Modeling the Dynamics of Correlations Among International Equity Volatility Indices

Moloud Rahmaniani[1]
Department of Accountancy and Finance
Otago Business School, University of Otago
Dunedin 9054, New Zealand
Email: moloud.rahmaniani@otago.ac.nz

Jin E. Zhang
Department of Accountancy and Finance
Otago Business School, University of Otago
Dunedin 9054, New Zealand
Email: jin.zhang@otago.ac.nz

Helen Roberts
Department of Accountancy and Finance,
University of Otago, PO Box 56,
Dunedin 9054, New Zealand
helen.roberts@otago.ac.nz

First Version: March 31, 2014

Current Version: November 28, 2014

Keywords: Volatility Index; Multivariate GARCH; Correlation; Financial Integration

JEL Classification: G13, F36, F37, G15

[1]Corresponding Author, Tel: +64 3 479 9138

Modeling the Dynamics of Correlations Among International Equity Volatility Indices

Abstract. This study introduces a one factor model to examine the correlation of volatility markets. We show that for markets where there is a higher level of stock market integration there is a correspondingly higher degree of volatility market integration. Our findings suggest that investors' expectations about future uncertainty are highly integrated. Applying the dynamic conditional correlation model developed by Engle (2002) to ten volatility indices across different countries we show that there is a positive and time varying correlation between all volatility indices and that the correlation with the underlying equity market index is one factor associated with volatility market integration.

1. Introduction:

This study investigates the time varying correlation of volatility markets. We measure dynamic asset correlations by applying the Dynamic Conditional Correlation (DCC) model (Engle, 2002) to implied volatility indices from around the world. We ask two research questions in this paper: (i) what is the relationship between stock market integration and volatility market integration? (ii) Is correlation in the underlying stock market index the only factor driving volatility market integration? We use a one factor GJR-GARCH/DCC model to examine the correlation between volatility and corresponding underlying equity indices. Our study uses a different methodological approach from those in the existing literature by modelling the dynamics of correlations of implied volatility indices jointly with the volatility of the series and the corresponding underlying market. We show that after controlling for the relationship between the volatility index and its corresponding underlying, there is still positive time-varying correlation within volatility markets.

Investors' expectations can change dramatically following significant decreases in the value of financial markets. Correlations between financial market changes have been considered as one factor in explaining the impact of shocks to financial markets during periods of greater turbulence (Silvennoinen and Thorp, 2013). While increased market integration gives investors greater opportunities for diversification; exposure to systemic risk also increases as market prices move more closely together in response to new information. Therefore models derived from investor expectations in volatility markets that use time varying correlations are more applicable during periods of greater uncertainty in markets.

Increasing financial market integration in response to globalization and the evolution of interdependence between and within markets is becoming an important consideration for investors and policy makers alike. Pressure to regulate markets to limit downside risk following the financial crisis has contributed to this response. Numerous studies examining stock market integration find that stock market integration has increased in recent years (Sharma & Seth 2012). In contrast, volatility market integration is a fairly new area of research in the finance literature. In particular, it is not yet clear how market participants interpret higher volatility in other markets when forming expectations about future fluctuation. Volatility prediction using information from other markets can result in better risk management and vega hedging (Siriopoulos and Fassas, 2013). Therefore time varying correlation models of volatility indices can provide market participants with relevant information to better hedge and manage portfolio positions.

Correlation increases during periods of high market volatility (Cappiello et al., 2006). As markets become more volatile investors require better diversification. Investment strategies based on simple correlation estimation techniques do not perform well during periods of market turbulence and expose market participants to correlation risk. Any financial position that incorporates returns from more than one underlying asset will have prices that are sensitive to the correlation between the underlying asset returns (Engle, 2002). Hence, the reliable forecasting of future correlations and volatilities is the basis of any pricing formula.

Worldwide volatility markets share several common characteristics. Siriopoulos and Fassas (2009) provide evidence of a positive correlation between volatility indices and a simultaneous negative correlation between volatility indexes and the corresponding underlying market index; we also report evidence consistent with these results. Increased integration between stock markets and the strong negative association between changes in the underlying equity market index and the volatility market index suggest that there should be a positive association between volatility indexes over time. Implied volatility indices are measures of aggregate investor expectation. They are a valuable source of information for international investors seeking to diversify existing portfolios. These indices can also be used by policy makers to estimate the effect of potential changes in macroeconomic policy on market risk and to their trade partners based upon adoption. The transmission of uncertainty has become increasingly important in developing an understanding of future market movement.

This study examines the correlation for ten implied volatility indices from six countries (USA, UK, Germany, France, Japan, and Netherlands). The countries and their associated volatility indices are given in Table IX in Appendix I. Figure I shows ten graphs, one for each of the implied volatility indices in our study covering the period January, 2004 to July, 2014. The shaded areas on each graph indicate global co-movement between the respective indices during periods of high volatility providing initial evidence of potential volatility market integration.

The paper proceeds as follows. Section II discusses the literature related to dynamic correlation estimation and introduces the model for the dynamic conditional correlations (DCC) between volatility indices; Section III discusses the data and the empirical results; Section IV concludes.

