Monetary policy and risk in the open economy

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[DRAFT]

Abstract

Under modest assumptions, variations in asset prices and returns mainly reflect variations in risk. I propose a general equilibrium, two-country model that incorporates risk premia (eg. sovereign, inflation, liquidity risk) in asset prices and returns. In the model with risk, monetary policy (i) determines the short-term nominal interest rate, but no longer directly drives the marginal rate of inter-temporal substitution (IMRS); (ii) delivers a Dornbusch 1976-type exchange rate response, but the mechanism works mainly through risk premia; (iii) shifts the expression of bond risk from the bond market to the foreign exchange market; and (iv) isolates the exchange rate from variation in the IMRS.

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1 Introduction

Under mild assumptions, variation in asset returns and asset prices are mainly variations in risk premia (Cochrane 2016, Cochrane 2001 and references therein). There is a broad disconnect between asset prices and expected means of macroeconomic variables. One approach to resolving that disconnect is accounting for the role of risk. Another is to explore market frictions and irrationality. This paper follows the the first approach. If variation in asset returns and prices are mainly variations in risk premia, and observed interest rates are mainly driven by monetary policy, we need a theory of how monetary policy affects risk premia (Alvarez et al. 2007a).

In this paper, I propose a general equilibrium model that incorporates both monetary policy and risk. The model departs from the standard approach by allowing for uncertainty about the value of ex-post payoffs, that generates risk premia in asset prices and returns, ex-ante, even for short-term government interest rates. The risk premia capture common risks such as default, inflation and liquidity risk, and term premia, that are priced into bond yields, ex ante. That departure is motivated by empirical evidence that asset prices and returns reflect risk premia rather than expected means of macroeconomic variables, such as consumption growth (Alvarez et al. 2007b, Backus et al. 2001, Cochrane 2001).[1] There is ample direct evidence of risk premia in short-term bond yields. Duffie (1996), shows that there are important differences in the risk characteristics of on-the-run and off-the-run US treasuries, related to their use as collateral. Lustig and Verdelhan (2007) provide evidence that high interest rates currencies tend to perform poorly in bad times, implying that there is a consumption risk-adjustment in short-term interest rates. Krishnamurthy and Vissing-Jorgensen (2012) provide evidence of a convenience yield in US Treasuries, that reflects, for example liquidity and collateral services. Della Corte et al. (2015) show that sovereign ratings have explanatory power for currency excess returns, implying a role for relative risk.

The model is also motivated by the need for a model that links monetary policy and risk (Alvarez et al. 2007a). Several empirical papers link short-term risk premia to monetary policy. Nagel (2014) shows that short-term liquidity risk[2] is positively correlated with the stance of US monetary policy. Canzoneri et al. (2007) show that, for five of six common specifications of preferences, the Euler equation interest rate, implied by consumption data, is negatively correlated with the stance of monetary policy. That implies positive correlation between the policy rate and the bond market premium (defined as the the spread between the policy rate and the Euler equation rate). Alvarez et al. (2007a) argue that, if exchange rates are

[1] A strand of the literature seeks to explain variation in asset prices and returns using .

[2] The spread between a 3-month repo backed by US government debt, that cannot be sold before maturity without a penalty, and a 3-month US Treasury bill that can be sold in a liquid market at any time.

roughly random walks, the variation in the interest differential is almost entirely accounted for by movement in the risk premium.

In a model without monetary policy, bond risk drives a wedge between the observed short-term interest rate and the *unobserved* inter-temporal marginal rate of substitution (IMRS or Euler equation interest rate).[3] In that case, the market-implied interest rate compensates the holder of the bond for delaying consumption (the Euler equation interest rate), for expected losses, and for the risk that losses materialise in bad times. The risk premium wedge breaks the direct link between the short-term policy rate rate and the Euler equation interest rate, that exists in the standard first-order model.

In the model with monetary policy, changes in the policy rate alter the risk-adjusted return on short-term bonds, and can be reflected in the IMRS, or the bond market premium, or both. Monetary policy affects the economy through the effect of nominal interest rates on budget constraints, balance sheets and relative prices; but no longer directly drives the IMRS and, in turn, the expected consumption path.[4]

A number of papers in the finance literature propose models that generate currency premia. The currency premium is usually limited to variances that arise from log-normality (for example, Alvarez et al. 2007b), or from currency revaluation risk (for example, Lustig and Verdelhan 2007). Those premia are present in this paper. In addition, uncertainty about the ex-post value of bond payoffs, in local currency terms, gives rise to risk premia in interest rates, ex ante. When incorporating bond risk, the assumption that the observed short-term benchmark rate is the "risk-free" Euler equation interest rate no longer holds. In the absence of monetary policy intervention in the local bond market, the shadow price of bond risk would be fully reflected in the bond market premium, and in the observed interest rate. Monetary policy intervention in the local bond market drives the observed short-term rate away from the market-implied rate.

Cochrane (2016) reviews a range of alternative preferences and market structures that drive a wedge between the Euler equation interest rate and expected consumption growth. In contrast, here the focus is on two wedges between the Euler equation interest rate and the observed short-term interest rate: bond risk and monetary policy intervention in bond markets. Some papers employ market segmentation to generate a currency premium (Chien et al. 2015 and Alvarez et al. 2007a). Like those papers, the model proposed here features market segmentation but, here, the source is monetary policy intervention in the local bond market, that,

[3]The IMRS is the inverse of the stochastic discount factor implied by the consumption Euler equation. It is the rate used, in theory, to discount the future and to price risk. It would be the rate on a state-contingent bond that pays off in units of consumption utility growth, if we had such a bond. It is also referred to as the Euler equation interest rate (Canzoneri et al. 2007), the discount rate, the risk-free rate, and the pricing kernel (Cochrane 2001).

[4]See Cochrane (2001) for a discussion of causality between consumption and the consumption Euler equation interest rate.

in turn, reflects the sticky price distortion.

The behaviour of the exchange rate, as the main asset price in the model, is of particular interest. In the model with risk, monetary policy affects the currency in three ways. First, higher home risk-adjusted returns initially appreciate the home currency, so it can subsequently depreciate to offset the higher risk-adjusted interest return, period by period. That Dornbusch (1976)-type response to policy is present, whether policy alters the Euler equation interest rate, the bond market premium, or both. Monetary policy control of the short-term interest rate has two other effects: it shifts the expression of bond risk from the bond market to the foreign exchange market, and it isolates the currency from variation in the (potentially volatile) Euler equation interest rate.

Model simulations shed light on some empirical exchange rate regularities:

- a high interest rate currency has excess returns (Fama 1984),
- a high interest currency is a strong currency (Engel 2016), and
- exchange rates are "too smooth" relative to volatile Euler equation interest rates (Brandt et al. 2006)

The next section sets out the model. Section 3 discusses calibration and model dynamics. Section 4 presents model dynamics. Sections 5 through 7 use simulated data to explore empirical exchange rate regularities.

2 The general equilibrium model

The general equilibrium model is a simplified, two-country version of the small open economy model of (Galí and Monacelli 2005). The basic model features a representative household that consumes a basket of home and foreign goods, receives wage income, issues home bonds, buys foreign bonds, and has a zero net debt. The capital stock is assumed to be fixed so that production depends only on productivity and labour input. In the benchmark model, labour input is also assumed to be fixed and is paid a constant CPI-based real wage.[5] Producers are assumed to have a degree of monopoly power in setting the prices of their specialised varieties, and to set prices in a staggered fashion, leading to stickiness in producer prices. In the interest of parsimony, I assume perfect competition in cross-border trade. The two-country model, is fairly standard, is set out in Appendix A.

The focus here is the treatment of interest rates and asset prices, and the role of monetary policy. In keeping with the proposition that variation in returns and asset prices mainly reflect variations in risk premiums, risk is accounted for in asset prices and returns. This is done by accounting for uncertainty about the ex-post value of

[5] That choice is motivated by parsimony, and the counter-cyclical behaviour of labour, in the absence of rigidities, in contrast to observed business cycle patterns.

bond payoffs in the Euler equations for home and foreign bonds. The ex-post value of bond payoffs may vary because of default, inflation, or a need to sell the bond before maturity to smooth consumption (liquidity risk), or changes in the state of the economy. Expected losses (or gains), and the risk that losses materialise in bad times, are priced into the contracted rate, ex-ante (Cochrane 2001). "Excess returns" in the uncovered interest parity condition, which is derived from the Euler equations for home and foreign bonds, reflect relative bond risk. Risk premia are treated as linear factors in the linear general equilibrium model, an approach that retains parsimony, while ensuring that risk is treated consistently across the asset prices and returns in the model.

The risk corrections break the direct link between the policy rate and inter-temporal substitution.[6] In the standard monetary policy model, monetary policy controls the Euler equation interest rate which, in turn, drives expected consumption growth. A higher policy interest rate requires an immediate fall in consumption, so that consumption growth is positive over the high interest period. Intuitively, higher interest rates encourage saving and discourage borrowing, so people shift consumption to the future.

In this model, the connection between monetary policy and consumption growth is indirect. Monetary policy intervention in the local bond market alters the risk-adjusted return on the short term bond. It affects the economy through the effect of nominal interest rates on household budget constraints and balance sheets and on relative prices. Inter-temporal substitution depends on expectations about future consumption growth.

2.1 Household

Two symmetrical economies are inhabited by representative households. Home household preferences are given by $U(C_t, N_t)$, where the household's consumption basket, C_t, is made up of home and foreign goods, and N_t is labour input. The home household chooses consumption, labour and home and foreign nominal bond holdings, B_t and B_t^* respectively, to maximise expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_{\beta,t} U(C_t, N_t)$$

subject to the period budget constraint:

$$\begin{aligned} \int_0^1 P_{H,t}(\iota) C_{H,t}(\iota) d\iota + \int_0^1 P_{F,t}(\iota) C_{F,t}(\iota) d\iota \ + \ & (1 + i_{t-1}) Z_t B_{t-1} + \varepsilon_t B_t^* \\ & = W_t N_t + B_t + \varepsilon_t (1 + i_{t-1}^*) Z^* t B_{t-1}^* \end{aligned} \tag{1}$$

where the parameter β is the subjective discount factor, $\varepsilon_{\beta,t}$ is a preference shock. On the left-hand side of (1), the household uses income to buy varieties (ι) of home

[6] See Cochrane (2001), chapters 1 and 2 for For a more detailed discussion.

and foreign goods, to repay, with interest, last period's nominal home bonds B_t, and to buy new foreign bonds, B_t^*. i_t is the home nominal interest rate and ε_t is the nominal exchange rate, defined as the value of the foreign currency in units of the home currency. On the right hand side of (1), the household receives labour income $W_t N_t$, the proceeds of the sales of home bonds, and interest income from last periods's holdings of foreign bonds $\varepsilon_t(1+i_t^*)Z_t^* B_{t-1}^*$. W_t is the nominal wage, and i_t^* is the foreign nominal interest rate. Z_t and Z_t^* capture uncertainty about value of ex-post bond payoffs. We will see, in the next section, that Z_{t+1} gives rise to expected losses/gains and to consumption risk corrections that incorporate sovereign risk, inflation risk, liquidity risk. Including Z_{t+1} is motivated by broad empirical support in the literature a large role for risk premia, even in short-term Treasuries.

The home household's real stochastic discount factor, M_{t+1}, used to discount period $t+1$ returns and to price risk, is defined as:

$$M_{t+1} \equiv \beta E_t \frac{U_{C,t+1}}{U_{C,t}} \tag{2}$$

and is assumed to be conditionally log-normal.[7] The unobserved IMRS, the Euler equation interest rate, is defined as $r_t^f = -log(M_{t+1}) = -m_{t+1} - \frac{1}{2}var_t m_{t+1}$, and is known in expectation at time, t. The Euler equation interest rate,

$$r_t^f \equiv -log E_t M_{t+1} = -m_{t+1} - \tfrac{1}{2} var_t m_{t+1} \tag{3}$$

represents the payoff required to postpone consumption from today until next period. It is the expected return on a bond that pays in units of consumption utility growth. In practice, we don't have such a bond, so the rate is assumed to be unobserved. The Euler equation interest rate is the rate used to discount future payoffs, and to price risk.

