Asset Fire Sales, the Threat of Bank Runs, and Contagion

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Abstract

This paper demonstrates that asset fire sales can happen when banks face the threat of a run by institutional depositors. Buying risky assets makes a bank's future asset value more volatile, thus increases the institutional depositors' incentives to withdraw early. Knowing this, banks may be reluctant to buy risky assets even if they have extra liquidity and these assets are undervalued, and an asset fire sale can occur as a result.

The paper implies that low returns can be contagious among risky assets. It also proposes that the asset fire sale problem will be more serious when the institutional depositors' financial performance is more volatile, when banks rely more heavily on deposits from institutional depositors, and when banks hold fewer liquid assets. As for policy implications, the paper suggests that using the marking-to-market rule to evaluate banks may be inappropriate during financial crises, and that the liquidity requirements should be stricter for banks with more wholesale deposits.

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1. Introduction

Liquidity plays an important role in the 2007-2009 financial crisis. Because of the sudden dry up of liquidity in asset and lending markets, many risky assets were sold in prices lower than their fundamental values, haircuts on collateral in the collateralized money markets soared sharply, and banks tightened their lending to borrowers. As discussed in Allen and Carletti (2008b), "One of the most puzzling features of the crisis has been the pricing of AAA tranches of a wide range of securitized products. It appears that the market prices of many of these instruments are significantly below what plausible fundamentals would suggest they should be." Using the U.S. data, Ivashina and Sharfstein (2010) report that "…new loans to large borrowers fell by 47% during the peak period of the financial crisis (fourth quarter of 2008) relative to the prior quarter and by 79% relative to the peak of the credit boom (second quarter of 2007)." In addition, Gorton (2009) documented that "financing in the repo market became very expensive and disappears for some asset classes, like subprime bonds." It is important to ask why liquidity that was abundant before the crisis occurred dried up so quickly for the asset and lending markets during the crisis.

This paper sheds light on this question. It establishes a simple model to study the conditions under which asset fire sales will occur, where asset fire sales refer to the phenomenon that risky assets are sold in prices below the highest prices that potential buyers would like to pay in normal times. In the model, banks collect deposits to invest in a long-term risky asset, and the asset's realized return is high in the good state and low in the bad state. A part of the deposits are made by sophisticated institutional depositors who face performance pressure. Banks need interim funding to satisfy the withdrawals by depositors with liquidity needs. However, the supply of liquidity is not evenly distributed among banks: some banks have extra liquidity and the others have to sell the risky asset to acquire the liquidity.

Under this setting, the paper demonstrates that an asset fire sale can happen even if the aggregate liquidity available in the financial system exceeds the aggregate liquidity needs. This is because banks with extra liquidity are reluctant to provide it due to the fear of a bank run started by institutional depositors. It is shown that institutional depositors at a bank will withdraw early if the bank's asset value in the bad state is lower than a cutoff value. As long as the price of the asset is higher than its return in the bad state, buying the risky asset will reduce a bank's asset value in the bad state, thus increases its institutional depositors' incentives to withdraw. This concern depresses the demand of banks with extra liquidity for the risky asset, and may result in a low "fire-sale" price of the risky asset. As will be seen, the fire sale problem can cause inefficient bank failures.

The model in this paper suggests a new contagion channel through which the low return of an asset can lead to low returns of other assets. It is shown that institutional depositors have stronger incentives to withdraw early when they are in poorer financial conditions. If institutional depositors invest in some assets that are not considered in the model and the returns of these assets turn out low, then according to the model, institutional depositors will become more eager to withdraw. Knowing this, to avoid a bank run, the banks with extra liquidity will decrease their demand for the risky asset considered in the model, which will result in a low return of this risky asset. Note that, in our model, a bank run need not really happen to damage the price of the risky asset. It is the threat of a bank run that affects the behavior of banks and causes a low asset price.

The paper has interesting implications. First, it implies that the asset fire sale problem will be more serious when the institutional depositors' financial performance becomes more volatile. To completely avoid a bank run, banks with extra liquidity have to behave as if the realized performance of the institutional depositors is at its lowest possible level when they make the asset purchase decisions. Therefore, the more volatile is asset returns, the more conservative will the asset buyers who face the threat of a creditor run become. As a result, the prices of risky assets will be lower.

Second, the model suggests that the composition of asset buyers' funding sources has an impact on the fire sale problem. When banks more heavily rely on non-core or wholesale deposits, the withdrawal pressure from institutional depositors is greater, so the fire sale problem will become more serious. Third, because the prices of risky assets can be affected by the degree of depositor panic, the model suggests that the market value accounting rule may be inadequate for evaluating banks during a financial crisis. It will be shown that banks may be identified as insolvent based on the marking-to-market rule even if they can survive in the good state.

Fourth, the model implies that requiring banks to hold more liquid assets alleviates the fire sale problem, although it comes at the cost of reducing the banks' profits in the good times. Because the fire sale problem is more important when banks rely more on non-core or wholesale deposits, this result also suggests that banks should be required to hold more liquid assets if non-core deposits are a more important funding source for them.

Finally, our model implies that the government's lending to healthy financial institutions in the hope that they will use the borrowed money to buy more risky assets will be an ineffective tool for solving the asset fire sale problem if these financial institutions are reluctant to buy due to the fear of a creditor run. By contrast, directly lending to financial institutions that have liquidity needs may be more effective because the lending reduces these institutions pressure to sell assets at low prices to acquire liquidity.

This paper is related to several strands of literature. Various papers explain why asset fire sales or market freeze may occur during a financial crisis. Brunnermeier and Pedersen (2009), Adrian and Shin (2010), and Gorton (2009) argue that the leverage constraints of financial institutions will be tightened when asset prices are low and asset returns are volatile. The leverage constraints will force these financial institutions to sell assets in the markets, thus further dampen asset prices. Shleifer and Vishny (2010) and Diamond and Rajan (2009) suggest that, by raising the expected return of hoarding cash, the expectation of a fire sale in the future will depress current asset prices and reduce the banks' incentives to lend or buy assets now. Diamond and Rajan also argue that, due to the asset substitution problem in Jensen and Mecking (1976), banks will refuse to sell risky assets in low prices to acquire the liquidity needed to guarantee their survival, and this seller's strike will result in market freeze. Shleifer and Vishny (1992), Allen and Gale (1998), Allen and Carletti (2008a), and Bolton, Santos, and Scheinkman (2009) demonstrate that, when the level of liquidity available is low, asset prices will be determined by the amount of cash in the market, and may be much lower than their fundamental values. Acharya, Gale, and Yorulmazer (2009), He and Xiong (2009), Brunnermeier and Oehmke (2009), and Huang and Ratnovski (2009) propose that the use of short-term debt financing is an important cause of the liquidity crisis. Bolton, Santos, and Scheinkman (2009) and Heider, Hoerova, and Holthausen (2010) show that adverse selection among players in the financial markets plays an important role in causing a liquidity crisis.

Complementary to these papers, our model points out that in addition to the asset sellers' leverage constraints, the composition of the asset buyers' funding sources is also an important determinant for whether asset fire sales will occur. If asset buyers heavily rely on short-term debt from sophisticated investors, they may lose the interest to acquire more risky assets when creditor runs becomes a concern. In addition, because the buyers' willingness to acquire risky assets is affected by the threat of creditor runs, the asset fire sale problem can occur even if the aggregate liquidity available in the system exceeds the aggregate liquidity needs. Therefore, asset prices may be lower than the cash-in-the-market prices defined in Allen and Gale (1998).