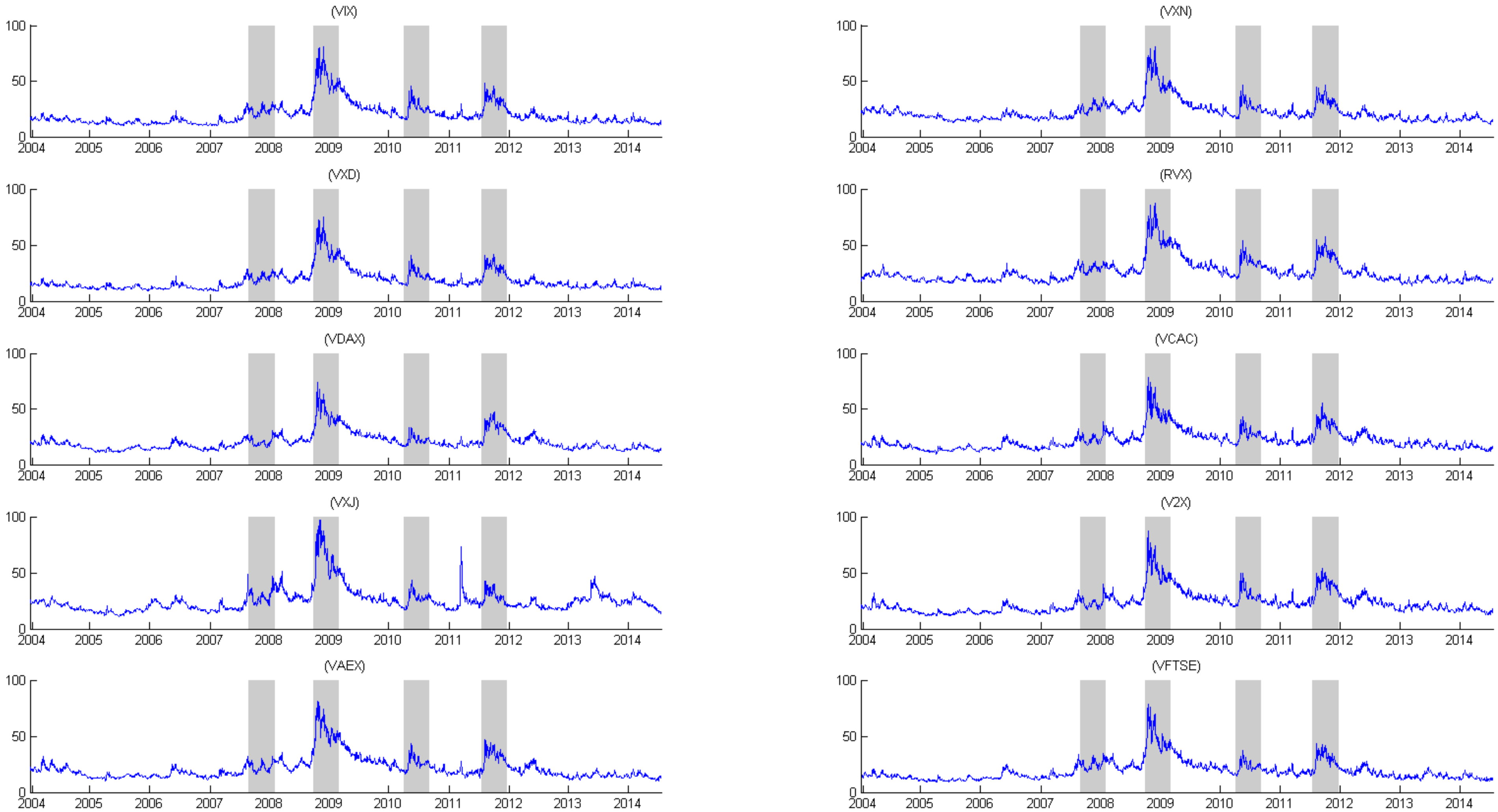

Figure I. Implied Volatility Indices, January, 2004- July, 2014

The shaded areas indicate a significant increase in volatility due to the bankruptcy of Lehman Brothers in 2008 and European financial turbulence in 2011, respectively.

2. Review of the literature

Implied volatility is a measure of market participants' expectations about future market uncertainty. The Black-Scholes (BS) option pricing model was first used to estimate implied volatility (IV) in 1993 by CBOE, but it was not successful. In 2003, CBOE re-launched VIX (the new VIX) using a model-free volatility index based on the work by Demeterfi et al. (1999).[2]

However, stochastic variation in volatility produces risk for a delta hedged position (Engle and Figlewski, 2014).[3] Market participants require models that can capture the dynamic nature of market volatility and also account for gamma risk. Using Dynamic Conditional correlation (DCC) methodology research has shown that market return correlation evolves over time. It is a dynamic mechanism, constantly responding to new information, impacting on market participant's expectations regarding future uncertainty. Examples include the work by Campell et al. (2002) who report that market correlations increased during the bear market. Yang (2005) uses the same correlation methodology applied to Japan and four Asian stock markets and finds that correlations increase during periods of market turbulence. Creti et al. (2013) report that the correlation between 25 commodities and the S&P 500 is time varying and experiences extreme fluctuations in some periods. Hau and Rey (2006) explain the observed correlation between 17 OECD markets using a risk-rebalancing hypothesis to model incomplete hedging of forex risk.

The EGARCH/DCC model proposed by Engle and Figlewski (2014) captures changes in the covariance between the implied volatility (IV) from stocks. Their study looks at the volatility dynamics and correlation of twenty-eight large cap stocks listed on the NYSE. They use VIX as a measure of aggregate market IV and form a one-factor model to explain idiosyncratic risk. An earlier DCC model developed by Siriopoulos and Fassas (2013) finds that volatility markets demonstrate significant integration. During periods of financial market turbulence they find that the conditional correlations across implied volatility indices increase. Badshah, Frijns and Tourani-Rad (2013) investigate contemporaneous spill-over effects among the implied volatility indices for stocks (VIX), gold (GVZ), and the exchange

[2] Siriopoulos and Fassas (2013) give a detailed account of the historic development of volatility indices based on option market prices.

[3] The realized volatility for the underlying stock may be larger or smaller than expected, causing hedging errors determined by the option's gamma. Second, shocks to the market's volatility risk premium also change option prices. An option writer who hedges delta, gamma and theta will still experience a mark to market loss on the position following an increase in implied volatility.

rate (EVZ) and find asymmetric volatility transmission from stock market to gold and exchange rate volatility markets.

Table I presents a chronological summary of papers that have looked at research questions about volatilities indices. The literature is relatively recent with the first paper by Äijö (2008) examining the linkage between volatility indices. All of the studies use vector autoregressive (VAR) models with different lags except for the one factor model introduced by Engle and Figlewski (2014). All of the studies prior to 2014 use relatively short data periods spanning at most 5 years. Our study, like the work by Engle and Figlewski (2014), looks at correlation dynamics rather than volatility transmission and spillover effects. We contribute to this literature by introducing a model to investigate correlation between volatility markets across countries.

This paper introduces a new one factor model to explain changes in volatility indices using data from January, 2004 to July, 2014 for ten different volatility indices. Our study shows that the correlation of volatility indices changes significantly over time and consistent with the work by Siriopoulos and Fassas (2013), we confirm that volatility markets are more correlated in periods of turbulence. We find that the correlation of volatility indices changes significantly over time and that volatility markets with higher correlation in the corresponding underlying stock market are highly integrated with their respective equity markets.

Table I. Summary of the literature concerning volatility market integration

Year	Author	Article	Subject	Model	Data	Terminology
2008	Äijö	Implied volatility term structure linkages between VDAX, VSMI and VSTOXX volatility indices	VDAX, VSMI, VSTOXX	VAR	Jan 2000 - Dec 2004	Linkage between volatility indexes
2009	Siriopoulos, Fassas	Implied Volatility Indices – A review	VIX, VXN, MVX, VSTOXX, VFTSE, VSMI	VAR	Jan 2004- Dec 2008	Volatility spillover effects/transmission effects of the volatility
2012	Siriopoulos, Fassas	An investor sentiment barometer — Greek Implied Volatility Index (GRIV)	GRIV,VIX,VDAX	VAR	Jan 2004– Dec 2009	Volatility transmission effects
2012	Kumar	A first look at the properties of India's volatility index.	IVIX, VIX, VFTSE, VXJ	VAR	Nov 2007 -May 2010	Volatility transmission
2012	López, Navarro	Implied volatility indices in the equity market: A review	VIX,VXN,VXD,RVX,VDAX,VAEX,VBEL,VCAC,VFTSE,VSMI,VSTOXX,MVX	No model	Jan 2004-Dec 2008	Spillover effect
2013	Liu, Ji, Fan	How does oil market uncertainty interact with other markets? An empirical analysis of implied volatility index	OVX,VIX,EVZ,GVZ	VAR	Jun 2008- Jul 2012	Uncertainty transmission
2013	Badshah, Frijns, Tourani Rad	Contemporaneous spill-over effects among the implied volatility indices for stocks (VIX), gold (GVZ), and the exchange rate (EVZ).	VIX,GVZ,EVZ	SVAR	Jun 2008 - Dec 2011	Spillover effect
2013	Siriopoulos, Fassas	Dynamic relations of uncertainty expectations: a conditional assessment of implied volatility indices	VIX, MVX, RVX, VXD,VXN, VAEX, VBEL, VCAC, VDAX, VFTSE, VSTOXX, VSMI, VXJ	VAR	Jan 2004- Jul 2009	Transmission of uncertainty/Volatility spillover effects
2014	Engle, Figlewski	Modelling the Dynamics of Correlations Among Implied Volatilities	VIX and 28 stocks listed on the NYSE	One- Factor Model	Jan1996- Feb 2009	Dynamics of correlations

3. Methodology

Constant correlation models assume that the correlation of each pair of assets is constant and time invariant. As a result they cannot capture any of the co-movement experienced by a financial time series typically reflected in real market data. Evidence of volatility market integration suggests that investors' expectations about future uncertainty move together when explained as a dynamic conditional correlation model (Siriopoulos and Fassas, 2013). Using the dynamic conditional correlation (DCC) model introduced by Engle (2002) we investigate the evolution across time of the correlation between different volatility indices. The DCC model is a generalization of Bollerslev's (1990) constant conditional correlation (CCC).