The Fisher equation relates the observed nominal interest rate, i_t, to the expected real interest rate, r_t, and expected inflation $E_t(1+\pi_{t+1}) = E_t P_{t+1}/P_t$:

$$i_t = r_t + E_t \pi_{t+1}$$

The real exchange rate, Q_t, is defined in terms of the home and foreign CPI baskets:

$$Q_t = \varepsilon_t \frac{P*_t}{P_t}, \quad \text{or, in logs,} \quad q_t = e_t + p_t^* - p_t \tag{4}$$

where $q_t = log(Q_t)$, $e_t = log(\varepsilon_t)$, $p_t = log(P_t)$ and $p_t^* = log(P_t^*)$.

The household's optimisation problem yields the following first order conditions for home and foreign bonds:

$$1 = E_t[M_{t+1}(1+r_t)Z_{t+1}] \tag{5}$$

$$Q_t = E_t[M_{t+1}Q_{t+1}(1+r_t^*)Z_{t+1}^*] \tag{6}$$

[7]If a variable, x_t is normal, then $X_t = e^{x_t}$ is log-normal. When X_t is known with certainty, $x_t = log(X_t)$, but $log E_t X_{t+1} = x_t + \frac{1}{2} var_{x_t}$, and $log E_t X_{1,t+1} X_{2,t+1} = x_{1,t} + \frac{1}{2} var(x_{1,t}) + x_{2,t} + \frac{1}{2} var(x_{2,t}) + cov(x_{1,t}, x_{2,t})$

Equation (5) equates the additional consumption from issuing a home bond today to the expected discounted consumption cost of principal and debt repayment next period. Equation (6) equates the cost of buying a foreign bond today with the expected discounted benefit of additional consumption next period from interest and principal receipts on the foreign bond, net of exchange rate revaluation. An equivalent set of equations holds for the foreign household with foreign variables denoted by an asterisk.

2.2 Returns and asset prices

In keeping with evidence that variation in asset returns and prices largely reflects variation in risk premia (Cochrane 2016, Alvarez et al. 2007a, Cochrane 2001), I account for risk in those variables. Risk premia reflect expected gains/losses and the risk that losses materialise in bad times. Such measures of risk include default risk, liquidity risk and inflation risk (see Appendix B). Risk is incorporated as a first-order factor in the linear model. This approach allows risk to be treated consistently in the bond market and in the foreign exchange market, while retaining clarity regarding transmission mechanisms.[8] In this section, I set out a model with no monetary policy intervention in bond markets. In the next section, I introduce monetary policy.

Bond premia

Assuming gross asset returns are log-normal, taking logs, equation (5) can be written as:

$$\begin{aligned} 0 &= logE_t\Big(M_{t+1}(1+r_t)Z_{t+1}\Big) \\ &= log\Big(e^{(E_t m_{t+1}+E_t r_t+E_t z_{t+1}+\frac{1}{2}var_t(m_{t+1}+r_t+z_{t+1}))}\Big) \\ &= \underbrace{E_t m_{t+1}+\tfrac{1}{2}var_t(m_{t+1})}_{-r_t^f}+r_t+\underbrace{E_t z_{t+1}+\tfrac{1}{2}var_t(z_{t+1})}_{logE_t Z_{t+1}}+cov_t(m_{t+1},z_{t+1}) \end{aligned}$$

where $m_t = log(M_t)$, $z_t = log(Z_t)$. Covariance terms in r_t are set to zero, because r_t is observed at time t. Rearranging:

$$r_t = r_t^f \underbrace{-logE_t Z_{t+1} - cov_t(m_{t+1},z_{t+1})}_{\lambda_t^H,\text{home bond risk}} \tag{7}$$

Equation (7) says that the contracted return on a bond compensates the holder for putting off consumption from today to next period; for expected losses,[9] and

[8]Ideally, higher order terms would arise endogenously in a model with time-varying ability to bear risk, but in practice, that has proved difficult. Most models do so by introducing a recession state variable (Cochrane 2016).

[9]In log terms, this includes a variance term because of log-normality: $logE_t(Z_{t+1}) = E_t z_{t+1} + \frac{1}{2}var_t(z_{t+1})$.

for consumption risk. The consumption risk correction, the final term, increases the yield on assets with payoffs that are expected to be positively correlated with consumption growth (negatively correlated with m_{t+1}). Holding such assets makes consumption more volatile (Cochrane 2001). Bond risk reflects differences in bond characteristics, such as their issuer, and their accepted uses, for example their acceptability as collateral.

Risk corrections drive a wedge between the observed short-term interest rate and the Euler equation interest rate. In the absence of uncertainty about the value of ex-post payoffs Z_{t+1}, the observed short-term rate would equal the Euler equation interest rate. This is the key departure from the standard approach in macroeconomic models and most finance models.

Applying the same method to the foreign equivalent of (5), we can write the observed short-term foreign interest rate as the foreign Euler equation rate plus the foreign risk premium:

$$r_t^* \quad = \quad r_t^{f*} \underbrace{-logE_t(Z_{t+1}^*) - cov_t(m_{t+1}^*, z_{t+1}^*)}_{\lambda_t^F\text{,foreign bond risk}} \tag{8}$$

In the absence of monetary policy intervention, the local currency bond premia reflect local currency pricing of bond risk. We will see later that policy intervention in the local bond market drives the bond premium away from the shadow price of risk. It is useful to define the relative (foreign) bond market premium:

$$\lambda_t^R \equiv (r_t^* - r_t^{f*}) - (r_t - r_t^f) \tag{9}$$

The currency excess return and uncovered interest parity

Applying the same approach to the home investor's Euler equation for the foreign bond (6),

$$r_t^* = r_t^f - logE_t(Z_{t+1}^*) - E_t\Delta q_{t+1} - \tfrac{1}{2}var_t(\Delta q_{t+1}) - cov_t(m_{t+1}, z_{t+1}^*) - cov_t(m_{t+1}, \Delta q_{t+1})$$

where $q_t = log(Q_t)$. The foreign contracted interest rate, r_t^* compensates the foreign investor for putting off consumption to next period, for expected losses (or gains), and for the risk that the value of the foreign payoff will be low or the foreign currency will be weak when consumption growth is low. Combining (7) and (10) gives the uncovered interest parity (UIP) condition:

$$E_t\Delta q_{t+1} \quad = \quad (r_t - r_t^*) + \lambda_t, \text{ where} \tag{10}$$

$$\begin{aligned} \lambda_t \quad = \quad & logE_tZ_{t+1} - logE_tZ_{t+1}^* + cov_t(m_{t+1}, (z_{t+1} - z_{t+1}^*)) \\ & -cov_t(m_{t+1}, \Delta q_{t+1}) - \tfrac{1}{2}var_t(\Delta q_{t+1}) \end{aligned} \tag{11}$$

The "currency excess return" or "carry trade return" ,λ_t , reflects the home investor's pricing of foreign bond risk relative to home bond risk. The terms in

Δq_{t+1} are present in a second-order approximation of a standard model (eg. Lustig and Verdelhan 2007, Obstfeld and Rogoff 1996). $\frac{1}{2}var_t(\Delta q_{t+1})$ captures the non-normality of the expected exchange rate change ΔQ_t, and $cov_t(m_{t+1}, \Delta q_{t+1})$ captures the risk that the home currency depreciates ($\Delta q_{t+}) > 0$) during bad times (m_t is high and r_t^f is low).

The terms in z_{t+1} reflect the home and foreign investors' consumption risk corrections on the foreign bond, capturing, uncertainty about the value of ex-post payoffs, Z_{t+1}, that generates default, liquidity and inflation risk.

When the home risk-adjusted interest rate is high relative to the foreign risk-adjusted interest rate, $(r_t - r_t^*) + \lambda_t > 0$, the the value of the home currency is expected to initially appreciate, so that it can subsequently depreciate to offset the higher risk-adjusted return, period by period à la Dornbusch (1976). There is no risk-adjusted excess return to holding the home or the foreign bond.

Risk sharing and the currency premium

Combining (8) and (10) gives the risk-sharing condition:

$$\begin{aligned} E_t \Delta q_{t+1} &= r_t^f - r_t^{f*} \\ &\quad \underbrace{-\tfrac{1}{2}var_t(\Delta q_{t+1}) - cov_t(m_{t+1}, \Delta q_{t+1}) - cov_t((m_t - m_{t+1}^*), z_{t+1}^*)}_{\lambda_t^{FX} \text{ (“currency premium”)}} \end{aligned} \tag{12}$$

Equation (12) equates the home and foreign investors' pricing of the foreign bond, that is, the foreign bond market clears at a yield r_t^*. The first line is the standard risk-sharing condition in a linear model. The first two terms on the second line capture currency revaluation risk, and are present in a second order approximation of the standard model (Lustig and Verdelhan (2007), Obstfeld and Rogoff (1996)). The final term on the second line captures the difference between the home and foreign household's pricing of other risks to the foreign bond payoff.

Combining equations 7, 8, 11, and 12 we can see that the currency premium, the currency excess return, λ_t, and the relative bond market premium, λ_t^R are related:

$$\lambda_t = \lambda_t^{FX} + \lambda_t^R \tag{13}$$

The UIP condition (10) can be written in terms of the observed, r_t, r_t^* and λ_t, or in terms of the unobserved Euler equation rates and currency premium r_t^f, r_t^{f*}, λ_t^{FX}:

$$\begin{aligned} E_t \Delta q_{t+1} &= (r_t - r_t^*) + \lambda_t &(14)\\ &= (r_t^f - r_t^{f*}) + \lambda_t^{FX} &(15) \end{aligned}$$

In the linear general equilibrium model, risk premia are incorporated as linear factors, in a way that is consistent across interest rates and the exchange rate (the key asset price in the model). This approach provides a tractable way of incorporating risk, enabling an understanding of the effects of the different risk wedges in a parsimonious framework.

In the absence of monetary policy, bond risk has no effect on the currency because the relative bond premium, λ_t^R, component of the interest differential if offset by the bond risk component of the currency excess return, λ_t in equation 10. Intuitively, the bond premium has no effect on the currency because the currency doesn't depreciate to offset a higher home premium, when the premium is compensation for risk. In the presence of policy intervention in the bond market, however, the bond premium may no longer offset the market price of risk.

2.3 Monetary policy

Monetary policy is assumed to be implemented through intervention in the local bond market to set the local interest rate according to a Taylor-type rule. In log terms:

$$i_t = \rho_r i_{t-1} + (1 - \rho_r)\phi_\pi + \epsilon_{r,t} \tag{16}$$

Policy intervention can affect the Euler equation interest rate, the bond market premium, or both.

In contrast to the standard model, the effect of policy on the Euler equation interest rate is now indirect, via the household budget constraint, balance sheet and relative prices. In this model, with zero steady-state debt and no durable good, the main channel from policy to consumption, and in turn the Euler equation interest rate, is through the effect of currency appreciation on relative prices.

Absent monetary policy intervention in the bond market, the home interest rate is the home Euler equation interest rate, r_t^f, plus the shadow price of home bond risk λ_t^H. In the presence of monetary policy, the bond premium no longer reflects the market price of risk,[10]

$$\tilde{\lambda}_t^H \equiv r_t - r_t^f \neq \lambda_t^H \tag{17}$$

The presence of bond risk fundamentally alters the monetary policy transmission mechanism. In the standard model, the policy rate and the Euler equation rate are perfectly correlated, giving a dominant inter-temporal substitution channel. Now, the monetary policy may transmit through the premium.

Regardless of whether policy affects the Euler equation interest rate or the bond market premium, a policy tightening increases the risk-adjusted return on the home bond. The higher risk-adjusted return appreciates the currency à la Dornbusch (1976). The home currency must appreciate until, either greater (currency revaluation) risk offsets the higher interest rate, or the home Euler equation rate rises relative to foreign. The adjustment mechanism depends on the model structure,

[10] Unless r_t^f moves one-for-one with the policy rate, which is unlikely. Empirically, Canzoneri et al. (2007) show that Euler equation interest rate implied by consumption growth tends to be negatively correlated with real policy interest rates. That implies that a policy tightening may reduce the Euler equation rate, and raise the bond market premium $\tilde{\lambda}_t^H$ by more than one-for-one.

and is discussed further in Section 4 in the context of the general equilibrium model employed here.