Our model also has contributions to the literature that studies financial contagion. Allen and Gale (2000), Shin (2008), Dasgupta (2004), and Cifuentes, Ferrucci, and Shin (2005) illustrate that contagion can occur when some financial institutions' liabilities are other financial institutions' assets. Goldstein and Pauzner (2004) show when two countries share the same depositors with a decreasing relative risk aversion utility function, bank runs in one country may trigger runs on banks in the other country even if the countries have independent fundamentals. This is because when depositors suffer losses in one country, the wealth effect will make them more eager to withdraw in the other country.

Our paper shares important common features with papers in this literature. In our model, the banks' liabilities are the institutional depositors' assets. Also, similar to the setting in Goldstein and Pauzner (2004), the institutional depositors in our model also become more panic when they have worse financial performance. We extend Goldstein and Pauzner's work by showing that contagion of low returns can happen even if bank runs do not really occur.

In the literature, Shleifer and Vishny (1997) show that speculators are unable to fully explore arbitrage opportunities if the sizes of funds they can control are increasing in their short-term financial performance. Our model is similar to theirs in that both suggest asset prices can deviate from fundamental values when traders face funding pressures. However, the main drivers of the pressure are different. In Shleifer and Vishny (1997), fund suppliers withdraw funds when the speculators' financial performance is poor. By contrast, in our model the threat of a bank run is more serious when fund suppliers (that is, institutional depositors) themselves are in poorer financial conditions. Also, unlike Shleifer and Vishny, we do not assume information asymmetry between traders and their fund suppliers.

Several articles in the literature discuss how the composition of financial institutions' funding sources affects them. Berlin and Mester (1999) report that banks relying more heavily on core deposits provide more loan rate smoothing in response to aggregate credit risk shocks. Shin (2009) points out that the run on Northern Rock Bank was not enacted by individual depositors, but by sophisticated institutional investors. He, Khang, and Krishnamurthy (2010) find that, while hedge funds and investment banks substantially reduce their holdings of securitized assets during the recent financial crisis, commercial banks (whose funding is much more stable and guaranteed by the government) increases the holdings of these assets. Huang and Ratnovski (2009) argue that wholesale funding has its dark side: wholesale funds lack the sufficient incentives to acquire information about banks, and are too eager to withdraw during bad times when they are uninformed. In contrast to Huang and Ratnovski (2009), we show that institutional depositors still have excessive incentives to withdraw even if they have the information about banks. More importantly, their incentives to withdraw are stronger when their financial performance is worse. Consistent with the empirical observation in He, Khang, and Krishnamurthy (2010), our model suggests that financial institutions with more stable funding sources will buy more risky assets.

The rest of the paper is organized as follows. Section 2 introduces the basic model. Section 3 analyzes the basic model. Section 4 discusses the implications of the model and the reasonableness of the key assumptions of the model. Section 5 contains concluding remarks.

2. The Basic Model

Consider a three-date model (dates 0, 1, and 2) with banks and depositors. All the players are risk neutral, and the risk-free discount rate is zero. For simplicity, assume that banks do not have capital; they collect deposits at date 0 to make investments. There are individual and institutional depositors. For each bank, the fractions of deposits from individual and institutional depositors are s and $1 - s$, respectively. To reflect the fact that deposits from individual depositors are a stable funding source for banks, assume that all the deposits of individual depositors mature at date 2, and they are fully insured by the government. Banks pay zero interest rate on fully insured deposits, and the government does not charge deposit insurance premiums on banks.[1]

As for deposits of institutional depositors, each institutional depositor is endowed with one dollar at date 0, and he also receives an income I at date 2. At date 0, institutional depositors can either deposit their endowments at banks or costlessly store the endowments. The income I is a continuous random variable at date 0 with density function h and accumulative function H, and the realized value of I is between $\underline{I}$ and $\overline{I}$, where $0 < \underline{I} < \overline{I}$. All the institutional depositors receive the same income, and the value of I is realized at date 1. More detailed assumptions about I will be introduced later.

Institutional depositors derive utility from consumption. They can consume at dates 1 and 2. Let c_1 and c_2 denote the consumption of an institutional depositor at dates 1 and 2, respectively. There are two types of institutional depositors. The type-1 institutional depositors have liquidity needs at date 1: they will incur a non-pecuniary liquidity cost L if $c_1 < d_1$, where L and d_1 are exogenously given constants with $L > 0$ and $d_1 > 1$. In contrast, the type-2 institutional depositors do not have liquidity needs at date 1.[2]

For each bank, among the institutional depositors, the fractions of type-1 and type-2 depositors are f and $1 - f$, respectively. The institutional depositors' types are revealed at date 1. At date 0, institutional depositors do not know their types. Once realized, a depositor's type is his private information.

Whatever his type is, an institutional depositor will suffer a penalty if his total consumption falls short of a target T. The type-2 depositors' utility function is

[1] The assumptions about deposits of individuals and deposit insurance are made to simplify the exposition. They are not critical; all of our qualitative results will hold if they are relaxed.

[2] Since only institutional depositors (but not individual depositors) are divided into two types, in the rest of the paper the terms type-1 and type-2 depositors refer to type-1 and type-2 *institutional* depositors.

$$U_2(c_1,c_2) \equiv \begin{cases} c_1 + c_2 & \text{if } c_1 + c_2 \geq T, \\ c_1 + c_2 - k(T - c_1 - c_2) & \text{if } c_1 + c_2 < T, \end{cases} \tag{1}$$

where k and T are both strictly positive constants. Equation (1) implies that an institutional depositor whose total consumption is lower than T will receive a penalty which is proportional to the amount of the shortfall. The type-1 depositors' utility function is

$$U_1(c_1,c_2) \equiv \begin{cases} U_2(c_1,c_2) & \text{if } c_1 \geq d_1, \\ U_2(c_1,c_2) - L & \text{if } c_1 < d_1, \end{cases} \tag{2}$$

which means the only difference between type-1 and type-2 depositors is that the former have liquidity needs at date 1.

In the real world, the income I can be interpreted as the institutional depositors' financial performance of investing in assets that are not considered in our model, and T can be interpreted as the performance target that they are supposed to achieve. It is either the amount of money that institutional investors promise to pay out to their claimholders, or the profit target they promise to achieve. Failing to meet these targets is costly to the institutions because it may lead to redemptions of funds, replacements of managers, and even failures of the institutions.

We further assume that

$$T < d_1 + \underline{I}. \tag{3}$$

Equation (3) is an important assumption. It implies that an institutional depositor can always satisfy the target if he successfully withdraws at date 1. Under this assumption, withdrawing early at date 1 is a "safer" strategy for institutional depositors in terms of achieving the target. As will be seen, this assumption implies that institutional depositors have stronger incentives to withdraw when they have a lower income. We will discuss whether this assumption is reasonable in Section 4.

To satisfy the institutional depositors' liquidity needs, banks offer them a deposit contract (d_1, d_2). For a dollar deposited at date 0, banks promise to pay d_1 if an institutional depositor withdraws at date 1, and pay d_2 if a depositor withdraws at date 2, where d_2 is a constant and $d_1 < d_2 < R$. The first-come, first-served rule is imposed, and a bank has to keep open before it runs out of money. Deposits of institutional depositors are not covered by deposit insurance. To simplify the exposition, we assume that the deposit contract is exogenously given. It can be easily endogenized by the arguments in Chen and Hasan (2008).