The volatility index return, $\log(\frac{v_{i,t}}{v_{i,t-1}})$ for the underlying equity index $S_{i,t}$ is assumed to be a zero-mean random variable such that changes in log $v_{i,t}$ follow the model:

$$\log(\frac{v_{i,t}}{v_{i,t-1}}) = \varphi_{i,t} + \sigma_{i,t}\varepsilon_{i,t} \qquad \varepsilon_{i,t} \sim IID(0,1) \tag{1}$$

Where

$\varepsilon_{i,t}$ are independent and identically distributed errors that follow a multivariate normal distribution for cross sectional markets

The functions $(\varphi_{i,t}, \sigma_{i,t})$ are the conditional mean and standard deviation of volatility index return and can be estimated using information at (*t-1*). In this paper we use two models to define $\varphi_{i,t}$. The first, simple model uses following equation;

$$log\left(\frac{v_{i,t}}{v_{i,t-1}}\right) = \beta_{1i} + \beta_{2i}log\left(v_{i,t-1}\right) + \sigma_{i,t}\varepsilon_{it} \tag{2}$$

Siriopoulos and Fassas (2009) show that there is a strong negative relationship between volatility indices and the corresponding underlying stock market index. Using this result we build a second model to measure the relationship between changes in the volatility index as a function of the market's corresponding underlying stock market index, and $\varphi_{i,t}$ for each index is defined by equation (3):

$$log\left(\frac{v_{i,t}}{v_{i,t-1}}\right) = \beta_{1i} + \beta_{2i}log\left(\frac{S_{i,t}}{S_{i,t-1}}\right) + \beta_{3i}log\left(v_{i,t-1}\right) + \sigma_{i,t}\varepsilon_{it} \tag{3}$$

The conditional variance of the log of the volatility index $\sigma_{i,t}^2$ is a measure of the volatility of the volatility index i.

Engle and Figlewski (2014) and Siriopoulos and Fassas (2013) both report evidence of an asymmetric effect when estimating the volatility of the implied volatility. To estimate conditional volatility of each index we use the GJR-GARCH volatility process:

$$\sigma_{i,t}^2 = \emptyset_{1i} + \emptyset_{2i}\varepsilon_{i,t-1}^2 + \emptyset_{3i}\varepsilon_{i,t-1}^2[\varepsilon_{i,t-1} < 0] + \emptyset_{4i}\sigma_{i,t-1}^2 \tag{4}$$

The conditional correlation for two assets based on the implied log volatility indices $\rho_{ij,t}$, is given by

$$\rho_{ij,t} = \frac{E_{t-1}(\log(v_{i,t}/v_{i,t-1}) - \varphi_{i,t})(\log(v_{j,t}/v_{j,t-1}) - \varphi_{j,t})}{\sqrt{Var_{t-1}\log(v_{i,t}/v_{i,t-1})\, Var_{t-1}\log(v_{j,t}/v_{j,t-1})}} \quad (5)$$

Substituting Equation (3) into Equation (1) allows us to derive equation (6):

$$\rho_{ij,t} = E_{t-1}(\varepsilon_{i,t}\varepsilon_{j,t}) \quad (6)$$

Using the DCC model (Engle, 2002) we can estimate the time varying correlation between volatility indices. This model consists of three stages. First, we use GJR-GARCH (1,1) to estimate volatilities and construct standardized residuals; this is also referred to as "DEGARCHING" the data. This standardized data has a volatility of one and by using the standardized residuals in the estimation we overcome any concerns regarding nonlinearity in calculating the correlation. Second, the quasi-correlations are estimated in a dynamic fashion using the standardized residuals.

$$\overline{R} = \frac{1}{T}\sum_{1}^{T} \varepsilon_i \varepsilon_j \quad (7)$$

$$Q_t = \overline{R} + \alpha(\varepsilon_{t-1}\varepsilon_{t-1}' - \overline{R}) + \theta(Q_{t-1} - \overline{R}) \quad (8)$$

Where

$\overline{\mathrm{R}}$ is the unconditional correlation of the ε_i's as presented in Table IV. Third, the estimated quasi-correlation matrix (8) must be rescaled as shown in equation (9):

$$\mathrm{R_t} = \mathrm{diag(Q)}^{-1/2}\mathrm{Q_t}\mathrm{diag(Q)}^{-1/2} \quad (9)$$

This rescaling is ensures that the values in the correlation matrix all lie within -1 and +1.

4. Data

This study uses a daily time series for 10 volatility indices covering the period from January 5, 2004 to July 22, 2014 (source: Bloomberg and DataStream, Thomson Financial). Using this sample we investigate the dynamic relationship between returns on volatility indices and the corresponding underlying equity indices for the USA, United Kingdom, Germany, France, Japan, and Netherlands.

4.1 Common features of volatility and equity indices

Table II summarizes sample statistics for the various indices included in our study. The reported statistics show that volatility indices are similar in terms of the mean, standard deviation, range, skewness and kurtosis measures exhibited over the sample period. The values reported in Table II show that the volatility indices exhibit positive skewness, high kurtosis and have a negative correlation with their own underlying equity indices. Mean volatility levels lie between 19.3 to 26.9and have a standard deviation of 8.0 to 12.9. These findings are common features of all volatility markets.[4]

Table II. Summary of volatility indices from January 2004 to July 2014

This table reports the mean, standard deviation, maximum, minimum values together with the skewness and kurtosis measures for each of the implied volatility indices in the study from January 5, 2004 to July 22, 2014. Panel B reports summary statistics for log return measures on each of the 10 volatility indices. The last column shows unconditional correlation between each volatility index and its corresponding underlying index