Intervention in the bond market leads to segmentation between the bond market and the currency market: $\tilde{\lambda}_t^R \neq \lambda_t^R$. As in Lustig and Verdelhan (2007), Backus et al. (2001) and Chien et al. (2015), mechanism described here involves asymmetric pricing of risk. Chien et al. (2015) propose a model in which heterogeneous trading strategies and home bias in consumption lead to market segmentation. Alvarez et al. (2007a) propose a model in which transactions costs between the goods market and the asset market lead to market segmentation. This paper is also concerned with market segmentation, but here, monetary policy intervention in short-term bond markets is the source of segmentation between the pricing of risk in the bond market and the currency market.

2.4 Utility specification

Period utility is assumed to take the form:

$$U(C_t) = \frac{C_t^{(1-\sigma)}}{1-\sigma} \tag{18}$$

where the utility function curvature parameter σ is determines the elasticity intertemporal substitution and φ determines labour supply elasticity.

Assuming log-normal consumption growth, the SDF (equation 2) is:

$$M_{t+1} = \beta E_t \big(\frac{C_{t+1}}{C_t}\big)^{-\sigma} \tag{19}$$

and the Euler equation interest rate $r_t^f = -E_t log(M_{t+1})$ is:

$$r_t^f = -log(\beta) + \sigma E_t \Delta log(C_{t+1}) - \frac{\sigma^2}{2} var_t(\Delta log(C_{t+1})) \tag{20}$$

To account for the second order precautionary savings term, I include an exogenous preference shock, $\varepsilon_{\beta,t}$:

$$U(C_t) = \varepsilon_{\beta,t} \frac{C_t^{(1-\sigma)}}{1-\sigma} \tag{21}$$

The SDF is

$$M_{t+1} = \beta E_t \frac{\varepsilon_{\beta,t+1}}{\varepsilon_{\beta,t}} \big(\frac{C_{t+1}}{C_t}\big)^{-\sigma} \tag{22}$$

In this linear model, the change in the exogenous preference shock, ε_{β_t}, can be interpreted as capturing precautionary savings, the final term in (20).[11]

[11] Cochrane (2016) shows a wedge in the consumption Euler equation can capture a range of models proposed in the macro-finance literature to reconcile asset prices and returns with the macroeconomy. In contrast to the Y_{t+1} in the models discussed in (Cochrane 2016), here Y_{t+1} is an exogenous shock process and has no underlying structure that generates a business cycle correlation.

Equation (20) can be read as consumption causing interest rates, or as interest rates causing consumption (Cochrane 2001). In the standard model, causality goes both ways (everything affects everything else in general equilibrium), but the dominant channel is that interest rates, set by monetary policy, drive consumption growth. A monetary policy tightening requires consumption to initially fall sharply, so that consumption grows over the high interest rate period. With the introduction of risk, that dominant inter-temporal substitution channel is weakened considerably. The Euler equation rate is still affected by monetary policy, but only indirectly, through the effect of policy on expected future consumption.

3 Model calibration

The baseline calibration of model parameters is standard (Table 1). The rate of time preference is calibrated at 0.99, implying a steady-state real interest rate of 4%.[12] The Calvo parameter, θ is set at 0.7, in the middle of the range estimated by Lubik and Schorfheide (2006). The elasticity of inter-temporal substitution and the price elasticity of substitution among traded goods are both set at one, as in Galí and Monacelli (2005) and Galí and Monacelli (2016). The trade openness parameter is set at 0.2, compared to 0.13 in Lubik and Schorfheide (2006), 0.3 in Galí and Monacelli (2016) and 0.4 in Galí and Monacelli (2005) for small open economies. The monetary policy smoothing parameter is set at 0.7, as in Galí and Monacelli (2005), and the monetary policy response to inflation is set at 2. That is a bit higher than the 1.5 in a standard Taylor rule, but is required for stability in this two-country model.

The productivity shock persistence is set at 0.8, which is within the estimated range of Galí and Monacelli (2005) and Galí and Monacelli (2016), and slightly below the 0.83-0.85 range in Lubik and Schorfheide (2006).[13] The variance of the technology shock is calibrated at 1%, which is above the 0.6-0.5% range in the Gali and Monacelli papers, and below the 2-3% range in Lubik and Schorfheide (2006). The IMRS shock persistence is set at 0.6 and the variance at 3% following estimated values of Galí and Monacelli (2016) for a their global preference shock. The relatively large variance is in keeping with the equity premium literature implies that IMRS is volatile. The variance of the monetary policy shock is calibrated at 0.15%, which is between the values used in Lubik and Schorfheide (2006) and Galí and Monacelli (2016).

The papers cited above do not provide guidance on the cost push shock variance, and the persistence of the UIP shock. The cost push shock is assumed to be i.i.d., as

[12] While that rate is perhaps high relative to post-crisis experience, it maintains consistency with the broader literature.

[13] Galí and Monacelli (2016) estimate shock persistence and variance outside the model from AR(1) models of time series for US and Euro area data. Lubik and Schorfeide estimate these parameters jointly with model parameters for small open economies.

Table 1: Model calibration

Parameter	Value	Description
β	0.99	rate of time preference
θ	0.75	fraction of prices not re-optimised
σ	1	elasticity of inter-temporal substitution
η	1	elasticity of substitution traded goods
α	0.2	share of imports in consumption basket
γ	0.3	decreasing returns to labour
ρ_r	0.7	monetary policy smoothing
ϕ_π	2.0	monetary policy response to CPI inflation
AR(1) coefficients		
ρ_a	0.8	persistence of productivity shock
ρ_p	0	persistence of cost push shock
ρ_λ	0.1	persistence of bond risk shock
ρ_β	0.6	persistence of discount factor shock
Shock standard deviations (%)		
σ_r	0.15	Std deviation of monetary policy shock
σ_a	1	Std deviation of productivity shock
σ_p	0.5	Std deviation of cost push shock
σ_λ	3	Std deviation of relative bond risk (UIP) shock
σ_β	3	Std deviation of discount factor shock

is standard, and its standard deviation is calibrated at 0.5%. An AR(1) process was estimated for the UIP residual $\lambda_{t+1} \equiv \Delta q_{t+1} - r_t^d$ for US-EU data (using nominal exchange rates and nominal Treasury bills), for the period Feb 1999-June 2017. The persistence was estimated at 0.1 and the residual standard deviation at 3%, which is a bit lower than the 4.5% estimated by Lubik and Schorfheide (2006) for the standard deviation of the UIP shock, which is akin to the bond premium shock.

Sensitivity to parameter values is considered in discussing simulation results, where relevant.

4 Model dynamics

In this section, I focus on the responses to three shocks: the monetary policy shock, the bond risk shock and the IMRS shock. The monetary policy shock is interesting because the monetary policy transmission mechanism is fundamentally altered by the presence of bond risk. The bond risk shock and the shock to the Euler equation rate are interesting because evidence suggests that they are volatile (section 3), because their effects in this model are non-standard, and because they provide interpretations of empirical exchange rate behaviour.

Those three shocks all contribute to the wedge between expected consumption growth and the observed interest rate. The IMRS shock is a wedge between expected consumption growth and the Euler equation interest rate. The bond premium is a wedge between the Euler equation interest rate and the market interest rate that would prevail in the absence of monetary policy intervention in the bond market. The policy shock (and the endogenous policy rule) drives a wedge between the observed interest rate and the market-implied interest rate. Despite that commonality, the three shocks have very different effects.

Impulse response functions to all unit shocks are shown in Figures 1 to 5.

4.1 Monetary policy

An exogenous tightening of home monetary policy increases the nominal home interest rate (Figure 3, top right graph). In this simple model, with zero steady state debt and no capital stock, the effect of the higher interest rate on the household budget constraint is modest. The main transmission mechanism is through the effect on the exchange rate. The higher risk-adjusted return on the home bond leads to an appreciation of the home currency à la Dornbusch (1976) (q_t falls then rises gradually to offset the higher home risk-adjusted return).

In contrast to the standard model with a dominant inter-temporal substitution effect, here consumption can rise of fall, depending on the structure of the economy. In this model, lower import prices reduce home inflation and enable the home household to increase consumption (tope left graph). In a broader class of models, with non-zero asset and debt positions, variable labour and long-lived capital, the mechanism might be quite different. With output unchanged, the foreign household correspondingly reduces consumption. Home goods inflation falls further because the real wage, which is constant in CPI terms, declines with CPI inflation reduces, reducing home production costs.

The home Euler equation interest rate falls (top middle graph), reflecting the expected subsequent fall in home consumption. As result, the higher policy rate is reflected, more than one for one, in the home bond market premium. Accordingly the relative foreign bond market premium $\lambda_t^R = \lambda_t^F - \lambda_t^H$ falls (bottom left). The asymmetric pricing of risk in the bond market (policy intervention) relative to the currency market (no intervention) is reflected in a higher foreign currency premium $\lambda_t^{FX} = \lambda_t - \lambda_t^R$ (relative to home, bottom middle graph), which appreciates the home currency. Although we see a familiar (Dornbusch 1976)-type exchange rate response, here, the transmission mechanism works mainly through risk premia.

The higher policy rate could translate to a higher IMRS in an economy with variable labour, and net debt positions. With unit elasticities, variable labour dampens, but does not reverse, the rise in consumption. In an economy with net debt position, or with constrained households with net debt, the higher interest rate can reduce consumption. The point here is not the sign of the consumption response, per se. The point is that, in a model with risk, the effect on inter-temporal substitution is indirect, rather than direct, and the transmission of monetary policy involves both the IMRS and the bond market premium, and the balance depends on the structure of the economy.

4.2 Bond risk

Bond risk is priced in the Euler equations for home and foreign bonds, and through the UIP condition, is priced into the currency. Higher home bond risk reduces the shadow price of foreign bond risk, relative to home, λ_t (Figure 4, bottom right graph). Higher home bond risk is not directly reflected in the observed nominal interest rate, which follows a Taylor-type policy rule, but affects the home interest indirectly via its effect on inflation. Therefore the home currency must initially depreciate (q_t rises, equation 10). The weaker home currency increases the cost of imports, increasing home inflation. The constant CPI-based real wage increases the home firm's marginal cost, putting additional upward pressure on home inflation, and a tightening of home monetary policy. The rise in the policy interest rate implies a higher relative home interest rate path, R_t, and home currency appreciation, but that effect is small compared to the effect of higher bond risk on the currency. The home interest rate is high relative to the foreign interest rate, and the home currency appreciates as the foreign bond premium dissipates.

That response is associated with two important dynamics. First, it generates the forward premium puzzle: the high interest rate currency appreciates (Engel 2016, Fama 1984), in contrast to the risk-free model (discussed in more detail in section 5 below). Second, when bond risk cannot be expressed in the bond market, because of monetary policy stabilisation of the observed interest rate, it is reflected in the currency via the currency premium.

4.3 Inter-temporal substitution

In a standard macroeconomic model, a positive preference shock (eg reduced precautionary saving motive) increases expected consumption growth and prices, leading to a monetary policy tightening. In this model, the IMRS shock affects only the bond market premium and the currency premium (Figure 5).[14] Absent a rise in prices, the higher IMRS is not initially reflected in the observed interest rate, which follows a Taylor-type policy rule. With a higher IMRS, policy intervention in the bond market, to maintain the Taylor rule interest rate, compresses the home bond market premium. The lower home bond market premium raises the (foreign) relative bond market premium, λ_t^R. The asymmetry in the pricing of risk in the home bond market and the currency market reduces the foreign currency premium, $\lambda_t^{FX} = \lambda_t - \lambda_t^R$, relative to home. The higher IMRS appreciates the home currency,

[14] In a model with durable assets, a higher discount factor might have a material effect on the economy, for example via wealth and collateral effects.

but the lower foreign currency premium depreciates the home currency, leaving the currency unchanged (equation 15). Policy intervention to stabilise the observed interest rate isolates the exchange rate from variation the IMRS. The currency is "too smooth" relative to variation in the Euler equation interest rate (Brandt et al. 2006, Chien et al. 2015). This dynamic is discussed, in the context of model simulations, in section 7 below.