Because $d_2 > d_1$, type-2 depositors will withdraw at date 2 if they feel that the deposits are risk-free. However, if type-2 depositors are concerned about the soundness of their banks, they may choose to withdraw early. We will say that a bank run occurs to a bank if all its institutional depositors (both type-1 and type-2) withdraw at date 1. Under the above setting, for each dollar of deposits collected at date 0, a bank's liability is

$$D \equiv s + (1 - s)\,[f\,d_1 + (1 - f)\,d_2] \tag{4}$$

if only type-1 depositors withdraw at date 1.

At date 0, banks invest all the deposits in a risky asset that matures at date 2.[3] A unit of the risky asset returns R with probability q and returns r with probability $1 - q$, where R and r are constants with $0 < r < 1 < R$. We will say that the state is good if the return of the risky asset is R, and is bad if the return is r. At date 0, q is a random variable; it equals 1 with probability η, and equals q_L with probability $1 - \eta$, where $0 < q_L < 1$. The value of q will be realized at date 1. We assume that η is sufficiently high, so that (i) the expected

[3] This assumption will be relaxed in Section 4. We will discuss how requiring banks to keep some cash at date 0 will affect the results.

return of the risky asset is positive, that is,

$$[\eta + (1 - \eta)\, q_L]R + (1 - \eta)(1 - q_L)\, r > 1,$$

and (ii) institutional depositors prefer making deposits to self-storage at date 0.

The risky asset can be liquidated at date 1. Whatever the state will be, a unit of the risky asset always returns κ if it is liquidated at date 1, and

$$0 < \kappa < (1 - s)\, f\, d_1. \qquad (5)$$

Equation (5) means that the proceeds from liquidating a bank's risky assets are not enough to satisfy its liquidity demands at date 1.

In addition to the return from investing in the risky asset, banks also generate profits from their banking operations (for example, fee incomes). Let N denote the total amount of deposits that each bank collects at date 0. The profits from banking operations that each bank will receive are $y\, N$, where y is a positive constant. Banks differ in the timing when the profits become available to them. Half of the banks receive the profits at date 1, and the other half of the banks receive the profits at date 2. We will call the banks that receive the profits at date 1 liquidity-rich banks, while the others liquidity-deficient banks because the former have the liquidity to pay off early withdrawing depositors at date 1 and the latter do not. The banks' types are revealed at date 1. It is assumed that

$$2\,(1 - s)\, f\, d_1 < y < (1 - s)\, d_1 - \kappa. \qquad (6)$$

Assuming $2\,(1 - s)\, f\, d_1 < y$ implies that the liquidity in the banking industry as a whole (which is $0.5\, y\, N$ per bank on average) is sufficient to satisfy the liquidity needs of type-1 depositors (which is $(1 - s)\, f\, d_1\, N$ per bank). The assumption $y < (1 - s)\, d_1 - \kappa$ means that even a liquidity-rich bank will fail at date 1 if a bank run occurs.

The risky asset can be traded among banks in the market at date 1 after the banks' types are revealed and before the depositors' types are revealed. All the banks are price takers in the market, and depositors can observe the trading among banks. By (5), liquidity-deficient banks cannot acquire enough liquidity from liquidating the risky asset. Therefore, they have to sell part of their risky asset to liquidity-rich banks to acquire the liquidity. Let b denote the fraction of the risky asset that a liquidity-deficient bank sells to a liquidity-rich bank, and let p denote the price of the risky asset. Both b and p will be endogenously determined. Assume that

$$(1-s) f d_1 < r. \tag{7}$$

Given equation (7), a liquidity-deficient bank can pay off type-1 depositors by selling the risky asset even if p is r, the asset's return in the bad state.

Banks may fail at either date 1 or date 2 if it cannot satisfy the withdrawals by depositors. Since bank failures are costly, assume that a bank suffers a non-pecuniary cost X if it fails, where X is a strictly positive constant. In addition, assume that at date 2 institutional depositors can always withdraw before individual depositors do. Withdrawing earlier allows institutional depositors to reduce their expected losses when their banks fail. This assumption reflects the fact that uninsured institutional depositors have superior information about banks and stronger incentives to withdraw early than insured individual depositors. It is consistent with the observation in Shin (2009) about the bank run on Northern Rock Bank during the subprime crisis.

To simplify the exposition, we impose the following conditions:

$$R + y > D; \tag{8}$$

$$\begin{aligned} y + \left(1 - \frac{(1-s) f d_1}{R}\right) r < (1-s)(1-f) d_2 < \\ y + \left(1 - \frac{(1-s) f d_1}{r}\right) R < s + (1-s)(1-f) d_2. \end{aligned} \tag{9}$$

Equation (8) means that a bank's asset value exceeds its liability in the good state. If (8) is violated, banks will be unable to survive even in the good state. As will be explained in the next section, the first inequality in equation (9) implies that type-2 depositors cannot get fully paid at date 2 in the bad state, while the second inequality implies that they can in the good state. The third inequality means that the liquidity-deficient banks will be unable to fully pay off depositors at date 2 if the price of the risky asset is r. The sequence of moves in the basic model is shown in Figure 1.

[Please insert Figure 1 here]

3. Analysis of the Basic Model

In this section, we analyze the equilibrium of the basic model. For simplicity, we will analyze only the pure-strategy Perfect Bayesian equilibria in which all the type-2 depositors at the same bank adopt the same strategy. As in Diamond and Dybvig (1983), the equilibrium in which all institutional depositors withdraw at date 1 is always an equilibrium, and it is always Pareto dominated by the equilibrium in which only type-1 depositors withdraw at date 1, if both can be supported as equilibria. Following Chen (1999), and Chen and Hasan (2006, 2008), we assume that institutional depositors will choose the Pareto dominant equilibrium when there are multiple equilibria. Under this assumption, for a bank, all the institutional depositors withdraw at date 1 only if "no type-2 depositor withdraws at date 1" cannot be sustained as part of an equilibrium.

As for the tie-breaking rules, assume that an institutional investor will not withdraw if he is indifferent between withdrawing and not withdrawing. Also, if a bank is indifferent between multiple actions, it will choose the one that allows it to survive the longest time.[4] In addition, a bank will not trade the risky asset at date 1 if trading will not allow it to avoid a bank run. Finally, without loss of generality, assume that if there are multiple equilibrium quantities (that is, b) given an equilibrium price p, the realized b will be the smallest one. This assumption greatly simplifies the exposition without changing the main results.

The model is solved backwards. Section 3.1 analyzes the institutional depositors' withdrawing decisions at date 1 when they learn their types, and Section 3.2 discusses the equilibrium.

3.1. The type-2 depositors' withdrawing decisions at date 1

To avoid the liquidity cost L, type-1 depositors always withdraw at date 1. Whether the type-2 depositors at a bank withdraw at date 1 depends on the realized values of q and I, as well as the financial conditions of the bank. To study whether "no type-2 depositor withdraws at date 1" can be sustained as part of the equilibrium, we consider the withdrawing decision of a representative type-2 depositor who believes that (i) none of the other type-2 depositors will withdraw at date 1, and (ii) he can always successfully withdraw at date 1, and (iii) he can always successfully withdraw and receive d_2 at date 2 in the good state. Let π denote the amount of money that the type-2 depositor expects to receive from the bank if he waits until date 2 and the return of the risky asset is r. We will discuss in more detail on how π is determined in Section 3.2.