INDEX	Mean	Std. Dev.	Min	Max	Skewness	kurtosis	Unconditional Correlation with underlying
Panel A: Volatility Index Level							
VIX Index	20.04	8.04	9.31	80.86	2.05	10.31	-
VXN Index	26.98	12.95	11.36	80.64	1.33	4.04	-
VXD Index	19.46	8.67	9.28	74.60	1.94	8.43	-
RVX Index	26.02	10.36	13.65	87.62	2.17	8.83	-
VDAX Index	20.46	8.00	10.88	74.00	2.13	9.01	-
VAEX Index	21.90	8.83	9.24	78.05	2.02	8.93	-
VCAC Index	25.19	10.62	10.97	97.27	2.76	13.71	-
VFTSE Index	23.30	9.60	11.60	87.51	2.01	8.68	-
VXJ index	21.48	9.93	10.12	81.22	2.29	9.71	-
V2X index	19.25	8.93	9.10	78.69	2.29	10.55	-
Panel B: Log returns for each Volatility Index							
VIX Index	-0.0001	0.0701	-0.4964	0.4960	0.61	7.95	-0.75
VXN Index	-0.0002	0.0598	-0.3533	0.3629	0.63	6.71	-0.73
VXD Index	-0.0001	0.0683	-0.3696	0.5281	0.70	7.05	-0.74
RVX Index	-0.0001	0.0553	-0.3508	0.3613	0.59	6.67	-0.74
VDAX Index	-0.0002	0.0539	-0.3304	0.2834	0.54	6.35	-0.74
VAEX Index	-0.0001	0.0653	-0.3718	0.4871	0.54	6.59	-0.69
VCAC Index	-0.0002	0.0619	-0.3340	0.5781	1.66	15.18	-0.63
VFTSE Index	-0.0001	0.0613	-0.2954	0.3277	0.68	5.89	-0.77
VXJ index	-0.0002	0.0621	-0.2402	0.3331	0.54	5.43	-0.68
V2X index	-0.0001	0.0639	-0.2427	0.2626	0.43	4.65	-0.68

[4] Similar results for gold volatility index (GVZ), oil volatility index (OVX) and exchange rate volatility index (EVZ) are available upon request, but they are not the focus of this paper.

Table III presents sample statistics for the various stock indices included in our study. The reported statistics show that stock indices exhibit less similarity compared to global volatility indices. In particular, the sign of the skewness of the stock indices and log returns on the indices varies across the different underlying markets confirming the presence of both positive and negative skewness in the data. The equity indices also display high kurtosis.

Table III. Summary of underlying stock indices from January 2004 to July 2014

This table reports the mean, standard deviation, maximum, minimum values together with the skewness and kurtosis measures for each of the stock indices in the study from January 5, 2004 to July 22, 2014. Panel B reports summary statistics for log return measures on each of the 10 stock indices.

INDEX	Mean	Std. Dev.	Min	Max	Skewness	kurtosis
Panel A: Level of Underlying Stock Index						
SPX Index	1304.12	240.87	676.53	1985.44	0.51	3.47
NDX index	2062.53	639.60	1036.51	3961.62	1.02	3.33
INDU index	11869.44	2056.62	6547.05	17138.20	0.45	3.00
RTY index	742.27	169.58	343.26	1208.65	0.75	3.52
DAX index	6300.36	1523.05	3646.99	10029.43	0.30	2.54
AEX index	4126.05	788.88	2519.29	6168.15	0.67	2.70
CAC index	12178.85	2878.20	7054.98	18261.98	0.41	1.92
UKX Index	3061.29	613.75	1809.98	4556.97	0.70	2.68
NKY index	373.87	75.73	199.25	560.93	0.49	2.74
SX5E index	5592.17	747.00	3512.09	6878.49	-0.39	2.36
Panel B: Log Return on the Underlying Stock Index						
SPX Index	0.0002	0.0130	-0.0947	0.1042	-0.38	13.34
NDX index	0.0004	0.0142	-0.1111	0.1037	-0.36	9.13
INDU index	0.0002	0.0119	-0.0820	0.1033	-0.16	12.99
RTY index	0.0003	0.0169	-0.1261	0.0863	-0.43	7.99
DAX index	0.0004	0.0143	-0.0774	0.1346	0.15	11.32
AEX index	0.0001	0.0149	-0.0947	0.1330	0.22	11.37
CAC index	0.0001	0.0160	-0.1211	0.1323	-0.59	11.21
UKX Index	0.0001	0.0148	-0.0821	0.1295	0.10	10.49
NKY index	0.0001	0.0141	-0.0959	0.0978	-0.16	11.97
SX5E index	0.0002	0.0124	-0.0927	0.1111	0.04	13.66

4.2 Unconditional correlations between volatility markets

Tests of stock market integration have been reported extensively in the literature (see Sharma and Seth, 2012 for a thorough review of this literature). These studies show that stock markets in different countries are correlated and their correlation is dynamic and changes over time. Conversely, volatility market integration is a relatively new research topic. Our study examines the time varying correlation of implied volatility markets with respect to the corresponding underlying equity markets. This is different to earlier research because we model volatility as a function of its corresponding underlying rather than lagged measures of other volatility indices (see for example Siriopoulos and Fassas, 2013).

Table IV reports the unconditional correlation for each of the volatility indices in our study. The high correlation coefficients reported across each of the pairwise indices provides evidence of regional integration in volatility markets.

Table IV. Unconditional Correlation, January, 2004- July, 2014)

This table reports Unconditional Correlations for the volatility indices calculated using $log(v_{i,t}/v_{i,t-1})$ *from January 5, 2004 to July 22, 2014.*

Index	VIX	VXN	VXD	RVX	VDAX	VAEX	VCAC	VFTSE	VXJ
VXN	0.90								
VXD	0.97	0.90							
RVX	0.90	0.89	0.89						
VDAX	0.54	0.56	0.57	0.56					
VAEX	0.50	0.52	0.52	0.50	0.80				
VCAC	0.16	0.20	0.16	0.19	0.34	0.35			
VFTSE	0.57	0.59	0.59	0.57	0.88	0.84	0.37		
VXJ	0.52	0.54	0.54	0.53	0.83	0.83	0.36	0.87	
V2X	0.51	0.53	0.53	0.53	0.81	0.79	0.36	0.84	0.85

5. Empirical results

Before modeling conditional correlation, we use 45 unconditional correlations to test for the effect of underlying equity market integration on volatility market integration. We use the unconditional correlation of the corresponding underlying equity indices for two different equity markets, ($COR\_S_{i,j}$), as the independent variable and the unconditional correlation of the volatility indices for the two different volatility markets, ($COR\_V_{i,j}$), as the dependent variable. The model is given in equation (10).

$$COR\_V_{i,j} = \beta_1 + \beta_2 COR\_S_{i,j} \tag{10}$$

Table VI. Effect of correlation in underlying on correlation of volatility indices

This table reports the coefficients for the regression model stated in equation (10). The results use the unconditional correlation of equity indices for two different markets, ($COR\_S_{i,j}$), as the independent variable and the unconditional correlation of two different volatility indices, ($COR\_V_{i,j}$), as the dependent variable.

$$COR\_V_{i,j} = \beta_1 + \beta_2 COR\_S_{i,j}$$

	Coefficient	S.D	t statistic	p-value
β_1	-0.022	0.013	-1.650	0.106
β_2	0.968	0.018	53.503	0.000

The coefficients for the model estimating the relationship between the unconditional correlation of the implied volatility to the unconditional correlation in the underlying equity market are reported in Table VI. The intercept and the slope coefficient are both highly significant at the 1% level. The significant intercept and slope coefficients are interpreted as evidence of stronger correlation between the underlying equity market indices is associated with greater correlation between implied volatility measures for the two markets. In the next part of the paper we expand this result by substituting the appropriate underlying indices into equation (3) of the DCC model, to create a one-factor model. Tests for correlation of the error terms from the one factor model will determine if there is any additional correlation that is still unexplained.

5.1 Estimation of the Variance of the Volatility Index

The estimated parameters for the two models presented in equations (2) and (3) are reported in Tables VII and VIII. The parameters are compared to critical values based on a t-distribution as the assumption that normal errors does not hold. The t statistics are given in parentheses below each estimated coefficient.