5 The forward premium puzzle

In a model without risk, theory predicts that a high interest rate currency (relative to others, relative to average), is strong in level terms, and depreciates each period to offset the higher home interest return. A large literature, summarised in Engel 2016, shows that, empirically, a high interest rate currency either does not depreciate to offset the higher home interest rate or even appreciates. In a risk-free model, that implies "excess returns" to holding the high interest currency. In te absence of risk, the value β_F in the following regression

$$\Delta q_{t+1} = \alpha_F + \beta_F(r_t - r_t^*) + \epsilon_{F,t+1} \tag{23}$$

is expected to be one. In contrast, empirical estimates of β_F are typically near zero or negative. A risk premium can potentially explain the disconnect between exchange rate movements and the interest differential: if the high interest rate is a risk premium, then the currency doesn't depreciate to offset the higher interest return because it is compensation for risk. While a risk premium can explain a high interest rate currency's failure to depreciate to fully offset the interest differential ($0 < \beta F < 1$), it cannot, on its own, explain negative values for β_F.

Estimates of equation (23), using simulated data for each model shock, are shown in Table 2. For most shocks, the prediction of the risk-free model, $\beta_F \sim 1$, holds. Consistent with empirical estimates, the explanatory power of the regression is weak, with an R^2 statistic of less than 0.2. Even in the standard risk-free model, exchange rate fluctuations are dominated by the initial Dornbusch 'overshooting' appreciation, in response to a high home interest rate, rather than by the subsequent gradual depreciation.

In contrast, for simulated data driven only by the IMRS shock, the estimated value of β_F is zero because there is no variation in r_t.

For data driven only by the bond risk shock, β_F is well below zero at -22.6.[15] For the combined shocks, for this calibration, the model delivers $\beta_F = -0.21$, well within the empirical range of $-1.4 \leq \beta_F \leq 0.6$ in (Engel 2016).

The risk shock can deliver large negative values for β_F because the bond risk shock and the monetary policy response have opposite affects on the currency,

[15] In a model without monetary policy, a risk shock delivers $\beta_F = 0$: the exchange rate doesn't adjust to offset the bond premium because it is compensation for risk. See (Engel 2016) for a detailed discussion.

and the bond risk dominates. A rise in home bond risk is not initially reflected in the nominal interest rate, which follows a Taylor-type rule. Therefore, higher home bond risk must, initially, be reflected in the currency. In equation (14), the change in λ_t is not offset by the change in λ_t^R, because monetary policy intervention to stabilise the nominal interest rate prevents the shadow price of risk from being expressed in the bond market premium. The higher home currency premium, $-\lambda FX_t = -(\lambda_t - \lambda_t^R)$, depreciates the home currency, increasing the import component of domestic inflation. The policy interest rate rises in response, slightly offsetting the effect of the bond premium on the currency. The relatively small policy change implies an initial appreciation of the home currency, followed by gradual subsequent depreciation. However, the bond risk shock implies a larger subsequent appreciation, as the bond shock dissipates, giving a potentially large negative value for β_F.

Sensitivity of the estimate of β_F to model parameters and shock persistence is shown in Figures (6) and (7) respectively. The simulated data is based on all model shocks, both home and foreign, in line with the calibration reported above. The parameter values are varied, one parameter at a time, over either the range commonly used or the range for which the model is determinate. Empirical estimates of β_F are typically near zero or negative. The values of β_F are most sensitive to the degree of openness, to price the Calvo stickiness parameter θ and to the degree of monetary policy smoothness. A more open economy, a low degree of stickiness and a monetary policy smoothing parameter of 0.3-0.5 deliver the most negative values for β_F. A high degree of price stickiness or policy smoothing moves β_F towards one. Negative values for β_F are seen for less persistent shocks to technology, prices (usually iid) and bond risk. Negative values for β_F do not require implausible values for model parameters.

Table 2: Fama Regressions

Fama Regression: $\Delta q_t = \alpha + \beta_F r^d_{t-1} + \epsilon_t$

	β_F	[95% confidence]	R^2
Fixed labour and constant real wage:			
Productivity	1.06	[1.01 , 1.11]	0.15
Cost push	0.96	[0.92 , 1]	0.19
Monetary policy	0.96	[0.92 , 1]	0.19
Bond risk	-22.6	[-23.4 , -21.8]	0.23
IMRS	0.00	[0 , 0]	0.00
All shocks	-0.21	[-0.43 , 0]	0.00

Table 3: The exchange rate level and the relative interest path

$$q_t = \alpha_R - \beta_R R_t^d + \epsilon_{R,t}$$
$$\text{where } R_t^d = E_t \sum_{t=0}^{\infty} r_t^d$$

	β_R	[95% confidence]	R^2
Fixed labour and constant real wage:			
Productivity	1.00	[1 , 1]	1.00
Cost push	1.00	[1 , 1]	1.00
Monetary policy	1.00	[1 , 1]	0.19
Bond risk	-9.15	[-9.3 , -9]	0.23
IMRS	0.00	[0 , 0]	0.00
All shocks	0.81	[0.78 , 0.85]	0.18

6 A high-interest rate currency is a strong currency

In the absence of monetary policy, a risk premium implies that a high interest currency should be weak relative to the sum of the relative interest rate path $-R_t = \sum_{t=0}^{\infty}(r_t - r_t^*)$. Since the currency does not need to depreciate to offset the higher interest return (it is compensation for risk), neither does it initially appreciate a là (Dornbusch 1976), leaving the currency unchanged, while the relative interest rate path R_t implies s strong currency. In contrast, Engel 2016 shows that a high interest rate currency tends to be a strong currency.

In Table 3, I report values for a regression of the exchange rate level on the relative interest rate path ($-R_t$):

$$q_t = \alpha_R - \beta_R R_t + \epsilon_{R,t}$$

R_t can be calculated directly in the model. The regression shows the value of the currency relative to the implied future relative interest rate path. When $\beta_R = 1$, The risk-free model predicts $\beta_R = 1$ - the currency is on the path implied by $-R_t$, and Engel's empirical results, based on VAR forecasts of $\Lambda_t = \sum_{t=0}^{\infty} \lambda_{t,t+1}$ suggest $\beta_R > 1$. For the first three shocks, the prediction of the risk-free model holds. Again, the exceptions are the IMRS shock and the bond risk shock. For the IMRS shock, there is no exchange rate response because monetary policy isolates the exchange rate from variation in the IMRS, as discussed in section 4.3.

In the absence of monetary policy, a rise in home bond risk would raise the home interest rate but have no effect on the currency. In that case, the currency doesn't depreciate to offset the higher return, because it is compensation for risk. Neither does the currency initially appreciate. In the absence of monetary policy, with no effect on the exchange rate, a bond risk shock, would be expected to deliver $\beta_R = 0$. That is, the currency would be weak relative to the interest rate path $-R_t$.

In the presence of monetary policy, the currency initially *depreciates* in response to a higher bond premium. Since the premium cannot be reflected in the bond yield, it must be reflected in the currency, as discussed in section 4.2. Therefore, the currency is not only weak relative to the expected relative interest rate path, but takes on the wrong sign, altogether. While a bond risk shock provides an interpretation of the the forward premium puzzle, it does not deliver Engel's result that a high interest currency tends to be a strong currency.

For the overall combination of shocks, however, a high interest rate currency is a strong currency. As Engel (2016) suggests, to deliver both empirical regularities, we need a combination of shocks: a persistent effect on monetary policy to deliver the exchange rate level, and a volatile bond risk shock to deliver the forward premium puzzle. The combination of shocks delivers an exchange rate that is strong (near the expected relative interest path, β_R, is near one) and that tends to appreciate ($\beta_F < 0$).

Sensitivity of estimates of β_R are show in figure 6 and 7. For some parameters - trade openness, and the persistence of the productivity, price and bond risk shocks - there is a tradeoff between the two empirical regularities, ie. when β_F is negative, β_R is low. Relative to the baseline calibration, achieving the two empirical regularities requires a transitory bond risk shock, an iid price shock (as is standard), a relatively persistent productivity shock, a Calvo parameter in the 0.3-0.7 range, and a monetary policy smoothing parameter below about 0.75.[16]

7 Exchange rates are "too smooth"

Unless risk aversion is implausibly high, the Euler equation interest rate, implied by the equity premium, is very volatile (Hansen and Jagannathan 1991). Brandt et al. (2006) show that, if exchange rates reflect relative Euler equation rates, then either exchange rates should be considerably more volatile than they are, or home and foreign Euler equation rates must be correlated, implying a higher degree of risk-sharing than is typically estimated.[17] In contrast, to explain the same puzzle, Chien et al. (2015) propose a model in which non-participation in financial markets drives down the cross-country correlation in aggregate consumption, implying a low degree of risk-sharing. Like Chien et al. (2015), the model proposed here involves market segmentation, but here, that segmentatino is the result of monetary policy intervention in short-term money markets. Policy intervention drives a wedge between the return on short-term bonds and the market-implied rate (the Euler equation rate plus the shadow price of bond risk).

[16] What this model does not deliver is a high interest rate currency that is stronger than the path implied by $-R_t$. Estimates of β_R are always less than or equal to one. Generating such a result requires some sort of over-adjustment, which may require a richer model.

[17] Full risk-sharing is rejected empirically (Backus and Smith 1993). Kose et al. (2003) show that cross-country consumption correlations did not increase in the 1990s, despite financial integration.

Table 4 reports the moments of simulated data from the model for each model shock and for a combination of shocks. For all shocks except the IMRS shock, the real exchange rate is more volatile than either Euler equation interest rate. There is no "too smooth" puzzle. In those cases (first four rows), the exchange rate is too volatile, compared to the volatility of the home and foreign Euler equation interest rates, because the Euler equation interest rates are negatively correlated (final column), and that negative correlation makes the currency more volatile than either Euler equation rate.[18]

Table 4: Exchange rate and Euler equation interest rate volatility

	$var(r_t^f)$	$var(r_t^{f*})$	$var(q_t)$	$cov(r_t^f, r_t^{f*})$
Fixed labour and constant real wage:				
Productivity	0.20	0.03	2.91	-0.06
Cost push	0.11	0.11	1.61	-0.11
Monetary policy	0.85	0.85	12.40	-0.85
Bond risk	0.42	0.39	1.08	-0.42
IMRS	1.21	0.00	0.00	0.00
All shocks	19.40	19.15	26.52	-7.70

In this model, the exchange rate is "too smooth" because policy intervention in the bond market to stabilise the observed interest rate isolates the exchange rate from variation in the Euler equation interest rate. A rise in the home Euler equation interest rate appreciates the currency (equation 10). However, with no direct effect on inflation,[19] monetary policy control of the observed interest rate means that the home bond market premium $\lambda_t^H = r_t - r_t^f$ must fall one-for-one with the rise in the Euler equation interest rate (Figure 5). That raises the relative foreign bond market premium $\lambda_t^R = \tilde{\lambda}_t^F - \tilde{\lambda}_t^H$. With the shadow price of relative bond risk, λ_t, unchanged, the currency premium $\lambda_t^{FX} = \lambda_t - \lambda_t^R$ must rise, depreciating the home currency, and fully offsetting the effect of the higher risk free rate in equation (10). The exchange rate is completely smooth, despite the increase in the home Euler equation interest rate. Monetary policy intervention in the local bond market isolates the currency from variation in the Euler equation interest rate.

Here, the IMRS shock is motivated by higher order terms in the Euler equation interest rate, such as precautionary savings. More generally, such a shock can capture a wide range range of alternative preferences and market structures (Cochrane 2016). In a broader class of models, the shock would be expected to have wider

[18] In a model with a richer transmission mechanism, for example, non-zero steady-state asset and debt holdings, that is not necessarily the case. The transmission mechanism in this model is intentionally parsimonious to clarify the effect of including bond risk.