Obviously, if $q = 1$, the type-2 depositor whom we consider will not withdraw early because he can always receive d_2 at date 2, which is greater than d_1. On the other hand, if $q = q_L$, the consumption target T may have an impact on the depositor's withdrawing

[4] This assumption is reasonable if there is a small chance that the government will bail out banks at date 2.

decision. In this case, if $\pi \geq T - I,$ the depositor's total consumption is always larger than T. His utility is $I + d_1$ if he withdraws at date 1, and is $q_L (I + d_2) + (1 - q_L) (I + \pi)$ if he does not withdraw at date 1. He will not withdraw if and only if $q_L d_2 + (1 - q_L) \pi \geq d_1$, or equivalently,

$$\pi \geq \frac{d_1 - q_L d_2}{1 - q_L}. \tag{10}$$

Equation (10) says that the type-2 depositor will not withdraw at date 1 if π, the payoff that he receives in the bad state for not withdrawing early, is high enough.

Alternatively, suppose that $\pi < T - I$. By (3), the depositor's total consumption is higher than T if he withdraws at date 1. On the other hand, if he waits until date 2, his total consumption is higher than T if and only if the state is good. So, his utility for withdrawing at date 1 is $I + d_1$, and his utility for not withdrawing at date 1 is

$$q_L (I + d_2) + (1 - q_L) [I + \pi - k (T - I - \pi)].$$

He will not withdraw at date 1 if and only if

$$\pi \geq \frac{d_1 - q_L d_2 + (1 - q_L) k (T - I)}{(1 - q_L)(1 + k)}. \tag{11}$$

From these results, we can derive the relationship between the depositors' income I and the cutoff value of π below which the type-2 depositor will withdraw at date 1. First consider the case where $I \leq T - \frac{d_1 - q_L d_2}{1 - q_L}$, which implies[5]

$$\frac{d_1 - q_L d_2}{1 - q_L} \leq \frac{d_1 - q_L d_2 + (1 - q_L) k (T - I)}{(1 - q_L)(1 + k)} \leq T - I.$$

[5] It can be shown that

$$\frac{d_1 - q_L d_2 + (1 - q_L) k (T - I)}{(1 - q_L)(1 + k)} = \frac{1}{1 + k} \frac{d_1 - q_L d_2}{1 - q_L} + \frac{k}{1 + k} (T - I).$$

Therefore, the value of $\frac{d_1 - q_L d_2 + (1 - q_L) k (T - I)}{(1 - q_L)(1 + k)}$ is always between $T - I$ and $\frac{d_1 - q_L d_2}{1 - q}$.

In this case, if $\pi \geq T - I$, the type-2 depositor will never withdraw at date 1 because π is also larger than $\frac{d_1 - q_L d_2}{1 - q_L}$. If $\pi < T - I$, the depositor will not withdraw if and only if (11) holds. Therefore, when $I \leq T - \frac{d_1 - q_L d_2}{1 - q_L}$, the type-2 depositor withdraws at date 1 if and only if (11) is violated.

Next consider the case where $I > T - \frac{d_1 - q_L d_2}{1 - q_L}$, which implies

$$T - I < \frac{d_1 - q_L d_2 + (1 - q_L) k (T - I)}{(1 - q_L)(1 + k)} < \frac{d_1 - q_L d_2}{1 - q_L}.$$

In this case, if $\pi \geq T - I$, the depositor will not withdraw if and only if (10) holds. If $\pi < T - I$, the type-2 depositor always withdraws at date 1 because π is also smaller than $\frac{d_1 - q_L d_2 + (1 - q_L) k (T - I)}{(1 - q_L)(1 + k)}$. Therefore, when $I > T - \frac{d_1 - q_L d_2}{1 - q_L}$, the type-2 depositor withdraws at date 1 if and only if (10) is violated. These discussions are summarized in the following proposition. The proofs of all the propositions are in the Appendix.

Proposition 1. Suppose that $q = q_L$, and a type-2 depositor believes that (i) none of the other type-2 depositors at his bank will withdraw at date 2, (ii) he can always successfully withdraw at date 1, and (iii) he can always successfully withdraw at date 2 in the good state.

(a) He will not withdraw at date 1 if and only if $\pi \geq \pi^*$, where

$$\pi^* \equiv \begin{cases} \frac{d_1 - q_L d_2 + (1 - q_L) k (T - I)}{(1 - q_L)(1 + k)} & \text{if} \quad I \leq T - \frac{d_1 - q_L d_2}{1 - q_L}, \\ \frac{d_1 - q_L d_2}{1 - q_L} & \text{if} \quad I > T - \frac{d_1 - q_L d_2}{1 - q_L}. \end{cases} \tag{12}$$

(b) π^* is decreasing in I if $I \leq T - \dfrac{d_1 - q_L d_2}{1 - q_L}$, and is independent of I if $I > T - \dfrac{d_1 - q_L d_2}{1 - q_L}$.

Proposition 1 can be explained as follows. In our model, π is the amount of money that a type-2 depositor receives if he waits until date 2 and the state is bad. He will not withdraw at date 1 if π is high enough. Parts (a) states π^*, the cutoff value of π below which the type-2 depositor will withdraw early. Part (b) documents the relationship between π^* and I. When I is sufficiently high, meeting the consumption target is not a concern, so π^* is independent of I. When I is low, not withdrawing at date 1 is a riskier strategy because the depositor may suffer the penalty for failing to meet the consumption target in the bad state if he chooses this strategy. As a result, the depositor will have stronger incentives to withdraw when I is lower.

According to Proposition 1, in our model a bank run will occur to a bank and all of its institutional depositors will withdraw at date 1 if and only if $q = q_L$ and $\pi < \pi^*$. We may say that π^* represents the institutional depositors' degree of panic: the higher the π^*, the more panic are institutional depositors. Proposition 1 states that institutional depositors will become more panic when their incomes are lower.

3.2. The trading of the risky assets and the equilibrium

In addition to profits and losses from trading, banks have two concerns when they trade the risky asset at date 1. First, liquidity-deficient banks have to acquire enough liquidity to pay off withdrawals by type-1 depositors. The amount of money that a liquidity-deficient bank has to raise is no lower than $N(1 - s) f d_1$. Second, if $q = q_L$, banks have to avoid bank runs. The trading of the risky asset may affect whether a bank run occurs because it changes the composition of the banks' assets, thus the π discussed in Section 3.1. By Proposition 1, "no type-2 depositor withdraws at date 1" can be supported as part of an equilibrium if and only if π is no lower than π^*.