The results in table VI show that the coefficient β_{2i} which measures the AR (1) effect of the volatility index is significantly negative. The coefficient is consistently negative and significant for each of the volatility indices in our sample and indicates that the dynamic implied volatility process is mean reverting for each volatility market.

Table VII. GJR-GARCH estimates for model with no factor

The table reports the estimated coefficients and t-statistics (in parentheses) for a model with no factor:

$$log\left(\frac{v_{i,t}}{v_{i,t-1}}\right) = \beta_{1i} + \beta_{2i} log\left(v_{i,t-1}\right) + \sigma_{i,t}\varepsilon_{it}$$

$$\sigma_{i,t}^2 = \emptyset_{1i} + \emptyset_{2i}\varepsilon_{i,t-1}^2 + \emptyset_{3i}\varepsilon_{i,t-1}^2[\varepsilon_{i,t-1} < 0] + \emptyset_{4i}\sigma_{i,t-1}^2$$

INDEX	$\boldsymbol{\beta_{1i}}$	$\boldsymbol{\beta_{2i}}$	$\boldsymbol{\emptyset_{1i}}$	$\boldsymbol{\emptyset_{2i}}$	$\boldsymbol{\emptyset_{3i}}$	$\boldsymbol{\emptyset_{4i}}$
VIX Index	0.0464	-0.0160	0.0004	0.1627	-0.1627	0.8241
	(4.36)	(-4.41)	(19881.81)	(134.49)	(-59.78)	(292.74)
VXN Index	0.0509	-0.0168	0.0002	0.1481	-0.1481	0.8475
	(4.44)	(-4.49)	(58049.21)	(128.26)	(-52.76)	(544.6)
VXD Index	0.0457	-0.0163	0.0004	0.1393	-0.1393	0.8403
	(4.39)	(-4.45)	(658.99)	(4.03)	(-0.65)	(3.53)
RVX Index	0.0460	-0.0144	0.0002	0.1330	-0.1330	0.8470
	(4.16)	(-4.19)	(37203.4)	(87.36)	(-39.87)	(215.28)
VDAX Index	0.0401	-0.0136	0.0001	0.0998	-0.0998	0.8947
	(4.02)	(-4.06)	(106009.02)	(263.33)	(-142.74)	(2422.73)
VAEX Index	0.0542	-0.0180	0.0002	0.1062	-0.0892	0.8795
	(4.64)	(-4.68)	(71856.96)	(256.88)	(-111.97)	(1873.02)
VCAC Index	0.0526	-0.0167	0.0002	0.1748	-0.1748	0.8360
	(4.44)	(-4.48)	(67319.11)	(53.45)	(-47.07)	(397.57)
VFTSE Index	0.0469	-0.0152	0.0003	0.1212	-0.1212	0.8591
	(4.27)	(-4.3)	(61560.18)	(211.32)	(-128.4)	(982.77)
VXJ index	0.0405	-0.0136	0.0002	0.1244	-0.1014	0.8583
	(4)	(-4.05)	(32793.33)	(102.46)	(-70.21)	(589.04)
V2X index	0.0399	-0.0139	0.0002	0.0898	-0.0898	0.8991
	(4.05)	(-4.1)	(53612.31)	(272.55)	(-159.75)	(1887.31)

The results for the one factor model estimated using GJR-GARCH are reported in Table VIII. The coefficient for the change in the corresponding underlying equity index, β_{2i} in equation (3) is consistently negative and significant. This result shows that, as we expected, there is a significant negative relationship between the implied volatility index and its corresponding underlying stock index. As expected the $\emptyset_{3i}$ coefficient, that tests for the asymmetric effect in volatility of the volatility index, is significant and negative, confirming the higher vega risk during equity market down turns.

Table VIII. GJR-GARCH Estimates for a One Factor Model

The table reports the estimated coefficients and t-statistics (in parentheses) for a model with one factor:

$$log\left(\frac{v_{i,t}}{v_{i,t-1}}\right) = \beta_{1i} + \beta_{2i}log\left(\frac{S_{i,t}}{S_{i,t-1}}\right) + \beta_{3i}log(v_{i,t-1}) + \sigma_{i,t}\varepsilon_{it}$$

$$\sigma_{i,t}^2 = \emptyset_{1i} + \emptyset_{2i}\varepsilon_{i,t-1}^2 + \emptyset_{3i}\varepsilon_{i,t-1}^2[\varepsilon_{i,t-1} < 0] + \emptyset_{4i}\sigma_{i,t-1}^2$$

INDEX	$\boldsymbol{\beta_{1i}}$	$\boldsymbol{\beta_{2i}}$	$\boldsymbol{\beta_{3i}}$	$\boldsymbol{\emptyset_{1i}}$	$\boldsymbol{\emptyset_{2i}}$	$\boldsymbol{\emptyset_{3i}}$	$\boldsymbol{\emptyset_{4i}}$
VIX Index	0.0424	-4.0454	-0.0144	0.0002	0.1562	-0.0673	0.7775
	(6.09)	(-56.92)	(-6.03)	(15284.23)	(49.78)	(-34.99)	(79.27)
VXN Index	0.0486	-3.0747	-0.0156	0.0001	0.0947	-0.0280	0.8280
	(6.25)	(-53.46)	(-6.15)	(51138.33)	(163.93)	(-23.08)	(359.17)
VXD Index	0.0416	-4.2100	-0.0145	0.0002	0.1145	-0.0305	0.8203
	(5.93)	(-53.92)	(-5.88)	(48676.3)	(148.81)	(-11.27)	(339.62)
RVX Index	0.0414	-2.4243	-0.0127	0.0001	0.1212	-0.0162	0.7964
	(5.62)	(-55.31)	(-5.56)	(51123.36)	(62.01)	(-1.44)	(170.08)
VDAX Index	0.0397	-2.7872	-0.0131	0.0000	0.1108	0.0041	0.8498
	(5.95)	(-54.9)	(-5.86)	(149710.44)	(119.58)	(1.22)	(716)
VAEX Index	0.0492	-3.0202	-0.0162	0.0001	0.1197	-0.0195	0.8241
	(5.81)	(-47.15)	(-5.84)	(52625.99)	(125.3)	(-21.71)	(374.95)
VCAC Index	0.0539	-2.4283	-0.0170	0.0001	0.0978	-0.0456	0.8745
	(5.87)	(-40.27)	(-5.89)	(71316.63)	(146.82)	(-21.8)	(1303.04)
VFTSE Index	0.0405	-3.1732	-0.0131	0.0002	0.1343	-0.0302	0.7814
	(5.79)	(-59.97)	(-5.82)	(21154.62)	(78.56)	(-15.33)	(91.13)
VXJ index	0.0419	-2.9846	-0.0140	0.0001	0.0841	-0.0237	0.8746
	(5.63)	(-45.68)	(-5.68)	(44113.83)	(124.03)	(-26.15)	(540.05)
V2X index	0.0363	-3.5085	-0.0124	0.0001	0.0664	-0.0299	0.9241
	(5.05)	(-46.48)	(-5.03)	(64522.87)	(154.16)	(-56.39)	(1405.47)

Figure II is a graph of the time series of the average correlation calculated over the 45 pairwise combinations of the 10 volatility indices in our sample using the two estimated models: (i) no factor model reported in Table VII and (ii) the one factor model reported in Table VIII in equation (2) and (3) respectively.[5] The one factor model given in equation (3) controls for the correlation between the two underlying stock indices. Figure II shows clearly that the correlation of the one factor model is lower and less volatile compared to the no factor model. The results also indicate that the correlation is not fully explained by our proposed one factor model and future research should consider other macroeconomic factors to explain the correlation between the implied volatility and underlying stock market.

