

$$\frac{1}{2}\sigma^2 V(t)^2 \frac{\partial^2 F_2(V(t))}{\partial V(t)^2} + \alpha V(t)\frac{\partial F_2(V(t))}{\partial V(t)} - rF_2(V(t)) = 0\,. \tag{2}$$

Equation (2) has an intuitive interpretation: If $F_2(V(t))$ is an asset value, then the normal return $rF_2(V(t))$ must be equal to its expected capital gain, given by:

$$\frac{E(dF_2(V(t)))}{dt} = \alpha V(t)\frac{\partial F_2(V(t))}{\partial V(t)} + \frac{1}{2}\sigma^2 V(t)^2 \frac{\partial^2 F_2(V(t))}{\partial V(t)^2}\,. \tag{3}$$

The solution to Equation (2) is given by:

$$F_2(V(t)) = A_1 V(t)^{\beta_1} + A_2 V(t)^{\beta_2}\,, \tag{4}$$

where A_1 and A_2 are constants to be determined,

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{(\frac{1}{2} - \frac{\alpha}{\sigma^2})^2 + \frac{2r}{\sigma^2}} > 1,$$

and

$$\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{(\frac{1}{2} - \frac{\alpha}{\sigma^2})^2 + \frac{2r}{\sigma^2}} < 0. \tag{5}$$

Suppose that V_2^* denotes the critical level of $V(t)$ that triggers the mortgagor to reclaim the property. This critical level and the two constants, A_1 and A_2, are solved from the boundary conditions given by:

$$\lim_{V(t)\to 0} F_2(V(t)) = 0, \tag{6}$$

$$F_2(V_2^*) = V_2^* - V(0), \tag{7}$$

and

$$\left.\frac{\partial F_2(V(t))}{\partial V(t)}\right|_{V(t)=V_2^*} = 1. \tag{8}$$

Equation (6) is the limit condition, which states that the option value to delay purchasing becomes worthless as the property value approaches its minimum permissible value, i.e., zero. Equation (7) is the value-matching condition, which states that, at the optimal timing of purchase, the mortgagor should be indifferent as to whether or not reclaim the property. Equation (8) is the smooth-pasting condition, which prevents the mortgagor from deriving any arbitrage profits.

Solving Equations (6) to (8) simultaneously yields:

$$V_2^* = \frac{\beta_1 V(0)}{(\beta_1 - 1)}. \tag{9}$$

Subtracting the purchase price $V(0)$ from V_2^* yields the net value from reclaiming the property as given by:

$$D = V_2^* - V(0) = \frac{V(0)}{(\beta_1 - 1)}. \tag{10}$$

Thus, for the hypothetical case where the statutory redemption period lasts forever, the mortgagor will not reclaim the property until the value of the property $V(t)$ reaches V_2^*. At that optimal exercise date, the mortgagor will gain at an amount equal to D shown by Equation (10).

Now consider the general case where the statutory redemption period is finite and the buyer has already purchased a property at time $t = 0$. The mortgagor then has a sequence of American-type call options that expire within a certain period because the mortgagor can reclaim the property at any time τ during the period from 0 to T. We denote this option value as $C_m(V(\tau), V(0), T - \tau)$ and follow Barone-Adesi and Whalley (1987) to find its pricing formula. Define $V_m^*(\tau)$ as the critical level of the property value that triggers the mortgagor to exercise the finite American call option at time τ, which is given by:[6]

$$\begin{aligned} C_m(V_m(\tau),V(0),T-\tau) &= V_m^*(\tau)-V(0) \\ &= e^{-(r-\alpha)(T-\tau)}V_m^*(\tau)N(d_1)-e^{-r(T-\tau)}V(0)N(d_1-\sigma\sqrt{(T-\tau)}) \\ &+[1-e^{-(r-\alpha)(T-\tau)}N(d_1)]\frac{V_m^*(\tau)}{\theta}, \end{aligned} \tag{11}$$

where

$$d_1=\frac{\ln\frac{V_m^*(\tau)}{V(0)}+(\alpha+\frac{\sigma^2}{2})(T-\tau)}{\sigma\sqrt{(T-\tau)}},\quad \theta=\frac{1}{2}-\frac{\alpha}{\sigma^2}+\sqrt{(\frac{1}{2}-\frac{\alpha}{\sigma^2})^2+\frac{2r}{\sigma^2[1-e^{-r(T-\tau)}]}},$$

and $N(\cdot)$ is the cumulative standard normal distribution function.