[19] In a model with capital, a lower Euler equation rate implies that the returns to investment in new capital are discounted by less. That would encourage investment. However, a precautionary savings response to uncertainty, by the firm, would tend to reduce investment and inflation.

economic effects, for example through labour choices and through the present value of investment in long-lived assets.

8 Conclusion

If variation in asset returns and prices are mainly variations in risk premia (Cochrane 2016, Cochrane 2001), and observed interest rates are mainly driven by monetary policy, then we need a theory of how monetary policy interacts with risk premia (Alvarez et al. 2007a). This paper proposes such a model.

The key departure from the standard model is accounting for uncertainties about the value of ex-post bond payoffs that give rise to premia in bond yields, ex-ante. Examples are default risk, inflation risk and liquidity risk. Those risks drive a wedge between the observed short-term interest rate and the “risk-free” rate of inter-temporal substitution. In the absence of monetary policy, the bond yield compensates the holder of the bond for putting off consumption, for expected losses, and for the risk that losses materialise in bad times.

Bond risk drives a wedge between the observed policy rate, or short-term government interest rate, and the unobserved rate of inter-temporal substitution, or Euler equation interest rate. That departure from the standard model is motivated by empirical evidence that (i) short-term interest rates, even in US Treasuries, reflect significant risk premia, (ii) short-term risk premia are positively correlated with the stance of monetary policy, and (iii) estimated Euler equation interest rates are negatively correlated with the stance of monetary policy.

The transmission mechanism of monetary policy is both the same and different. The policy rate no longer directly drives inter-temporal substitution. The policy rate affects the economy, and in turn the Euler equation rate, through its effect on budget constrains, balance sheets and relative prices. Monetary policy intervention in the local bond market can affect the Euler equation interest rate, the bond market premium, or both. In this model, monetary policy works mainly the bond market premium. Despite working mainly through the bond market premium, monetary policy generates a familiar Dornbusch (1976) exchange rate response: a higher *risk-adjusted* home interest rate initially appreciates the home currency, which subsequently depreciates, period by period, to offset the higher *risk-adjusted* home bond return. Monetary policy has two additional effects on the currency: it shifts the expression of bond risk to the currency, and isolates the currency from variation in the Euler equation interest rate.

Model simulations shed light on some aspects of empirical exchange rate behaviour. The model delivers a high interest rate currency that is both strong in level-terms (Engel 2016) and exhibits excess returns on short-term bonds (Fama 1984). As proposed by Engel, a volatile bond premium delivers the forward premium puzzle, while persistent, familiar model shocks (eg, monetary policy, productivity, and price shocks) deliver an exchange rate level that is near the expected relative interest rate path. The model can deliver an exchange rate that is"too smooth" relative to Euler equation interest rates (Brandt et al. 2006). Here, the exchange rate is "too smooth" because monetary policy intervention in bond market isolates the exchange rate from variation in the (volatile) Euler equation interest rate.

This framework provides a fertile environment for future work. The central question of models used for monetary policy analysis is, How the stance of policy affect the economy? Without the direct inter-temporal substitution channel, the monetary transmission mechanism reflects the effects of policy interest rates on budget constraints, balance sheets and relative prices.

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Figure 1: IRF: Home productivity shock