[5] All 45 pairwise correlations for both models have been presented in Figure IV.

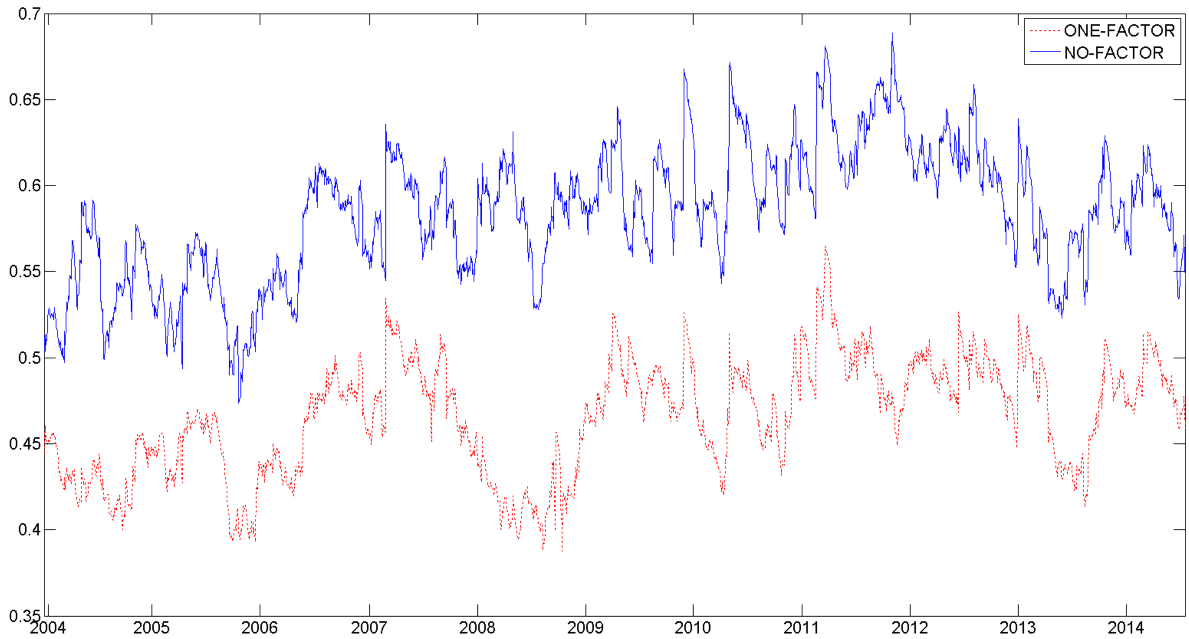

Figure II. Mean estimated correlation across pairs of volatility and equity indices for the no factor and one factor models reported in Tables VII and VIII.

6. Conclusion:

We use 10 volatility indices to examine the correlation of volatility markets in six countries. Our main result is that we find evidence of a significant time varying correlation between volatility expectations in different countries. Models that estimate implied volatility using an assumption of constant correlation have poor performance during periods of market volatility when extreme changes occur. Empirical evidence indicates that correlation is higher during periods of high market uncertainty (Engle and Figlewski, 2014). Investors require models to estimate a dynamic correlation relationship between markets and the impact of that information on volatility indices. Accurate and timely information about changes in the correlation of volatility indices can be applied to adjust hedge positions in the wake of future anticipated periods of high volatility. Investor's desires for more risk diversification or risk-rebalancing is one reason for developing correlation estimators of volatility indices.

Our results confirm common features of volatility indices include a demonstrated negative relationship with the corresponding underlying equity market, fat tail distributions and positive skewness (Siriopoulos and Fassas, 2009). Studies by Engle and Figlewski (2014) and Siriopoulos and Fassas (2013) have examined various features of volatility index correlation. However explanations for the correlation measures and magnitude are still unclear. Our study investigates time varying correlation using 10 different volatility indices in relation to the underlying equity index for each market. We introduce the measure of underlying dynamic correlation as a first explanation for this relationship in a one factor model.

Consistent with the work by Siriopoulos and Fassas (2013) we confirm that volatility markets are more correlated in periods of turbulence. Our study shows that the correlation of volatility indices changes significantly over time and that volatility markets with higher correlation in the underlying stock market are highly integrated with their respective volatility markets. While our study finds that the correlation with the underlying equity market is one reason for this dynamic correlation future research should examine other reasons that may also explain this effect. Our study opens the way for further investigation into underlying macroeconomic reasons behind this co-movement in volatility markets. This research shows that the underlying equity market explains part of the implied volatility story however there may be other factors that are also important in explaining integration within volatility markets. These might include openness of the economy, level of trade between countries, foreign exchange volatility and interest rate differentials. The relationship between these factors and market volatility has direct relevance for policy makers at both a macro and global level together with global investors.

References

Aboura, S. (2003). International transmission of volatility: A study on the volatility indexes VX1, VDAX and VIX. Working paper, Université Paris IX Dauphine.

Äijö, J. (2008). Implied volatility term structure linkages between VDAX, VSMI and VSTOXX volatility indices. Global Finance Journal, 18(3), 290–302.

Badshah, I. U., Frijns, B., & Tourani Rad, A. (2013). Contemporaneous Spill Over Among Equity, Gold, and Exchange Rate Implied Volatility Indices. Journal of Futures Markets, 33(6), 555-572.

Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model. The Review of Economics and Statistics, 498-505.

Cappiello, L., Engle, R. F., & Sheppard, K. (2006). Asymmetric dynamics in the correlations of global equity and bond returns. Journal of Financial econometrics, 4(4), 537-572.

Creti, A., Joëts, M., & Mignon, V. (2013). On the links between stock and commodity markets' volatility. Energy Economics, 37, 16-28.

Demeterfi, K., Derman, E., Kamal, M., & Zou, J. (1999). More than you ever wanted to know about volatility swaps. Goldman Sachs quantitative strategies research notes, 41.

Engle, R. (2009). Anticipating correlations: a new paradigm for risk management. Princeton University Press. 43-73.

Engle, R. &Figlewski, S, (2014) Modeling the Dynamics of Correlations Among Implied Volatilities. The Review of Finance, 1-28.

Hau, H., & Rey, H. (2006). Exchange rates, equity prices, and capital flows. Review of Financial Studies, 19(1), 273-317.

Kumar, S. S. S. (2012). A first look at the properties of India's volatility index. International Journal of Emerging Markets, 7(2), 160-176.

Liu, M. L., Ji, Q., & Fan, Y. (2013). How does oil market uncertainty interact with other markets? An empirical analysis of implied volatility index. Energy, 55, 860-868.

Lopez, R., & Navarro, E. (2012). Implied volatility indices in the equity market: A review. African Journal of Business Management, 6(50), 11909-11915.

Sharma, A., & Seth, N. (2012). Literature review of stock market integration: a global perspective. Qualitative Research in Financial Markets, 4(1), 84-122.