We can use Equation (11) to numerically solve $V_m^*(\tau)$. Barone-Adesi and Whalley (1987) note that an analytically tractable solution, which is approximately equal to $C_m(V_m(\tau),V(0),T-\tau)$ in Equation (11), is given by:

$$C_m(V_m(\tau),V(0),T-\tau)=V_m^*(\tau)-V(0)=(V_2^*-V(0))(1-e^{-h(\tau)})=\frac{V(0)}{(\beta_1-1)}(1-e^{-h(\tau)}), \tag{11'}$$

where

$$h(\tau)=\left[(r-\alpha)(T-\tau)+2\sigma\sqrt{(T-\tau)}\right](\beta_1-1).$$

Solving from Equation (11') yields

$$V_m^*(\tau)=g(\tau)V(0), \tag{12}$$

where

$$g(\tau)=1+\frac{1}{(\beta_1-1)}(1-e^{-h(\tau)}). \tag{13}$$

[6] Consider $V(t)$ as an asset value. Those who purchase this asset should require a compensation equal to $(r-\alpha)V(t)$, given that $V(t)$ is expected to grow at a rate equal to α. Consequently, the term $r-\alpha$ replaces the asset yield in the formula developed in Barone-Adesi and Whalley (1987).

Equation (12) shows the time path of the property value that triggers the mortgagor to reclaim the property over the redemption period. We may use it to calculate the potential loss at the foreclosure sale incurred by a buyer, who must pay the purchase price $V(0)$ and the transaction cost, K. As a result, the buyer will never purchase the property unless the buyer is able to operate the property better than the mortgagor who defaulted the loan. We thus assume that the buyer can increase the value of the property to $\varepsilon V(0)$, where $\varepsilon > 1$. At each point of time during the statutory redemption period, the potential loss for the buyer is the value of an European call option, with $\varepsilon V(0)$ as the price of the underlying asset and $V_m^*(\tau)$ as the strike price at the expiration date τ. Denoting this option value as $C_b(\varepsilon V(0), V_m^*(\tau), \tau)$ yields

$$C_b(\varepsilon V(0), V_m^*(\tau), \tau) = e^{-(r-\alpha)\tau}[\varepsilon V(0) N(d_2) - V_m^*(\tau) N(d_2 - \sigma\sqrt{\tau})], \tag{14}$$

where

$$d_2 = \frac{\ln\frac{\varepsilon}{g(\tau)} + (\alpha + \frac{\sigma^2}{2})(T-\tau)}{\sigma\sqrt{(T-\tau)}}.$$

Consider the decision rule for a buyer attending the foreclosure sale. Under the assumption that the buyer does not need to purchase the property during any certain period of time, the buyer thus has an option value of waiting similar to $F_2(V(t))$ stated before.[7] Let us denote this option value of waiting as $F_1(V(t))$, which is given by $F_1(V(t)) = B_1 V(t)^{\beta_1} + B_2 V(t)^{\beta_2}$, where B_1 and B_2 are constants to be determined and β_1 and β_2 are defined in Equation (5). Suppose that V_b^* denotes the critical level of $V(0)$ that triggers the buyer to purchase the property. This critical level and the two constants, B_1 and B_2, are solved from the boundary conditions as follows:

[7] It is more plausible to assume that the foreclosure sale lasts for a finite period of time. However, this will complicate the analysis. Our main result will hold no matter whether the mortgage sale lasts for a finite period of time or not.

$$\lim_{V(0)\to 0} F_1(V(t)) = 0, \tag{15}$$

$$F_1(V_b^*) = \varepsilon V_b^* - V_b^* - \int_0^T C_b(\varepsilon V(0), V_m^*(\tau), \tau) d\tau - K. \tag{16}$$

and

$$\left.\frac{\partial F_1(V(0))}{\partial V(0)}\right|_{V(0)=V_b^*} = \varepsilon - 1 - \int_0^T \frac{\partial C_b(\varepsilon V(0), V_m^*(\tau), \tau)}{\partial V(0)} d\tau. \tag{17}$$

Using Equation (11') and solving Equations (15)-(17) simultaneously yields the explicit solution for V_b^* as given by

$$V_b^* = \frac{K\beta_1}{(\beta_1 - 1)(\varepsilon - 1 - G_1)}, \tag{18}$$

where

$$G_1 = \int_0^T e^{-(r-\alpha)\tau} [\varepsilon N(d_2) - g(\tau) N(d_2 - \sigma\sqrt{\tau})] d\tau.$$

Substituting V_b^* into the right-hand side of Equation (16) yields the gain from purchasing the property at the date of purchasing as given by

$$F_1(V_b^*) = \frac{K}{(\beta_1 - 1)}. \tag{19}$$

One may argue that competitive pressure at the foreclosure will lead the property value that triggers purchasing to depart from that shown by Equation (18). However, as long as the actual trigger price is equal to a constant factor multiplied by the trigger level shown by Equation (18), then all our main results will remain unchanged.[8]

We are in a position to investigate how changes in various exogenous forces affect a buyer's decision to purchase and the associated gain from purchasing. First, differentiating V_b^* in Equation (18) with respect to T, K, and ε yields the results stated below.