First consider the case where $q = q_L$. Define

$$Z_{LD} \equiv \begin{cases} 1 & if (1-b)R + b\,p + y - D \geq 0; \\ 0 & if (1-b)R + b\,p + y - D < 0. \end{cases} \tag{13}$$

The Z_{LD} in equation (13) is an indicator variable which equals 1 if the asset value of a liquidity-deficient bank is no lower than its liability when the state is good.[6] When $q = q_L$, if a liquidity-deficient bank expects that a bank run will not occur, its optimization problem can be written as[7]

$$Max_b\, V_{LD} \equiv q_L\,\{Z_{LD} N[(1-b)R + b\,p + y - D] - (1 - Z_{LD})\,X\} - (1 - q_L)\,X\,. \tag{14}$$

subject to

$$b\,p \geq (1 - s)\,f\,d_1; \tag{14a}$$

$$\pi_{LD} \equiv \frac{(1-b)\,r + y + b\,p - (1-s)\,f\,d_1}{(1-s)(1-f)} \geq \pi^*\,. \tag{14b}$$

In the above equations, V_{LD} is the payoff for a liquidity-deficient bank. Note that the possibility that liquidity-deficient banks cannot survive even in the good state has been considered when defining V_{LD}. As mentioned, if $Z_{LD} = 0$ in equilibrium, liquidity-deficient banks will take the actions that allow them to survive the longest time. Also, the first inequality in (9) guarantees that all the liquidity-deficient banks fail in the

[6] Equation (13) can be explained as follows. For each unit of risky asset it invests at date 0, a liquidity-deficient bank sells b units at the price p during the trading. After paying off type-1 depositors, the date-2 asset value of the liquidity-deficient bank is $N\,[(1 - b)\,R + b\,p + y - (1 - s)\,f\,d_1]$ if the state is good, and its liability at date 2 is $N\,[s + (1 - s)\,(1 - f)\,d_2]$. Therefore, the bank's asset value exceeds its liability if $(1 - b)\,R + b\,p + y \geq D$.

[7] As mentioned, a liquidity-deficient bank can pay off the deposits at date 2 in the good state if $N[(1 - b)\,R + b\,p + y - D]$. Also, by equation (9), the bank fails and suffers the bankruptcy cost X in the bad state. The probabilities of the good and bad state are q_L and $1 - q_L$, respectively. Using these facts, we have (14).

bad state.[8]

Condition (14a) requires that the proceeds from selling the risky asset are enough for paying off type-1 depositors. Condition (14b) states that π_{LD}, the π for a type-2 depositor at a liquidity-deficient bank, is no lower than π^*, so that a bank run will not occur.[9] Note that when computing π_{LD}, we have used the assumption that institutional depositors can withdraw before individual depositors at date 2.

Applying the similar logic, if a liquidity-rich bank expects that a bank run will not occur, its optimization problem can be written as[10]

$$Max_b\, V_{LR} \equiv q_L\, N[(1+b)R + y - b\,p - D] - (1-q_L)\,X \tag{15}$$

subject to

$$b\,p \leq y - (1-s)\,f\,d_1; \tag{15a}$$

$$\pi_{LR} \equiv \frac{(1+b)r + y - b\,p - (1-s)f\,d_1}{(1-s)(1-f)} \geq \pi^*. \tag{15b}$$

Equations (15a) and (15b) are similar to (14a) and (14b) except that liquidity-rich banks buy rather than sell the risky asset at date 1 and that liquidity-rich banks have the profits $y\,N$ from banking operations. The V_{LR} in (15) and π_{LR} in (15b) are the payoff for liquidity-rich banks and the π for their type-2 depositors, respectively.

Define

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8 As will be shown, the price p is never higher than R. The first inequality in (9) implies that liquidity-deficient banks will fail in the bad state even if they can sell the risky asset at the price R at date 1.

9 Equation (14b) can be explained as follows. At date 2, if the state turns out bad, the asset value in the bank is $N\,\{[y + b\,p - (1-s)\,f\,d_1] + (1-b)\,r\}$, and its liability to type-2 depositors is $N\,(1-s)\,(1-f)\,d_2$. It can be easily shown that given (9), the bank's asset value is lower than $(1-s)\,(1-f)\,d_2$. Therefore, under the assumption that institutional depositors withdraw before individual depositors do, the payoff for a type-2 depositor in the bad state is $\{[y + b\,p - (1-s)\,f\,d_1] + (1-b)\,r\}\,/\,(1-s)\,(1-f)$.

10 By (8) and the fact that the price of the risky asset will not exceed R, the liquidity-rich banks are always solvent in the good state if a bank run does not happen at date 1. Therefore, we need not consider the possibility that they cannot survive in the good state at date 2.

$$\pi_1 \equiv \frac{(y+r)-(2-\frac{r}{R})(1-s)\,f\,d_1}{(1-s)(1-f)}; \qquad (16)$$

$$\pi_2 \equiv \frac{(y+2r)-2(1-s)\,f\,d_1-\frac{r}{R}[s+(1-s)(1-f)d_2-y]}{(1-s)(1-f)}; \qquad (17)$$

$$\pi_3 \equiv \frac{(y+r)-(1-s)\,f\,d_1}{(1-s)(1-f)}. \qquad (18)$$

It can be easily shown that $\pi_1 < \pi_2 < \pi_3$ under our assumptions. By (14) and (15), we have the following proposition.

Proposition 2. Suppose that $q = q_L$.

(a) If $\pi^* > \pi_3$, no trading of the risky asset occurs at date 1, and a bank run occurs to all the banks.

(b) Trading of the risky asset occurs and no bank run happens if and only if $\pi^* \leq \pi_3$. The equilibrium price p is $p^*(\pi^*)$, where

$$p^*(\pi^*) \equiv \begin{cases} R & \text{if } \pi^* \leq \pi_1; \\ \dfrac{r(1-s)\,f\,d_1}{(1-s)[2\,f\,d_1+(1-f)\pi^*]-(y+r)} & \text{if } \pi_1 < \pi^* \leq \pi_3. \end{cases} \qquad (19)$$

p^* is decreasing in π^* and is increasing in s. Moreover, $p^*(\pi_1) = R$ and $p^*(\pi_3) = r$. The equilibrium b is $(1 - s)\,f\,d_1 / p^*$.

(c) If $\pi^* \leq \pi_2$, all the bank survive at date 2 in the good state, and fail in the bad state.

(d) If $\pi_2 < \pi^* \leq \pi_3$, liquidity-rich banks survive at date 2 if and if the state is good, while liquidity-deficient banks always fail at date 2.

Part (a) of Proposition 2 suggests that a bank run will occur to all the banks when institutional depositors are excessively panic. If they are not, then no bank run will occur at date 1, and banks will trade the risky asset. Even though no bank run occurs when $\pi^* \leq \pi_3$, the threat of bank runs has an impact on banks. Part (b) states that when the trading occurs, the equilibrium price p is between r and R, and is decreasing in π^*, the institutional depositors' degree of panic. This result can be explained as follows. For liquidity-rich banks, buying the risky asset at a price higher than r will reduce the type-2 depositors' payoff in the bad state. That is, by (15b), π_{LR} is decreasing in b when $p > r$. We may say that buying more of the risky asset makes liquidity-rich banks "look worse" in the bad state, thus increases their type-2 depositors' incentives to withdraw early. This concern reduces the willingness of liquidity-rich banks to buy the risky asset at date 1, and it is more serious when π^* is higher. As a result, the higher the π^*, the lower the liquidity-rich banks' demand for the risky asset, so the equilibrium price becomes lower. The depositor panic is not a concern when $\pi^* \leq \pi_1$. In this case, the price of the risky asset at date 1 equals R, the asset's return in the good state.

In the rest of the paper, we will say that a fire sale problem happens when $p^* < R$. By (14) and (15), banks always fail in the bad state, so they care only about the asset's return in the good state. Therefore, when $q = q_L$, the highest price that liquidity-rich banks are willing to pay for the asset in this case is still R even though the expected return of the risky asset is lower than R.[11] Following our definition, a fire sale problem occurs when $\pi^* > \pi_1$.