Siriopoulos, C., & Fassas, A. (2009). Implied Volatility Indices-A Review. University of Patras, Rion, Greece.

Siriopoulos, C., & Fassas, A. (2013). Dynamic relations of uncertainty expectations: a conditional assessment of implied volatility indices. Review of Derivatives Research, 16(3), 233-266.

Silvennoinen, A., & Thorp, S. (2013). Financialization, crisis and commodity correlation dynamics. Journal of International Financial Markets, Institutions and Money, 24, 42-65.

Soriano, P., & Climent, F. J. (2005). Volatility transmission models: a survey. Available at SSRN 676469.

Yang, S. Y. (2005). A DCC analysis of international stock market correlations: the role of Japan on the Asian Four Tigers. Applied Financial Economics Letters, 1(2), 89-93.

Appendix I. Stock Market Volatility Indices in World

Table IX. Volatility Indices in the data sample

Volatility Index (v_i)	**Underlying Index (S_i)**	**Exchange**	**Country**
VIX Index	SPX Index	CBOE	USA
VXN Index	NDX index	CBOE	USA
VXD Index	INDU index	CBOE	USA
RVX Index	RTY index	CBOE	USA
VDAX Index	DAX index	Deutsche Börse	Germany
VCAC Index	CAC index	Euronext (Paris)	France
VXJ index	NKY index	Unofficial	Japan
V2X index	SX5E index	Euronext	Euro Zone
VAEX Index	AEX index	Euronext (Amsterdam)	Netherlands
VFTSE Index	UKX Index	Euronext	UK

We only consider volatility indices in stock markets that are well established in major markets. To see a comprehensive review of all volatility indices refer to Siriopoulos (2009).
All of the volatility indices are model free and based on the CBOE methodology introduced in 2003.

Figure III. Volatility of Volatility Indices with no factor

This graphs present result of GJR-GARCH (1,1) volatility model for each index using the one-factor model. Parameters of the model are given in Table VII.

$$log\left(\frac{v_{i,t}}{v_{i,t-1}}\right) = \beta_{1i} + \beta_{2i}log\left(\frac{S_{i,t}}{S_{i,t-1}}\right) + \beta_{3i}log(v_{i,t-1}) + \sigma_{i,t}\varepsilon_{it}$$
$$\sigma_{i,t}^2 = \emptyset_{1i} + \emptyset_{2i}\varepsilon_{i,t-1}^2 + \emptyset_{3i}\varepsilon_{i,t-1}^2[\varepsilon_{i,t-1} < 0] + \emptyset_{4i}\sigma_{i,t-1}^2$$

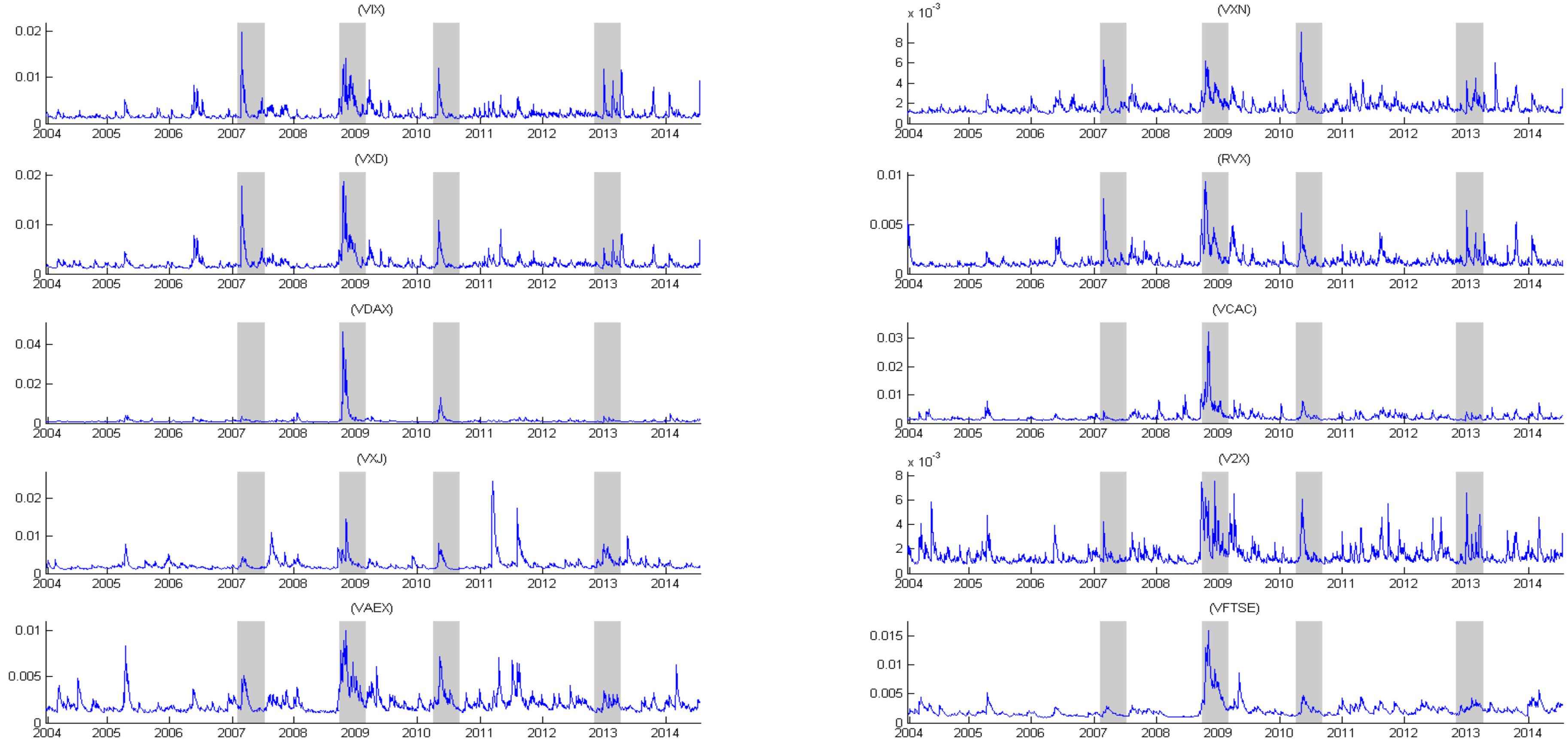

Figure IV. Time Varying Correlation of Volatility Indices for one-factor model and no-factor model

This graphs present result of DCC correlation model for each index in both the no-factor model and the one-factor model. Parameters for each model are given in Table VII and Table VIII, respectively.