Proposition 1: *A buyer at the foreclosure sale will wait for a better state of nature to purchase (*V_b^* *increases) if (*i*) the statutory redemption period lasts longer (*T *increases); (*ii*)*

[8] Grenadier (2002) has shown that, for an oligopolist industry, there exists such a constant factor, which is a function of the number of firms in the industry.

the buyer incurs larger less transaction costs (K *increases*)*; and* (*iii*) *the buyer is less capable of operating the property* (ε *decreases*).

Proposition 1(i) follows because a buyer will expect the loss associated with the reclaim option exercised by the mortgagor to increase if the statutory redemption period lasts longer. Proposition 1(ii) follows because a buyer who incurs larger transaction costs will gain less from purchasing the property immediately. Proposition 1(iii) follows because waiting is more valuable for a buyer who is less capable of operating the property.

Second, differentiating $F_1(V_b^*)$ in Equation (19) with respect to K, r, α, and σ yields the results stated below.

Proposition 2: *A buyer will gain more at the date of purchasing* ($F_1(V_b^*)$ *increases*) *if* (*i*) *the buyer incurs larger transaction costs* (K *increases*)*;* (*ii*) *the buyer is far-sighted* (r *decreases*)*;* (*iii*) *the housing price is expected to appreciate at a higher rate* (α *increases*)*; and* (*iv*) *the volatility of the housing price inflation increases* (σ *increases*).

Equation (19) indicates the buyer's gain evaluated at the date of purchasing is given by $K/(\beta_1 - 1)$. We may, however, consider the time value of money, i.e., the gain from purchasing based on some reference date. This can be done for changes in T, K, ε, and r because the Brownian motion specified in Equation (1) will then remain unchanged. Let us take V_b^* in Equation (18) as the reference date, and consider the case where a buyer purchases a property at a date, $\bar{V}$, which is later than that shown by V_b^*. The probability that the buyer will purchase at the date when the value of property $V(t)$ being equal to V_b^* is given by $(V_b^*/\bar{V})^{\beta_1}$, and thus the expected gain at $V(t) = V_b^*$ is equal to $(V_b^*/\bar{V})^{\beta_1} F_1(\bar{V})$. We then find three main results as stated below.

Proposition 3: *After considering the time value of money, a buyer at the foreclosure sale will gain less when* (*i*) *the statutory redemption period lasts longer* (T *increases*)*;* (*ii*) *the buyer incurs larger transaction costs* (K *increases*)*; and* (*iii*) *the buyer is less capable of operating the property* (ε *decreases*).

Propositions 3(i) and (iii) are obvious because without considering the time value of money, a buyer will gain the same amount at the date of purchasing no matter how long the statutory redemption period lasts or how capable the buyer is. After considering the time value of money, however, the buyer will gain less when the statutory redemption period lasts longer or the buyer is less capable because the buyer will then collect the proceeds from the purchase at a later date.

Proposition 3(ii) needs further explanation. Suppose that the transaction cost incurred by the buyer increases from K to λK, where $\lambda > 1$. As a result, the property value that triggers the buyer to purchase will be equal to λV_b^*, and the gain from purchasing will be equal to $\lambda F_1(V_b^*)$. Evaluating this gain at $V(t) = V_b^*$ yields $(V_b^* / \lambda V_b^*)^{\beta_1} \lambda F_1(V_b^*) = \lambda^{1-\beta_1} F_1(V_b^*) < F_1(V_b^*)$ because both $\lambda > 1$ and $\beta_1 > 1$. The result is also intuitive because an increase in the transaction cost implies that the strike price for purchasing a property, i.e., $V(0) + K$, increases, and thus the call option value on purchasing the property will decrease as a result.

However, it is ambiguous regarding how an increase in either the expected appreciation rate of the housing price, α, or the volatility of that expected appreciation rate, σ, affects the buyer's incentive to purchase. We thus employ the numerical analysis in the next section to clarify this ambiguity.