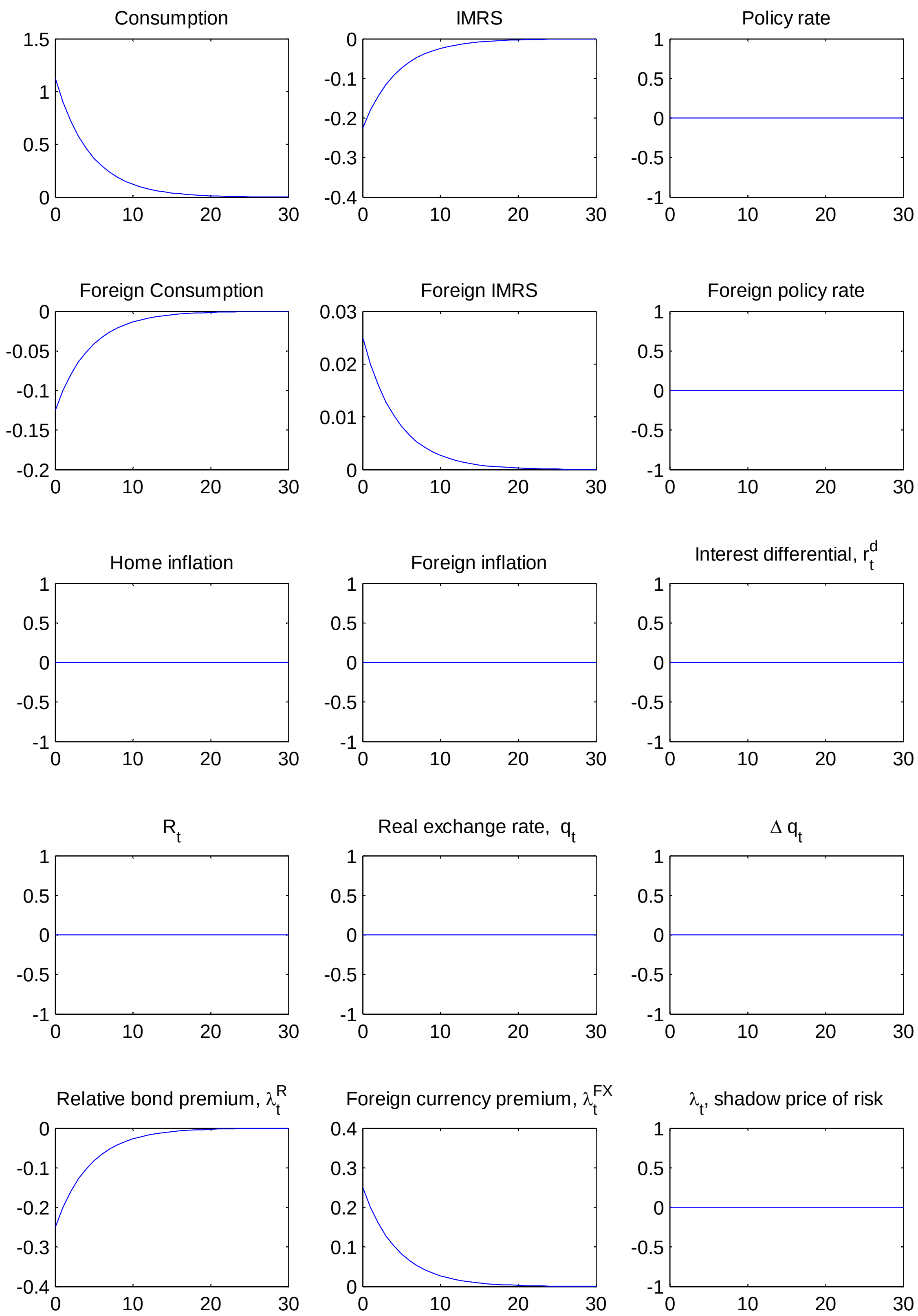

Figure 2: IRF: Home cost push shock

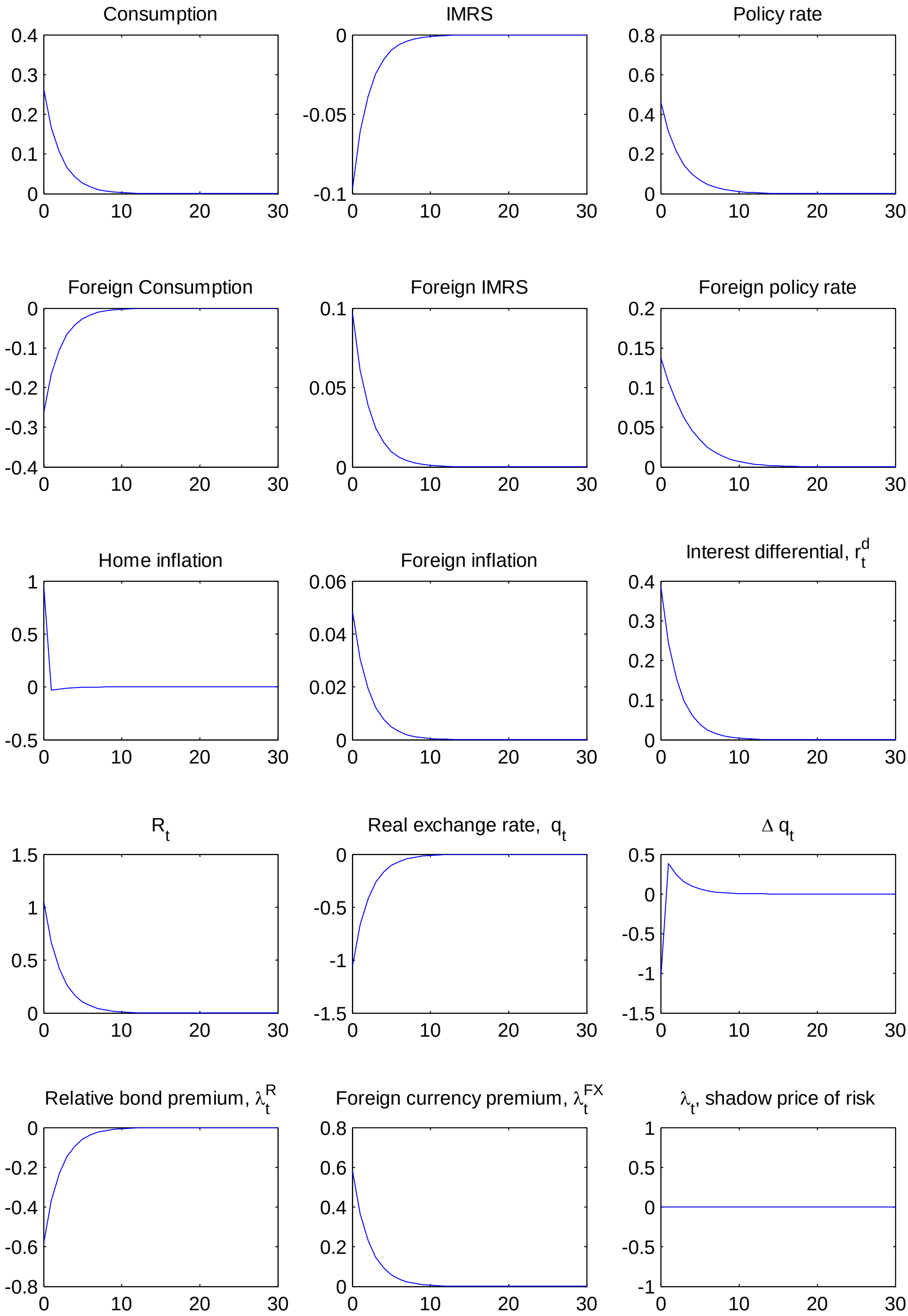

Figure 3: IRF: Home monetary policy shock

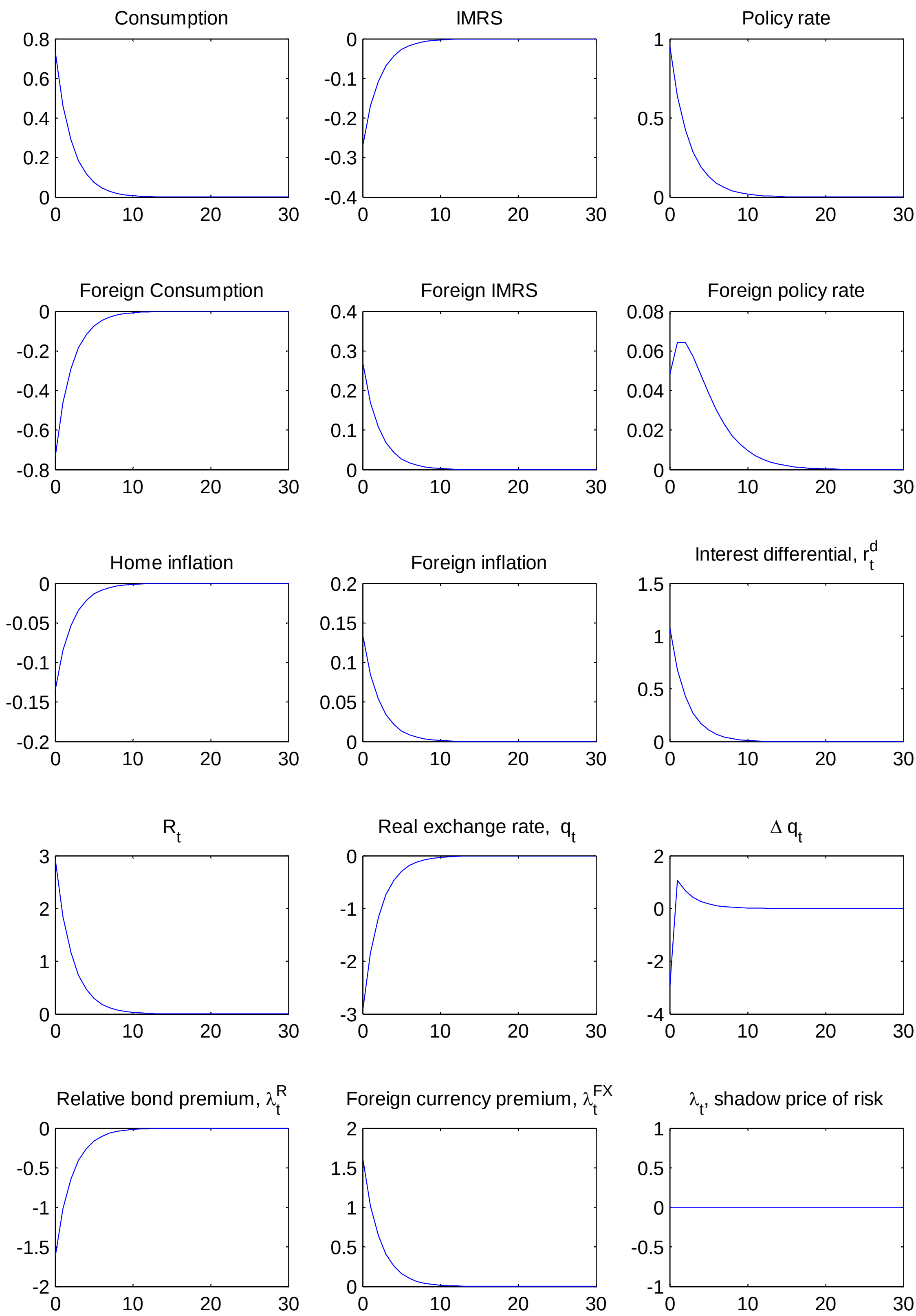

Figure 4: IRF: Home bond risk shock

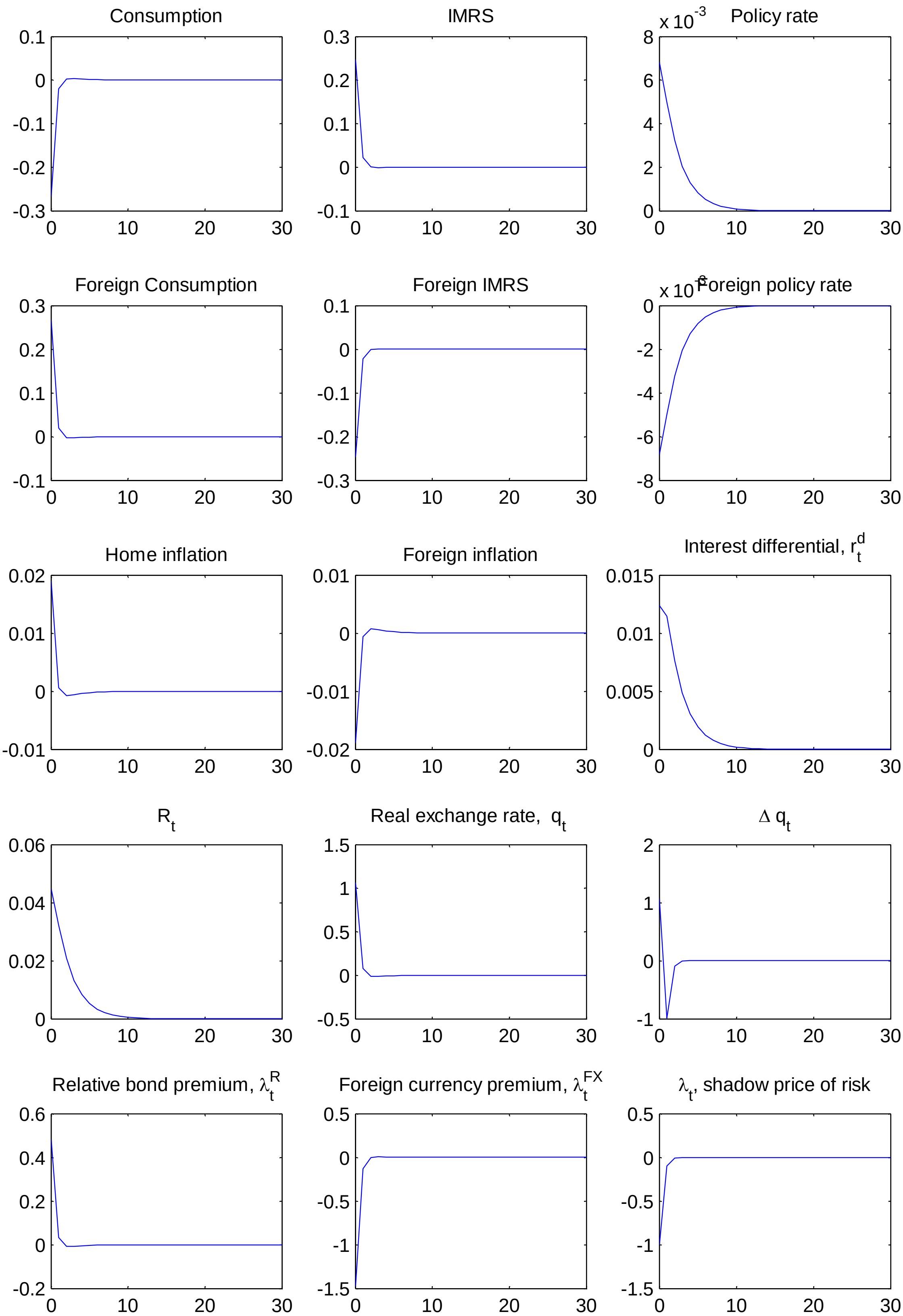

Figure 5: IRF: Home discount factor shock

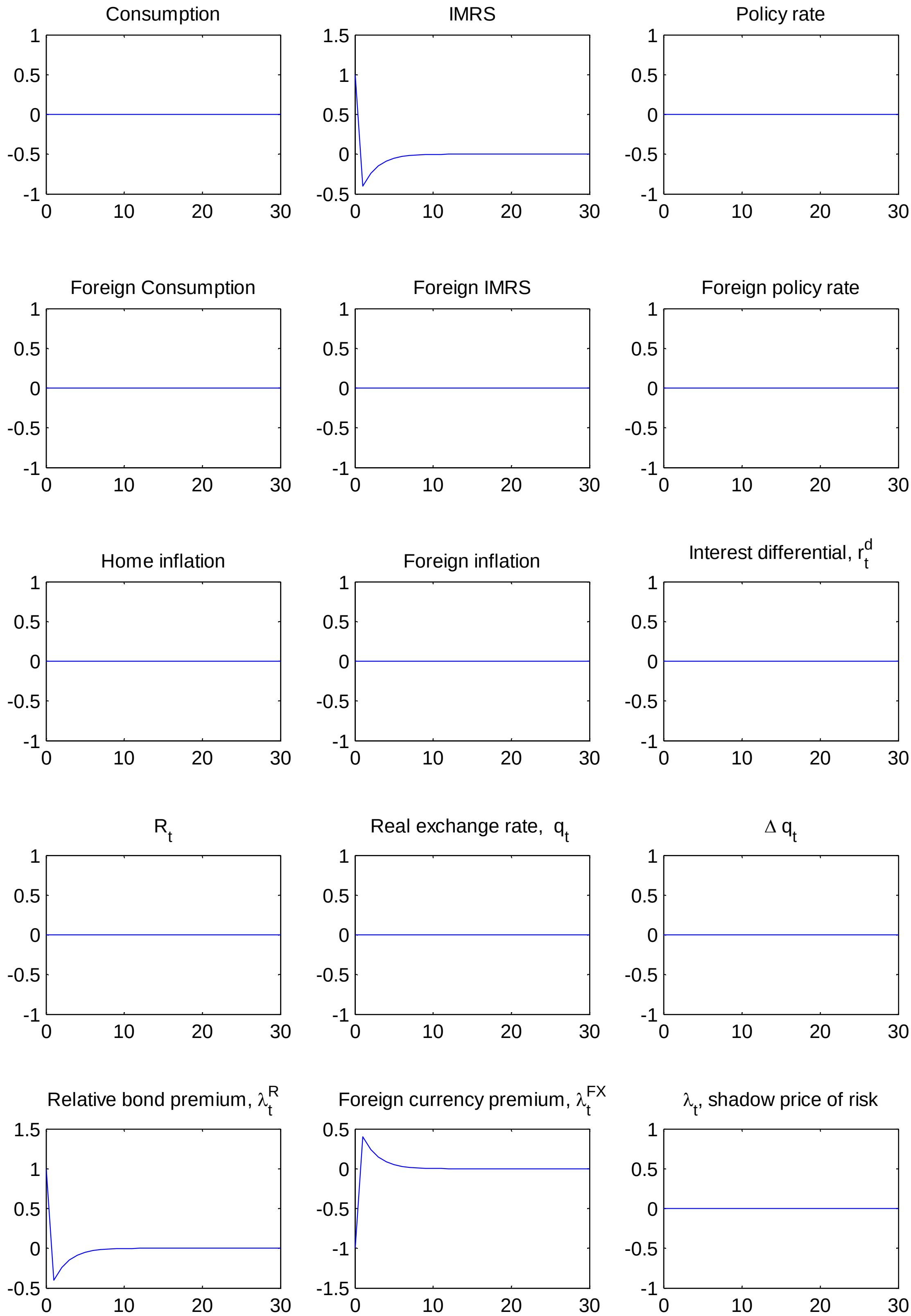

Figure 6: Parameter sensitivity

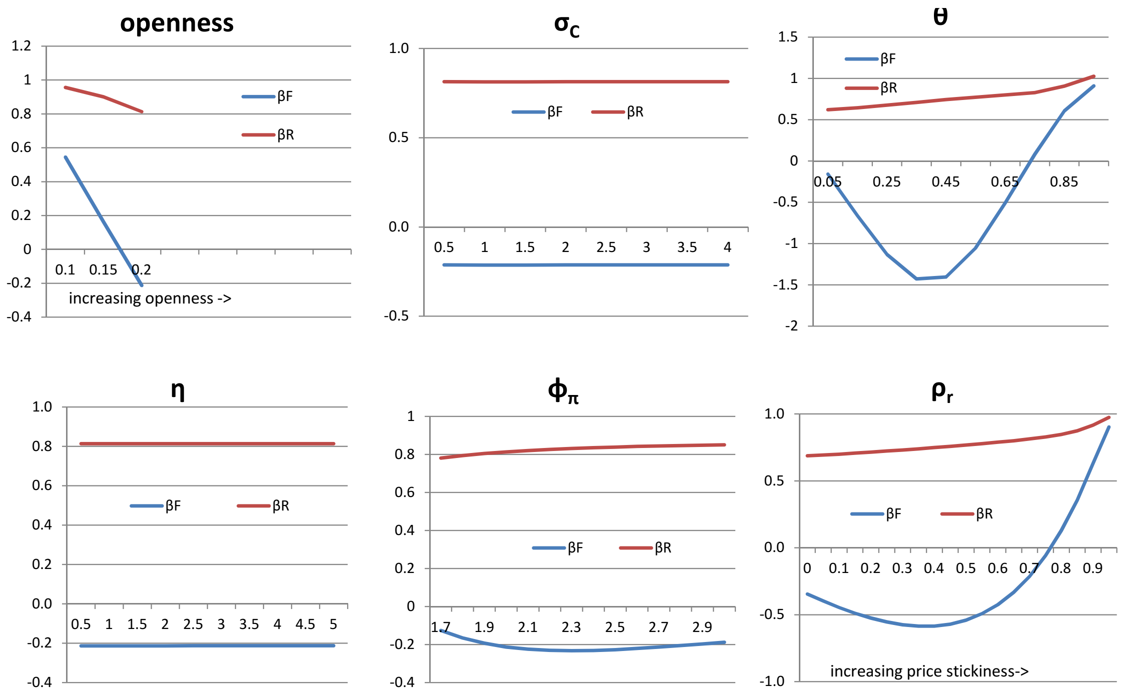

Figure 7: Sensitivity to shock persistence

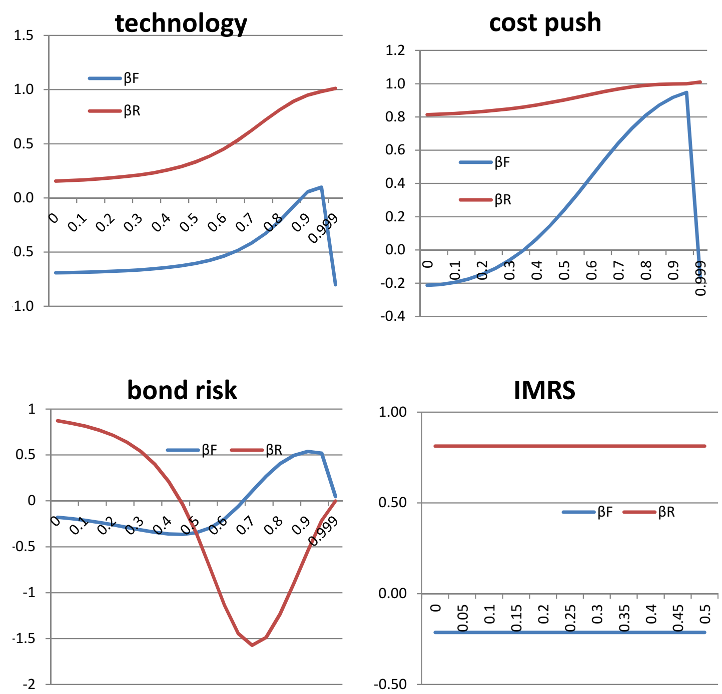

Table 5: Variance Decomposition

	Home shocks					Foreign shocks				
	Prod.	Cost push	MP	Bond risk	IMRS	Prod.	Cost push	MP	Bond risk	IMRS
c_t	16.5	1.7	1.2	38.1	0	1.6	1.7	1.2	38.1	0
r_t^f	1.0	0.1	0.1	20.2	57.9	0.1	0.1	0.1	20.2	0
i_t	64.0	12.4	4.6	0.1	0.0	17.4	1.4	0.1	0.1	0
π_t	52.2	41.7	0.1	0.6	0.0	4.5	0.2	0.1	0.6	0
r_t^d	11.7	21.9	15.2	1.2	0.0	11.7	21.9	15.2	1.2	0
R_t	34.8	8.6	6.0	0.6	0.0	34.8	8.6	6.0	0.6	0
q_t	7.2	1.8	1.2	39.8	0.0	7.2	1.8	1.2	39.8	0
Δq_t	0.6	0.9	0.6	47.9	0.0	0.6	0.9	0.6	47.9	0
λ_t^R	0.8	0.6	0.4	27.9	20.3	0.8	0.6	0.4	27.9	20.3
λ_t^{FX}	0.4	0.3	0.2	39.9	9.3	0.4	0.3	0.2	39.9	9.3
λ_t	0	0	0	50.0	0	0	0	0	50.0	0

Note: 5-year horizon.

A General equilibrium model

The basic general equilibrium model is a symmetrical, two-country version of the small open economy setup in (Galí and Monacelli 2005). The aim here is to describe the key ingredients of the model since the baseline model is fairly standard, referring the reader to the (Galí and Monacelli 2005) for detailed derivations.

1 Households

Two symmetrical economies are inhabited by representative households. The home household's consumption basket, C_t, is made up of home and foreign goods according to:

$$C_t = \left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{A1}$$

where $C_{H,t}$ is the basket of home goods consumed in the home country $C_{F,t}$ is the basket of foreign goods consumed in the home country, α is the steady-state import ratio, a measure of trade openness, η is the elasticity of substitution between home and foreign goods. The consumption baskets of home and foreign goods are made up of varieties, ι, given by:

$$C_{H,t} \equiv \Big(\int_0^1 C_{H,t}(\iota)^{\frac{\epsilon-1}{\epsilon}} \Big)^{\frac{\epsilon}{\epsilon-1}} \quad and \quad C_{F,t} \equiv \Big(\int_0^1 C_{F,t}(\iota)^{\frac{\epsilon-1}{\epsilon}} \Big)^{\frac{\epsilon}{\epsilon-1}} \tag{A2}$$

where $\epsilon > 0$ is the elasticity of substitution between varieties.

Home household preferences are given by $U(C_t, N_t)$, N_t is labour input. The household chooses consumption, labour input and bond holdings to maximise expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_{\beta,t} U(C_t, N_t)$$

subject to the period budget constraint:

$$\int_0^1 P_{H,t}(\iota) C_{H,t}(\iota) d\iota + \int_0^1 P_{F,t}(\iota) C_{F,t}(\iota) d\iota + (1+i_{t-1}) B_{t-1} + \varepsilon_t B_t^*$$
$$= W_t N_t + B_t + \varepsilon_t (1+i_{t-1}^*) B_{t-1}^* \tag{A3}$$

where the parameter β is the subjective discount factor, and $\varepsilon_{\beta,t}$ is a preference shock. On the left-hand side of (A3), the household uses income to buy varieties (ι) of home and foreign goods, to buy new nominal foreign bonds, B_t^*, and to repay, with interest, last period's nominal home bonds B_t, where i_t is the home nominal interest rate and ε_t is the nominal exchange rate, defined as the value of the foreign currency in units of the home currency. On the right hand side of (A3), the household receives labour income $W_t N_t$, the proceeds of the sales of home bonds, and interest income from last periods's holdings of foreign bonds $\varepsilon_t(1+i_t^*)B_{t-1}^*$, where i_t^* is the foreign nominal interest rate, and W_t is the nominal wage.

Preferences are assumed to take the functional form:

$$U(C_t) = \varepsilon_{\beta,t}\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{N_t^{1+\varphi}}{1+\varphi}, \tag{A4}$$

where the curvature parameter σ determines the elasticity of inter-temporal substitution, and φ is the inverse elasticity of labour supply. Given the constant elasticity of substitution aggregator (A1), and assuming symmetry across all ι goods, the optimal allocation of expenditure between home and imported goods is:

$$C_{H,t} = (1-\alpha)\Big(\frac{P_{H,t}}{P_t}\Big)^{-\eta}C_t \quad \text{and} \quad C_{F,t} = \alpha(\frac{P_{F,t}}{P_t})^{-\eta}C_t \tag{A5}$$

where $P_{H,t}$ is the price index of home-produced goods and $P_{F,t}$ is the price index of foreign-produced goods. The home consumer price index is

$$P_t = \Big((1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\Big)^{\frac{1}{1-\eta}} \tag{A6}$$

The household's real stochastic discount factor, to discount period $t+1$ returns, M_{t+1} is defined as:

$$M_{t+1} = \beta E_t \frac{\varepsilon_{\beta,t}}{\varepsilon_{\beta,t+1}}\frac{U_{C,t}}{U_{C,t+1}} \tag{A7}$$

The inter-temporal marginal rate of substitution (IMRS) is $r_t^f = -log(M_{t+1})$. The IMRS, or Euler equation interest rate, is the *unobserved* return on a hypothetical bond that pays off in units of consumption utility growth. It is the expected compensation required to put off a unit of consumption from today until tomorrow. It is known, in expectations, at time t.

Fisher equations relate observed nominal interest rates to expected real interest rates r_t and expected inflation $E_t\pi_{t+1} = log(E_tP_{t+1}/P_t)$:

$$i_t = r_t + E_t\pi_{t+1}$$

and the real exchange rate, Q_t, is defined in terms of the home and foreign CPI baskets:

$$Q_t = \varepsilon_t\frac{P_t^*}{P_t}, \quad \text{or, in logs,} \quad q_t = e_t + p_t^* - p_t \tag{A8}$$

where $q_t = log(Q_t)$, $e_t = log(\varepsilon_t)$, $p_t = log(P_t)$ and $p_t^* = log(P_t^*)$.

The household's optimisation problem yields the set of first order conditions in (5) and (6), and the following equation:

$$\frac{W_t}{P_t} = \frac{U_{N,t}}{U_{c,t}} \tag{A9}$$

that equates the marginal rate of substitution between labour and consumption to the CPI-based real wage. In log terms,

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{A10}$$

In the baseline model, I assume labour supply to be constant and the CPI-based real wage to be constant.

An equivalent set of equations holds for the foreign household, indicated by an asterisk, *. The degree of openness, α, the elasticity of substitution among varieties, ϵ, and the elasticity of substitution between home and foreign goods, η, are assumed to be the same for home and foreign households.

2 Inflation and the terms of trade

For simplicity, the law of one price is assumed to hold for individual goods that are traded. That is, import costs are completely passed through, so that:

$$P_{F,t} = \varepsilon_t P^*_{F,t} \quad \text{and} \quad \varepsilon_t P^*_{H,t} = P_{H,t}$$

where, $P^*_{H,t}$ and $P^*_{F,t}$ are the foreign currency prices of home and foreign goods consumed in the foreign country.

Define the terms of trade, S_t as the relative price of foreign goods:

$$S_t = \frac{P_{F,t}}{P_{H,t}}, \qquad \text{or, in logs,} \qquad s_t = p_{F_t} - p_{H,t} \tag{A11}$$

where $s_t = log(S_t)$. A rise in the terms of trade represents an increase in competitiveness of home goods.

Log-linearising the CPI formula around the steady state gives:

$$\begin{aligned} p_t &= (1-\alpha)p_{H,t} + \alpha p_{F,t} \\ &= p_{H,t} + \alpha(p_{F,t} - p_{H_t}) \\ &= p_{H,t} + \alpha s_t \end{aligned}$$

The difference between CPI inflation and home inflation is proportional to the terms of trade multiplied by the degree of openness. Combining (A8), linear versions of (A5) and its foreign equivalent, the real exchange rate, in this symmetrical 2-country setup, is equal to the terms of trade:[20]

$$\begin{aligned} q_t &= e_t + p^*_t - p_t \\ &= e_t + (1-\alpha)p^*_{F,t} + \alpha p^*_{H,t} - (1-\alpha)p_{H,t} - \alpha p_{F,t} \\ &= (1-\alpha)(e_t + p^*_{F,t}) + \alpha(e_t + p^*_{H,t}) - (1-\alpha)p_{H,t} - \alpha p_{F,t} \\ &= (1-\alpha)p_{F,t} + \alpha p_{H,t} - (1-\alpha)p_{H,t} - \alpha p_{F,t} \\ &= p_{F,t} - p_{H,t} \\ &= s_t \end{aligned}$$

[20]In contrast to the standard Gali and Monacelli small open economy setup where the two are proportional.

3 Production and domestic price setting

Domestic households produce home goods according to the production function,

$$Y_t = A_t N_t \bar{K} \tag{A12}$$

where Y_t is home output, A_t is productivity and $\bar{K}$ is the capital stock, assumed to be constant. Log productivity, $a_t = log(A_t)$ is assumed to evolve according to an AR(1) process $a_t = \rho_a a_{t-1} + \epsilon_{a,t}$ where, $0 < \rho_a < 1$ and $\epsilon_{a,t} \sim N(0, \sigma_a^2)$.