$$Q_t = \overline{R} + \alpha(\varepsilon_{t-1}\varepsilon'_{t-1} - \overline{R}) + \theta(Q_{t-1} - \overline{R})$$

$$\mathrm{R_t} = \mathrm{diag(Q)}^{-1/2}\mathrm{Q_t}\mathrm{diag(Q)}^{-1/2}$$

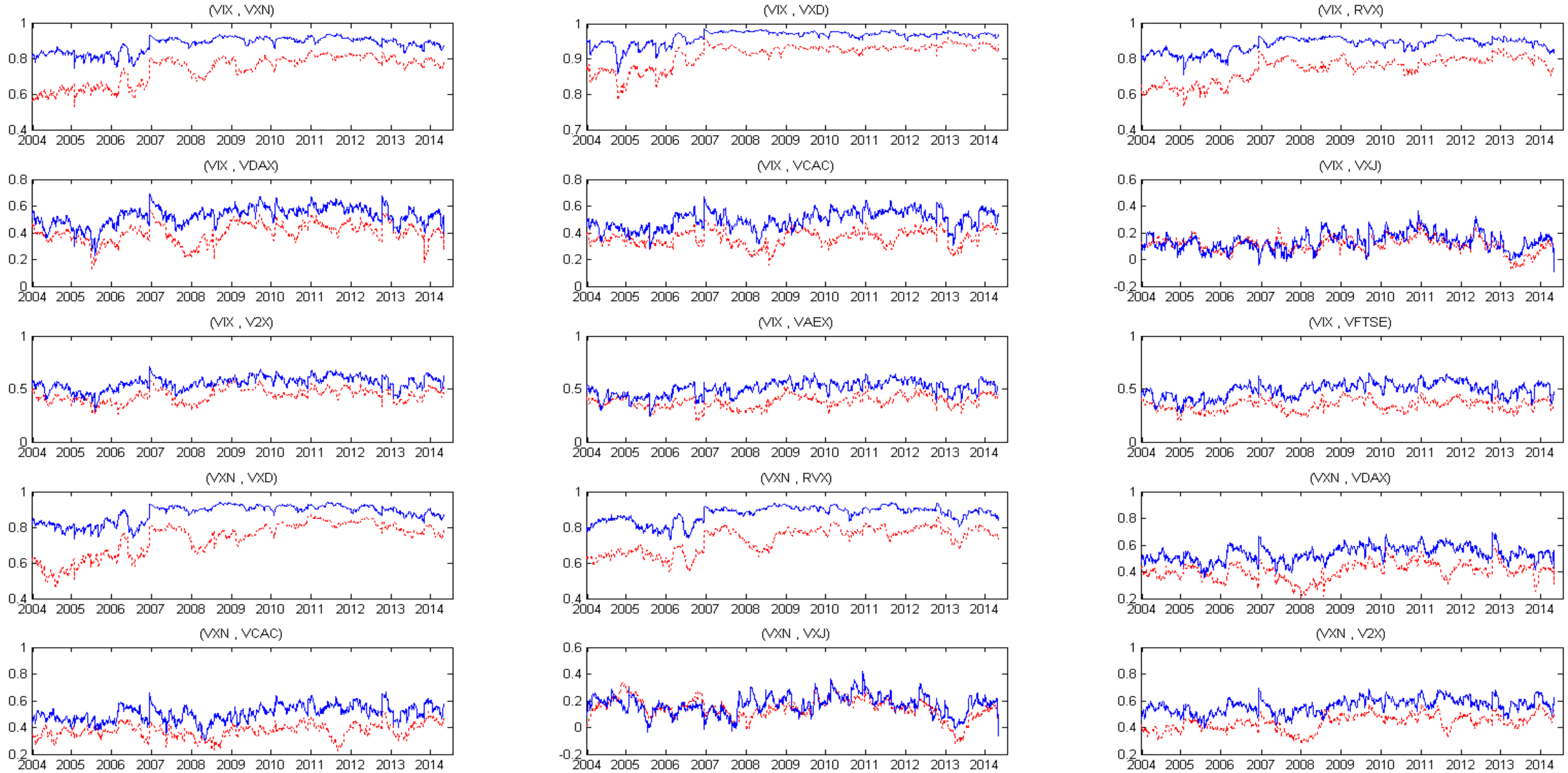

Figure IV. (continued)

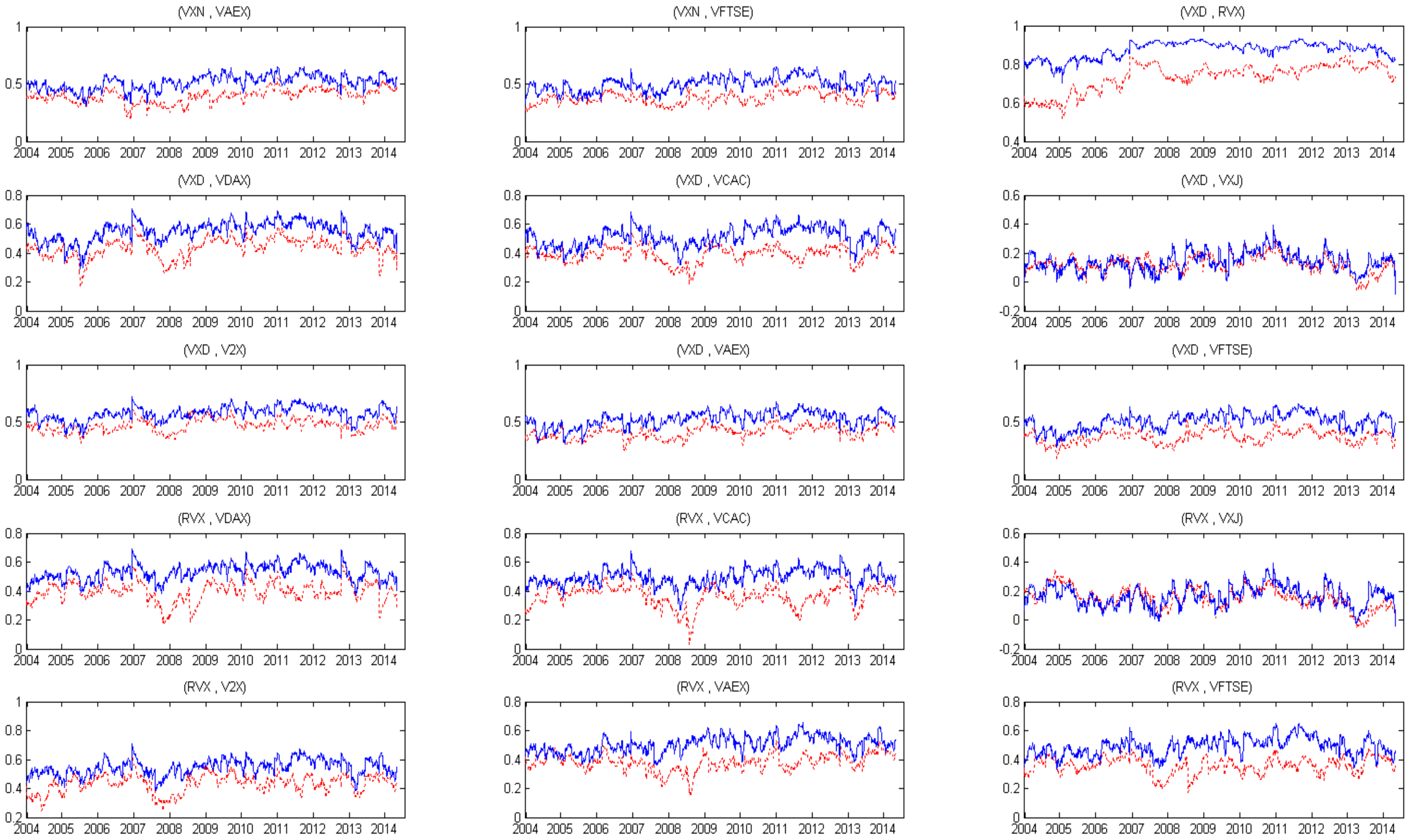

Figure IV. (Continued)