Numerical Analysis

We choose a set of parameter values as the benchmark case to make our results in the last section more vivid. We assume that the statutory redemption period lasts for one year, i.e., $T = 1$, which is implemented by most states in the U.S. (Clauretie, 1987). A buyer expects the housing price to increase 1% per year, i.e., $\alpha = 1\%$, and this inflation rate to evolve stochastically with a volatility equal to 20% per year, i.e., $\sigma = 20\%$.[9] Both the mortgagor and a buyer at the foreclosure sale have a common discount rate equal to 6% per year, i.e., $r = 6\%$. The buyer incurs an irreversible transaction cost equal to one unit, i.e., $K = 1$, and can improve the value of the property by 5%, i.e., $\varepsilon = 1.05$. Under this benchmark case, the buyer will not purchase the property until its value reaches 59.3 units.[10]

Insert Figure 1 here

Figure 1 shows the result for the case where the volatility of the housing price inflation, σ, changes in a region between 0% to 40% per year. We see that there is a turning point at $\sigma = 3\%$ per year. As the volatility increases, the potential loss resulting from the reclaim option exercised by the mortgagor increases. Consequently, for a buyer at the foreclosure sale, both the value from purchasing a property immediately and the option value of waiting decrease. When uncertainty is very insignificant (i.e., the volatility is smaller than 3% per year), the reduction of the former will be outweighed by the reduction of the latter, and thus the buyer will accelerate purchasing. This contrasts with the situation when uncertainty is significant because the buyer will then delay purchasing. Nevertheless, without considering the time value of money, the gain for the buyer at the date of purchasing will unambiguously

[9] According to Goetzmann and Ibbotson (1990), during the period of 1969 to 1989, the annual standard deviation for REITs on commercial property was equal to 15.4%. The volatility of the housing price inflation in our benchmark case was a little higher than this value.

[10] The transaction cost relative to the purchase price is equal to 1/59.3 = 1.68%, which is smaller than that spent in purchasing a property not subject to the foreclosure sale, i.e., 5% – 6% (Stokey, 2009).

increase with the volatility.[11]

Insert Table 1 here

Table 1 presents the comparative-statics results, in which only one parameter is changed around its benchmark value, while the other parameters are held at their benchmark values. The results conform to the theoretical results stated in Propositions 1, 2, and 3. Panel A shows the case where the expected appreciation rate of the housing price, α, varies from -1% to 3% per year. It shows that as the expected appreciation rate increases, the buyer will lose more if purchasing immediately than if delaying the purchase. The buyer will thus gain more at the date of purchasing if we do not consider the time value of money.[12]

Panel B shows the results for the case where the irreversible transaction cost, K, varies from 0.5 units to 1.5 units. An increase in the transaction cost unambiguously encourages a buyer to delay purchasing, and thus the buyer gains more at the date of purchasing. However, after considering the time value of money, the gain associated with purchasing will decrease with the transaction cost.

Panel C shows the results for the case where the statutory redemption period, T, varies from a half year to one and half years. As expected, a longer statutory redemption period encourages the buyer to delay purchasing, but leaves unchanged the gain at the date of purchasing. However, a longer statutory redemption period will reduce the gain from purchasing after we take the time value of money into account.

[11] When the volatility changes, the value of property as shown by the Brownian motion given by Equation (1) also changes. As such, choices of the reference may affect our conclusion when we take the time value of money into account. If we use the benchmark parameter values as the reference point, then we find that the buyer will gain if the volatility increases from zero to 11% per year, but will lose if the volatility further increases from 11% per year.

[12] If we take into account the time value of money by using the benchmark parameter values as the reference point, then we find that the buyer will gain when the expected appreciation rate increases from -1% per year to 1% per year, but will lose if that rate further increase from 1% to 3% per year.

Panel D shows the results for the case where the buyer can improve the performance of the purchased property by 3% to 7%, i.e., ε varies from 1.03 to 1.07. As the buyer's managerial ability improves, the gain from purchasing also increases, which, however, is exactly offset by the increase of the potential loss resulting from the reclaim option exercised by the mortgagor. This implies that the benefit for the buyer to purchase the property immediately is independent of the buyer's managerial ability. However, as the buyer's managerial ability improves, the buyer's option value from waiting will decrease. Thus the buyer will purchase earlier, and gain more from purchasing after we take into account the time value of money.

Panel E shows the results for case where the buyer's discount rate, r, varies from 4% to 8% per year. It shows that a far-sighted buyer (low r) will purchase later, and thus gain more from purchasing. However, when we take the time value of money into account, the gain from purchasing is almost the same when the discount rate varies between 4% to 8% per year.