The labour market is assumed to be perfectly competitive. The nominal wage is equal to the marginal product of labour:

$$\frac{(1-\gamma)P_{H,t}Y_t}{N_t} = W_t$$

Following Gali and Monacelli 2005, producers are assumed to have a degree of monopoly power in setting the prices of their varieties, and to set their prices in a staggered fashion à la Calvo (1983). In each period, a fraction $(1-\theta)$ of firms re-optimise their prices ($\theta \in [0,1]$), subject to a series of constant elasticity demand schedules. The remaining fraction θ increase their prices by steady-state inflation (which is set to zero). The Calvo pricing structure yields a Philips Curve, whereby inflation is a weighted average of expected future inflation and real marginal cost $mc_{H,t}$:

$$\pi_{H,t} = p_{H_t} - p_{H,t-1} = \beta E_t(\pi_{H,t+1}) + \frac{(1-\beta\theta)(1-\theta)}{\theta} mc_{H,t} \tag{A13}$$

The firm's total nominal cost is $W_t N_t$ and the marginal cost is:

$$MC_t = \frac{d}{dY} W_t \frac{Y_t}{A_t} = \frac{W_t}{A_t}$$

The firm's real marginal cost, in log terms, is

$$\begin{aligned} mc_{H,t} &= w_t - p_{H,t} - a_t \\ &= (w_t - p_t) + (p_t - p_{H,t}) - a_t \\ &= (w_t - p_t) + \alpha s_t - a_t \end{aligned}$$

Marginal cost is low when consumption is low (people want to work more), when foreign prices are low relative to home (the terms of trade is low or the foreign currency is weak) or when home productivity is high.

For the baseline model, I assume that labour supply is constant, and that the firm pays a constant CPI-based real wage. In that case, production is only a function of productivity $Y_t = A_t$, and the firm's marginal cost is $mc_{H,t} = \alpha s_t - a_t$.

4 Market clearing

Goods market clearing requires that domestic output is the sum of home goods consumed at home and abroad, and foreign output is the sum of foreign goods consumed at home and abroad. In log-linear terms,

$$Y_t = C_{H,t} + C^*_{H,t} \quad \text{and} \quad Y^*_t = C_{F,t} + C^*_{F,t} \tag{A14}$$

Substituting A5 and their foreign equivalents into A14,

$$\begin{aligned} Y_t &= C_{H,t} + C^*_{H,t} && \text{(A15)} \\ &= (1-\alpha)\big(\frac{P_{H,t}}{P_t}\big)^{-\eta} C_t + \alpha(\frac{P_{H,t}}{\varepsilon_t P^*_t})^{-\eta} C^*_t && \text{(A16)} \\ & && \text{(A17)} \end{aligned}$$

In log-linear terms, using $\bar{Y} = \bar{Y^*} = \bar{C} = \bar{C^*} = 1$

$$\begin{aligned} y_t &= (1-\alpha)(-\eta)(p_{H,t} - p_t) + (1-\alpha)c_t + \alpha(-\eta)(p_{H,t} - e_t - p^*_t) + \alpha c^*_t && \text{(A18)} \\ &= (1-\alpha)(-\eta)(-\alpha s_t) + (1-\alpha)c_t + \alpha(-\eta)(p_{H,t} - p_t + p_t - e_t - p^*_t) + \alpha c^*_t && \text{(A19)} \\ &= (1-\alpha)(-\eta)(-\alpha s_t) + (1-\alpha)c_t + \alpha(-\eta)(-\alpha s_t - q_t) + \alpha c^*_t && \text{(A20)} \\ &= (1-\alpha)c_t + \alpha c^*_t + \eta s_t(\alpha(1-\alpha) + \alpha^2 + \alpha) && \text{(A21)} \\ &= (1-\alpha)c_t + \alpha c^*_t + 2\alpha\eta s_t && \text{(A22)} \end{aligned}$$

For the foreign economy, the same manipulations yield

$$y^*_t = (1-\alpha)c^*_t + \alpha c_t - 2\alpha\eta s_t \tag{A23}$$

B Risk

This appendix provides examples of common risks that arise from uncertainty about the value of ex-post payoffs.

Default premium

Consider a 1-period bond that is contracted at a gross rate $(1 + r^c_t)$, payable at $t+1$, but that defaults with a non-zero probability. The pricing equation can be written

$$1 = E_t[M_{t+1}(1 + r^c_t)(1 - d_{t+1})]$$

where $d_t \in [0,1]$ captures both the probability of default and loss in the event of default. The log pricing equation is:

$$\begin{aligned} 0 &= log[E_t\big(M_{t+1}(1 + r^c_t)(1 - d_{t+1})\big)] \\ &= log\big(e^{(E_t m_{t+1} + E_t r^c_t - E_t d_{t+1} + \frac{1}{2} var_t(m_{t+1} + r^c_t - d_{t+1}))}\big) \\ &= -r^f_t + r^c_t + logE_t(1 - d_{t+1}) - cov_t(m_{t+1}, d_{t+1})) \\ r^c_t &= r^f_t - logE_t(1 - d_{t+1}) + cov_t(m_{t+1}, d_{t+1}) \end{aligned}$$

The contracted rate is known ex-ante, so $logE_t(1 + r^c_t) \sim r^c_t$ and $cov_t(r^c_t, m_{t+k}) = 0$. Defining the default premium as the difference between the contracted rate and the risk-free rate,

$$\begin{aligned}\text{default premium} &\equiv r_t^c - r_t^f \\ &= -log(E_t(1-d_{t+1})) + cov_t(m_{t+1}, d_{t+1}),\end{aligned}$$

The default premium reflects the expected loss and a risk correction that increases the contracted yield if losses from default are expected to be higher when the marginal utility of consumption rises. The default premium is part of the bond premium, λ_t^H.

Inflation premium

Inflation risk is part of the default premium above. Writing the bond Euler equation in terms of the nominal interest rate:

$$1 = E_t[M_{t+1}\frac{(1+i_t^c)}{(1+E_t\pi_{t+1})}(1-d_{t+1})]$$

where, d_{t+1} now excludes the effects of inflation.

$$\begin{aligned}0 &= log[E_t\big(M_{t+1}\frac{(1+i_t^c)}{(1+E_t\pi_{t+1})}(1-d_{t+1})\big)] \\ &= log\big(e^{(E_t m_{t+1} + E_t i_t^c - E_t\pi_{t+1} - E_t d_{t+1} + \frac{1}{2}var_t(m_{t+1}+i_t^c-\pi_{t+1}-d_{t+1}))}\big) \\ &= -r_t^f + i_t^c - log(E_t(1+\pi_{t+1}) + cov_t(m_{t+1},\pi_{t+1})) + logE_t(1-d_{t+1}) - cov_t(m_{t+1},d_{t+1}) \\ r_t^c &= i_t^c - E_t\pi_{t+1} = r_t^f \underbrace{+\tfrac{1}{2}var_t(\pi_{t+1}) - cov_t(m_{t+1},\pi_{t+1})}_{\text{inflation risk}} - logE_t(1-d_{t+1}) + cov_t(m_{t+1},d_{t+1})\end{aligned}$$

The inflation captures thee risk that inflation erodes the ex-post value of the nominal payoff, and the risk that it does so in bad times.

Term premium

While the assumption that the term premium is small is relevant for overnight securities, for the monthly or quarterly returns that are often used in empirical analysis, term premia may be material. Consider a two-period fixed-rate bond that pays a certain $(1+r_{2,t}^c)^2$ at $t+2$. The pricing equation, $1 = E_t[M_{t+1}M_{t+2}(1+r_{2,t}^c)^2]$, equates the cost of buying the bond today with the expected value of the payoff at $t+2$. The log of the pricing equation is:

$$\begin{aligned}0 &= logE_t[M_{t+1}M_{t+2}(1+r_{2,t}^c)^2] \\ &= log\big(e^{(E_t m_{t+1} + E_t m_{t+2} + 2E_t r_{2,t}^c + \frac{1}{2}var_t(m_{t+1}+m_{t+2}+2r_{2,t}^c))}\big) \\ &= -E_t r_t^f - E_t r_{t+1}^f + 2E_t r_{2,t}^c + cov_t(m_{t+1},m_{t+2}) \\ 2r_{2,t}^c &= E_t r_t^f + E_t r_{t+1}^f - cov_t(m_{t+1},m_{t+2}) \qquad \text{(B1)}\end{aligned}$$

Since the contracted rate $r_{2,t}^c$ is known with certainty at time t, $cov_t(r_{2,t}^c, m_{t+k}) = 0$. However, the state of the economy, and so the marginal utility of consumption in subsequent periods is not known with certainty, so $cov_t(m_{t+1}, m_{t+2}) \neq 0$. Defining the term premium as the holding-period return on the 2-period bond net of the return on rolling over a 1-period risk-free bond,

$$\begin{aligned} \text{term premium} &\equiv 2r^c_{2,t} - r^f_t - r^f_{t+1} \\ &= -cov_t(m_{t+1}, m_{t+2}) \end{aligned}$$

The term premium compensates the holder for uncertainty about the marginal utility of consumption in the future. To generate a positive term premium, the stochastic discount factor must have negative serial correlation: $cov_t(m_{t+1}, m_{t+2}) < 0$. Negative serial correlation in the risk-free rate means that holding a multi-period bond with a fixed nominal payoff makes consumption more volatile. If the payoff $r^c_{2,t}$ helps to smooth consumption today, the negative serial correlation between m_{t+1} and m_{t+2} means that it is unlikely to help to smooth consumption next period. A term premium doesn't require uncertainty about bond payoffs, but uncertainty about the state of the economy that affects the value of bond payoffs. The term premium is part of the bond premium, λ^H_t.

Liquidity premium

The price of a short-term bond can deviate considerably from its hold-to-maturity value because of collateral value, demand and supply, and short-term safety factors.

Consider holding the 2-period bond described above, but with a non-zero probability that the bond will be sold, at $t+1$, to smooth consumption, subject to a liquidation cost. The pricing equation is

$$1 = E_t[M_{t+1}M_{t+2}(1 + r^s_{2,t})^2(1 - d_{t+1})]$$

where $(1 + r^s_{2,t})$ is the gross yield on the bond that may need to be sold, and d_{t+1} captures both the probability that the bond will be sold at $t+1$ and the expected discount if the bond is sold, relative its hold-to-maturity value. The log of the pricing equation is:

$$\begin{aligned} 2r^s_{2,t} &= E_t r^f_t + E_t r^f_{t+1} - logE_t(1 - d_{t+1}) - cov_t(m_{t+1}, m_{t+2}) \\ &\quad + cov_t(m_{t+1}, d_{t+1}) + cov_t(m_{t+2}, d_{t+1}) \end{aligned}$$

The observed, contracted rate on the bond reflects expected risk-free returns, expected losses from selling the bond before maturity, a term premium, and a risk correction that increases the yield on the bond if losses are expected to be greater when the marginal utility of consumption is expected to rise.

Defining the liquidity premium as the yield on the bond that is sold at a discount at $t+1$, net of the yield on the 'liquid' bond (B1),

$$\begin{aligned} \text{liquidity premium} &\equiv r^s_{2,t} - r^c_{2,t} \\ &= \tfrac{1}{2}(logE_t(1 - d_{t+1}) + cov_t(m_{t+1}, d_{t+1})) \end{aligned}$$

the liquidity premium captures the expected loss from selling the bond, $logE_t(1 - d_{t+1})$, and a risk correction. If investors are more likely to liquidate bonds to smooth consumption when the marginal utility of consumption rises, the expected loss from selling illiquid bonds is likely to be positively correlated with m_{t+1}. The expected discount d_{t+1} can also be interpreted as a transaction cost associated with selling the bond or 'specialness' (Krishnamurthy and Vissing-Jorgensen 2012, Vayanos 1998 and Aiyagari and Gertler 1991). If a bond is expected to sell at a premium in bad times $E_t d_{t+1} < 0$, for example when the market wants to hold high quality assets and collateral − a 'flight to quality' response − then the yield on the bond will be lower, reflecting its expected liquidity value. Feldhütter and Lando (2008), Duffie (1996) and Amihud and Mendelson (1991) estimate short-term safety factors, to be substantial for US Treasuries. (Nagel 2014) links a liquidity premium to monetary policy. The liquidity premium is part of the bond premium, λ_t^H.