The fire sale problem may create inefficiencies. The lower the price, the higher is the quantity of the risky asset that liquidity-deficient banks have to sell to acquire the liquidity needed to meet the withdrawals by type-1 depositors. If the price is too low,

[11] When $q = q_L$, the expected return of the risky asset is $q_L R + (1 - q_L) r$.

liquidity-deficient banks would not have enough assets to pay off depositors at date 2 even if the state is good. As shown in part (d) of the proposition, when $\pi_2 < \pi^* \leq \pi_3$, liquidity-deficient banks will fail and incur the bankruptcy cost X at date 2 even in the good state.[12] Note that type-2 depositors at liquidity-deficient banks will not start a bank run in this case. By equation (9),

$$(1-s)(1-f)d_2 < y + (1 - \frac{(1-s)f d_1}{r})R,$$

which implies that type-2 depositors can be fully paid off at date 2 in the good state given the assumption that they can withdraw before individual depositors.

Proposition 2 has interesting implications. First, it proposes a new channel through which the low asset returns are contagious. By Proposition 1, the institutional depositors' degree of panic is decreasing in their incomes. Suppose that institutional depositors invest in other risky assets not considered in our model, and the returns of these assets are low. By reducing the institutional depositors' income and raising their degree of panic, the low returns of these assets will decrease the price of the risky asset in our model. A puzzle about the recent financial crisis is that it seems the subprime losses suffered by investors are not large enough to justify the seriousness of the crisis. Proposition 2 provides a possible explanation for the puzzle. It demonstrates the possibility that the low returns on subprime securities can become contagious and depress the returns of other assets.

Second, by part (b) of the proposition, p^* is increasing in s, which suggests that the fire sale and contagion problems stated above become more serious when deposits of institutional depositors are a more important funding source for banks. This result is intuitive. When s is higher, liquidity-rich banks are less worried about the withdrawals by

[12] However, in this case, liquidity-deficient banks will still trade at date 1 because doing so allows them to survive one more period.

institutional depositors, so their demand for the risky asset is higher. This result is consistent with the empirical results in He, Khang, and Krishnamurthy (2009) that financial institutions with more stable funding sources (that is, commercial banks) are net buyers of risky assets during the financial crisis of 2007-2009, while other financial institutions, such as investment banks, are net sellers.

Having analyzed the case of $q = q_L$, now consider the date-1 equilibrium when $q = 1$. If the state is always good, institutional depositors are not worried about the viability of banks, so they never start a bank run. In this case, the equilibrium (p, b) is $(R, (1 - s) f d_1 / R)$, and all the banks can survive at date 2.

4. Discussions

In this section, we explore the extensions of the model, and discuss several critical assumptions of the model. We first demonstrate how the fire sale problem will be affected by the volatility of the institutional depositors' income. We next show that the marking-to-market rule may be inadequate during the financial crisis period. In addition, we will study the possibility of reducing the fire sale problem by requiring banks to hold more liquid assets at date 0. Finally, we discuss key assumptions of the model and argue that they are reasonable.

In the following, we will show that the fire sale problem will become worse when the institutional depositors' income becomes more volatile. To illustrate this point, we modify the assumptions about the institutional depositors' income as follows. Suppose that I is a random variable when banks trade the risky asset at date 1: banks only know that $I = I_0 + v$ with probability 0.5 and $I = I_0 - v$ with probability 0.5, where v and I_0 are positive constants that satisfy

$$T - d_1 < I_0 - v < T - \frac{d_1 - q_L d_2}{1 - q}. \tag{20}$$

After banks trade the risky asset, the value of I is realized and institutional depositors decide whether to withdraw according to the realized I. All the other assumptions are kept unchanged.

Let π_0^* and π_L^* denote the π^* when I equals I_0 and $I_0 - v$, respectively. By (20) and Proposition 1, we know that $\pi_0^* < \pi_L^*$. The following proposition demonstrates that the uncertainty about the institutional depositors' income worsens the fire sale problem.

Proposition 3. Suppose that the assumptions about I are modified as above. If X is sufficiently large and $\pi_1 < \pi_L^* < \pi_3$, the equilibrium price of the risky asset is $p^*(\pi_L^*) < p^*(\pi_0^*)$. Moreover, the equilibrium price of the risky asset is decreasing in v.

Proposition 3 is intuitive. For liquidity-rich banks, when the bankruptcy cost X is sufficiently large, the profits from trading the risky asset are a second-order consideration compared with preventing a bank run. As a result, when liquidity-rich banks face uncertainty about π^*, they will behave as if the realized π^* is its highest possible value, π_L^*. If a liquidity-rich bank assumes a lower π^* when trading the risky asset, it will suffer a bank run once the realized π^* is π_L^*. Since π_L^* is increasing in v by Proposition 1, liquidity-rich banks will become more conservative when v is larger. Therefore, the equilibrium price of the risky asset is lower when the institutional depositors' income is more volatile.

Our model also implies that market value accounting may be inappropriate for evaluating banks when the fire sale problem is present. Suppose that all the assumptions are the same as those in Section 2, and the government applies the marking-to-market rule to judge whether banks are insolvent at date 1 after banks trade the risky asset. The following proposition shows that banks may be identified as insolvent even if they can survive at date 2 in the good state.

Proposition 4. Suppose that all the assumptions in Section 2 hold. Also, $\pi^* < \pi_3$, so banks trade the risky asset at date 1. If the government applies the marking-to-market rule to judge whether banks are insolvent after banks trade the risky asset, both liquidity-deficient and liquidity-rich banks will be identified as insolvent if and only if $\pi^* > \pi_{MM}$, where

$$\pi_{MM} \equiv \frac{(y+r)-(2-\frac{r}{D-y})(1-s)f\,d_1}{(1-s)(1-f)}. \tag{21}$$

Moreover, $\pi_1 < \pi_{MM} < \pi_2$.

By Proposition 4, liquidity-rich banks will be identified as insolvent when $\pi^* > \pi_{MM}$ even if they can survive at date 2 in the good state. Also, when $\pi_{MM} < \pi^* < \pi_2$, liquidity-deficient banks can survive at date 2 in the good state, but they will be identified as insolvent by market value accounting. If the fire sale problem is present and allowing banks to keep in operation does not trigger serious moral hazard problems on the bank side, it is inadequate to close the banks just because the market values of their assets are smaller than their liabilities.

In our model, we assume that banks invest all the deposits in the risky asset at date 0.

If they keep some of the deposits as cash rather than investing all of them in the risky asset, there will be more liquidity in the system, so the fire sale problem should become less serious. To investigate this issue, suppose that at date 0, each bank holds fraction m of the deposits as cash, and invest the remaining deposits in the risky asset, where

$$0 < m < (1 - s) f d_1. \tag{22}$$

If m is greater than $(1 - s) f d_1$, liquidity-deficient banks will no longer need to sell the risky asset at date 1. Also, equations (8) and (9) are modified as

$$(1 - m)R + m + y > D; \tag{23}$$

$$\begin{aligned} y + m + (1 - m - \frac{(1-s) f d_1}{R}) r < (1-s)(1-f) d_2 < \\ y + m + (1 - m - \frac{(1-s) f d_1}{r}) R < s + (1-s)(1-f) d_2. \end{aligned} \tag{24}$$

Define

$$\pi_{1m} \equiv \pi_1 + \frac{m(2 - r - \frac{r}{R})}{(1-s)(1-f)}; \tag{25}$$

$$\pi_{2m} \equiv \pi_2 + \frac{m[2(1-r) + \frac{r}{R}]}{(1-s)(1-f)}; \tag{26}$$

$$\pi_{3m} \equiv \pi_3 + \frac{m(1-r)}{(1-s)(1-f)}. \tag{27}$$

It can be easily shown that $\pi_{1m} < \pi_{2m} < \pi_{3m}$. Applying the similar logic for proving Proposition 2, we have the following proposition.