Insert Figure 1 here

Conclusion

Statutory redemption offers the right for a mortgagor who defaulted a loan to reclaim his collateralized property within a certain period of time. This repurchase option benefits the mortgagor, but harms a buyer attending at the foreclosure sale. We derive a closed-form solution of the decision rule for the buyer, which indicates that the buyer will delay purchasing and gain less if (i) the redemption period lasts longer, (ii) the buyer is less capable of operating the property; and (iii) the buyer incurs more transaction costs in purchasing the property. However, we are not sure how uncertainty in housing price inflation affects the buyer's incentive to purchase.

This article builds a simplified model and thus can be extended in the following ways. First, we may take the equitable right of redemption into account. Second, we may allow more buyers to compete at the foreclosure sale rather than assume that there exists only one single buyer. We leave these extensions to future research.

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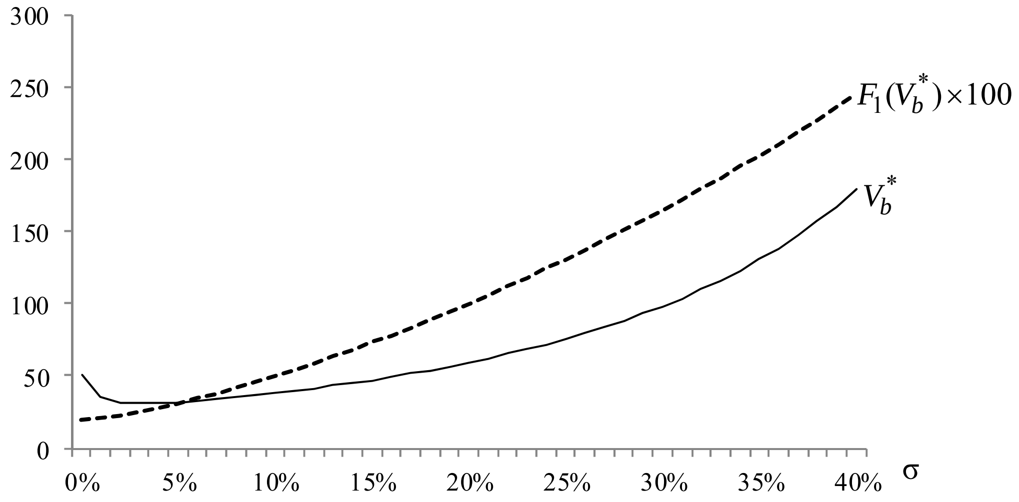

Figure 1: The purchase timing and the associated gain for various levels of volatility

This figure shows the purchase timing and the associated gain for a buyer at the foreclosure sale for various levels of volatility. The solid curve shows the trigger point V_b^*, which is defined in Equation (18), and the dotted curve shows $F_1(V_b^*)$ (scaled up 100 times), which is the gain at the date of purchasing as defined in Equation (19). The benchmark parameter values are given by $K = 1$, $T = 1$, $\varepsilon = 1.05$, $\alpha = 1\%$, and $r = 6\%$.

Table 1: Optimal purchase timing and the associated gain at the date of purchasing

Benchmark case: $\sigma = 20\%$, $\alpha = 1\%$, $K = 1$, $T = 1$, $\varepsilon = 1.05$, and $r = 6\%$					
	Panel A: Variation in α				
	-1%	0%	1%	2%	3%
V_1^*	48.7	52.9	59.3	69.1	87.4
$F_1(V_1^*)$	0.61	0.77	1.00	1.37	2.00
	Panel B: Variation in K				
	0.5	0.75	1.0	1.25	1.5
V_1^*	29.6	44.5	59.3	74.1	88.9
$F_1(V_1^*)$	0.50	0.75	1.00	1.25	1.50
	Panel C: Variation in T				
	0.5	0.75	1	1.25	1.5
V_1^*	46.0	51.4	59.3	71.6	92.5
$F_1(V_1^*)$	1.00	1.00	1.00	1.00	1.00
	Panel D: Variation in ε				
	1.03	1.04	1.05	1.06	1.07
V_1^*	121.1	79.4	59.3	47.4	39.6
$F_1(V_1^*)$	1.00	1.00	1.00	1.00	1.00
	Panel E: Variation in r				
	0.04	0.05	0.06	0.07	0.08
V_1^*	72.9	64.5	59.3	55.6	52.9
$F_1(V_1^*)$	1.46	1.18	1.00	0.88	0.79

Note: The terms σ, α, K, T, ε, and r denote the volatility of the expected appreciation rate of the housing price, the expected appreciated rate of the housing price, the transaction cost, the statutory redemption period, the buyer's managerial ability, and the buyer's discount rate, respectively.