Proposition 5. Suppose that $q = q_L$, and each bank keeps fraction m of the deposits as cash at date 0, where m satisfies equation (22).

(a) If $\pi^* > \pi_{3m}$, no trading of the risky asset occurs at date 1, and a bank run occurs to all the banks.

(b) Trading of the risky asset occurs and no bank run happens if and only if $\pi^* \leq \pi_{3m}$. The equilibrium price p is $p_m^*(\pi^*)$, where

$$p_m^*(\pi^*) \equiv \begin{cases} R & \text{if } \pi^* \leq \pi_{1m}; \\ \dfrac{r[(1-s)f d_1 - m]}{(1-s)[2 f d_1 + (1-f)\pi^*] - (y+r) - m(2-r)} & \text{if } \pi_{1m} < \pi^* \leq \pi_{3m}. \end{cases} \quad (28)$$

p_m^* is decreasing in π^* and is increasing in s and m. Moreover, $p_m^*(\pi_{1m}) = R$ and $p_m^*(\pi_{3m}) = r$. The equilibrium b is $[(1-s) f d_1 - m] / p_m^*$.

(c) If $\pi^* \leq \pi_{2m}$, all the bank survive at date 2 if the state is good, and fail if the state is bad.

(d) If $\pi_{2m} < \pi^* \leq \pi_{3m}$, liquidity-rich banks survive at date 2 if and if only the state is good, while liquidity-deficient banks always fail at date 2.

(e) For i = 1,2,3, π_{im} is increasing in m, so $\pi_{im} > \pi_i$. Moreover, $p_m^*(\pi^*) > p^*(\pi^*)$ if $\pi_1 < \pi^* < \pi_3$.

Proposition 5 is a generalization of Proposition 2, where the latter is a special case of Proposition 5 when $m = 0$. Comparing these two propositions, we know that requiring banks to hold cash or other liquid assets at date 0 alleviates the fire sale problem. Because $p_m^*(\pi^*) > p^*(\pi^*)$, the price of the risky asset becomes higher. Also, by the fact that $\pi_{2m} > \pi_2$, liquidity-deficient banks are more likely to survive at date 2 in the good state. The liquidity requirement affects the banks' payoffs in the following ways. First, by reducing the amount of money that liquidity-deficient banks have to finance from selling the risky asset, the requirement lowers the supply of the risky asset. Second, by increasing the proportion of the risk-free asset for liquidity-rich banks, the requirement reduces the institutional depositors' degree of panic, thus increases the liquidity-rich banks' demand for the risky asset. Third, because the rate of return on cash is zero, requiring banks to hold cash reduces the banks' profits at date 2 in the good state. The first two effects increase the price of the risky asset, while the third one makes banks (especially liquidity-deficient banks) more vulnerable in the good state. Proposition 5 says that the first two effects dominate, so liquidity-deficient banks are more likely to survive in the good state when banks are required to hold cash or other liquid assets at date 0.

The determination of the optimal m is beyond the scope of this paper. It will depend on the tradeoff between the social gains from alleviating the fire sale problem and the lower bank profits in the good state. The optimal m will be higher if the fire sale problem is more serious (that is, η and/or q_L is lower) or R, the return of the risky asset in the good state, is lower. An interesting implication of our model is that the optimal m should be decreasing in s. Because both keeping funding sources more stable and holding more liquid assets can reduce the fire sale problem, our model implies that the liquidity requirement should be stricter for banks that rely more heavily on non-core or wholesale deposits.

Having discussed the applications of the model, in the rest of this section we discuss certain critical assumptions of our model. We make several assumptions to simplify the exposition. Relaxing them will not change our main results. By equation (3), we assume that $T < d_1 + \underline{I}$, so $I + d_1$ is always higher than T. By this assumption, we get the result that π^* is decreasing in I when I is low. This result will no longer hold if equation (3) is relaxed. To see this, consider the case where $q = q_L$ and $I + d_1 < T < I + d_2$. In this case, a type-2 depositor's payoff is $(1 + k)(I + d_1 - T)$ if he withdraws early, and is

$$q_L (I + d_2 - T) + (1 - q_L) (1 + k) (I + \pi - T)$$

if he waits until date 2. He will not withdraw early if and only if

$$\pi \geq \frac{d_1 - q_L d_2 + k[d_1 - q_L(T - I)]}{(1 - q_L)(1 + k)}. \tag{29}$$

Note that the right-hand side of (29) is *increasing* rather than *decreasing* in I, which means a decrease in I may *reduce* the type-2 depositors' incentives to withdraw early. This is because given $I + d_1 < T < I + d_2$, a type-2 depositor always suffers the penalty for not achieving the consumption target if he withdraws at date 1, and he can avoid this penalty if he waits until date 2 and the state is good. In this case, a decrease in I increases the disadvantage of withdrawing early, thus reduces the depositor's incentives to withdraw at date 1.[13]

However, we think that it is reasonable to assume equation (3) so that depositors are more "panic" when I is lower. For institutional depositors such as hedge funds, a lower I means worse financial performance and therefore greater redemption pressure. To make sure that they do not suffer further losses from deposits would become more important. If I is so low that $I < T - d_1$, the redemption pressure may push the institutional depositors to withdraw early anyway whatever the financial conditions of their banks are. Therefore, for our purpose, equation (3) is a reasonable assumption.[14]

In the paper, we assume that liquidity-deficient banks acquire liquidity through selling the risky asset. Our results will not change if liquidity-deficient banks borrow

[13] If I becomes even lower so that $I + d_2 < T$, type-2 depositors will be penalized for not achieving the consumption target whatever they do at date 1. In this case, I will not affect the cutoff value of π under which type-2 depositors will withdraw early. Type-2 depositors will not withdraw early if and only if (10) holds.

[14] As Goldstein and Pauzner (2004), an alternative way to make institutional depositors more panic when their income is lower is to assume that institutional depositors have a decreasing relative risk aversion utility function.

from liquidity-rich banks at date 1. Liquidity-rich banks will suffer losses if they lend to liquidity-deficient banks at date 1 and the state turns out bad.[15] Therefore, for liquidity-rich banks, lending to liquidity-deficient banks also make them "look worse" in the bad state. As a result, they may be reluctant to lend when the threat of a bank run by institutional depositors is a concern.

An important assumption of the model is that institutional depositors will choose the Pareto dominant equilibrium when there are multiple equilibria. If this assumption is relaxed, our main results will still hold as long as institutional depositors have stronger incentives to withdraw early when π is lower and they choose the same cutoff value of π when deciding whether to withdraw early. In this case, a bank run will occur if and only if $\pi < \pi^* + \varepsilon$, where $\varepsilon \geq 0$ represents the degree of inefficiency of the bank runs. When ε is higher, the liquidity-rich banks' demand for the risky asset will be lower, and the fire sale problem will be more serious.

5. Concluding Remarks

This paper demonstrates that asset fire sales can happen even if the aggregate supply of liquidity in the financial system exceeds the aggregate liquidity demand. It identifies the conditions under which the fire sale problem will be more serious, and provides an explanation for why low asset returns are contagious.

This paper can be extended in several directions. In this paper, we investigate the asset fire sale problem. The model can be easily modified to study other important

[15] Suppose that a liquidity-rich bank lends money to a liquidity-deficient bank at date 1. If the liquidity-deficient bank fails at date 2, the lending liquidity-rich bank has to compete with other creditors (especially institutional depositors) of the liquidity-deficient bank for getting paid. It is likely that the liquidity-rich bank will not be fully paid, and will suffer losses.

features of liquidity crises. For example, our model can be used to study market freeze. If the probability that a bank will need the liquidity to pay off early withdrawing depositors is strictly less than 1, then the liquidity-deficient banks may be reluctant to sell the risky asset at low prices. This will result in market freeze.

Another promising direction to extend the model is to discuss which measures the government should use when a liquidity crisis occurs. If asset fire sales and market freeze happen because potential asset buyers are reluctant to buy due to the threat of a creditor run, then according to our model, providing liquidity to potential asset sellers may be more effective than providing liquidity to potential asset buyers for alleviating the crisis. In addition, improving the funding stability of financial institutions (such as offering government guarantees on the financial institutions' liabilities) may also help for reducing liquidity problems. However, these mechanisms have their costs. Studying the optimal mechanisms for solving liquidity crises will require a careful investigation that takes into consideration all the important benefits and costs of the possible mechanisms.

Appendix

Proof of Proposition 1.

Part (a) of the proposition follows directly from the discussions in the text preceding the proposition. Part (b) is obvious from equation (12). **Q.E.D.**

Proof of Proposition 2.

For part (a), when $\pi^* > \pi_3$, (14b) cannot be satisfied if $p \le r$, and (15b) cannot be satisfied if $p \ge r$. Therefore, no trading will occur at date 1. A bank run will occur to all the banks because both (14b) and (15b) are violated when $b = 0$.

Now consider the case where $\pi^* \le \pi_3$, so it is possible that both (14b) and (15b) are satisfied. Note that the equilibrium price is never higher than R, the risky asset's return in the good state. Also, it will not fall below r. If it is strictly smaller than r, by (15), (15a), and (15b), buying the risky asset will increase the liquidity-rich banks' payoff without violating any constraint. Given the assumption that $y > 2\,(1-s)\,f\,d_1$, there will be excess demand for the risky asset and the price will rise.

Given the result that $r \le p \le R$, we know that (14b) does not bind when (15b) is satisfied. Also, because all the banks will fail in the bad state, the value of a unit of risky asset is R to both banks. Therefore, when $p < R$, the quantity of the risky asset that each liquidity-rich bank is willing to buy is the largest one that does not violate (15a) and (15b), and the quantity that each liquidity-deficient bank is willing to sell is the smallest one that can satisfy (14a). Using these results, it can be easily verified that the binding constraints are (14a) and (15b) when $p < R$, which implies

$$p^*(\pi^*) = \max\{\min\{R, \frac{r(1-s)\,f\,d_1}{(1-s)[2\,f\,d_1 + (1-f)\,\pi^*] - (y+r)}\}, r\}\,. \tag{A1}$$

By (A1), we have equation (19). The remaining results in part (b) are obvious from (19).

For parts (c) and (d), suppose that $\pi_1 \le \pi^* \le \pi_3$. Liquidity-deficient banks will not fail in the bad state if and only if

$$(1-b^*)\,r + y + b^*\,p^* - D \ge 0. \tag{A2}$$

Using the results that $b^* = (1-s)\,f\,d_1 / p^*$ and that $p^* \le R$, the first inequality of (9) implies that (A2) does not hold, so liquidity-deficient banks will fail in the bad state. Because the date-2 asset value of a liquidity-rich bank is no larger than that of a liquidity-deficient bank in the bad state given $p^* \ge r$, liquidity-rich banks will also fail at date 2 in the bad state.

As to whether banks can survive at date 2 in the good state, liquidity-rich banks can by equation (8) and the fact that $p^* \le R$. Liquidity-deficient banks can survive in the good state if and only if

$$(1 - b^*)\, R + y + b^*\, p^* - D \geq 0,$$

which is equivalent to $\pi^* \leq \pi_2$. This completes the proof of Proposition 2. **Q.E.D.**

Proof of Proposition 3.

As shown in the proof of Proposition 2, (15b) binds in equilibrium when $\pi_1 < \pi_L^* < \pi_3$, so a bank run will occur to a liquidity-rich bank if the bank assumes that $I > I_0 - v$ when trading the asset but the realized I is $I_0 - v$. When X is sufficiently large, in order to avoid the bank failure caused by a bank run, liquidity-rich banks will behave as $I = I_0 - v$ when trading the risky asset. This completes the proof of the proposition. **Q.E.D**.

Proof of Proposition 4.

When evaluated at the market price p^*, both liquidity-rich and liquidity-deficient banks are insolvent if and only if $p^* + y - D < 0$, which is equivalent to $\pi^* < \pi_{MM}$. By (16), (17), and (21), it can easily verified that $\pi_1 < \pi_{MM} < \pi_2$. This completes the proof of the proposition. **Q.E.D**.

Proof of Proposition 5.

Given m, equations (13) to (15) become

$$Z_{LD-m} \equiv \begin{cases} 1 & \text{if } m + (1-b-m)R + b\,p + y - D \geq 0; \\ 0 & \text{if } m + (1-b-m)R + b\,p + y - D < 0. \end{cases} \tag{A3}$$

$$Max_b\, V_{LD} \equiv q_L\, \{Z_{LD} N[m + (1-b-m)R + b\,p + y - D] - (1 - Z_{LD})X\} - (1 - q_L)\, X\,. \tag{A4}$$

subject to

$$m + b\,p \geq (1 - s)\, f\, d_1; \tag{A4a}$$

$$\pi_{LD-m} \equiv \frac{m + (1-b-m)\,r + y + b\,p - (1-s)\,f\,d_1}{(1-s)(1-f)} \geq \pi^*. \tag{A4b}$$

$$Max_b\, V_{LR} \equiv q_L\, N[m + (1+b-m)R + y - b\,p - D] - (1 - q_L)\, X \tag{A5}$$

subject to

$$b\,p \leq m + y - (1 - s)\, f\, d_1; \tag{A5a}$$

$$\pi_{LR-m} \equiv \frac{m + (1+b-m)\,r + y - b\,p - (1-s)\,f\,d_1}{(1-s)(1-f)} \geq \pi^*. \tag{A5b}$$

Proposition 5 can be easily proved by applying the same logic for proving Proposition 2.

Date 0	Date 1	Date 2
- Institutional depositors receive the endowed deposits. - Each bank collects deposits of N dollars, and invests the deposits in a risky asset.	- The value of q, the banks' types, and the institutional depositors' income I are revealed. Each liquidity-rich bank receives profits $y\,N$. - Banks trade the risky asset. - The types of the institutional depositors are revealed. Institutional depositors decide whether to withdraw and how much they consume at date 1. A bank fails if it is unable to satisfy withdrawals.	- Each liquidity-deficient bank receives $y\,N$ if it does not fail at date 1. - The risky asset matures. The return of the risky asset becomes common knowledge. - All the depositors who have not withdrawn withdraw. - Institutional depositors receive their incomes, and consume all the money they have.

Figure 1. The Time Line of the Basic Model

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