Revisiting the linkages between real estate and equity markets through the lens of a wavelet analysis

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Abstract

Understanding the relationship between real estate and equity markets is of paramount importance, even more after the U.S. subprime mortgage crisis in 2008 and as witnessed by the Global Financial Crisis (GFC). In this paper, we take a comprehensive perspective of those linkages by focusing on the Real Estate Investment Trust (REIT) segment and relying on continuous wavelet analysis and frequency causality.Using monthly data for the U.S. over the period January 1972 - April 2014, we show that the co-movement between U.S. real estate and equity returns varies both across frequencies and over time, which can be informative about the existence of diversification benefits. Our empirical evidence also provides important insights about the direction of causality between the two asset returns, which sheds light about the hedging opportunities between them. It means that a dedicated equity-real estate portfolio would have to consider the effects of time and frequencies, in order to maximize the diversification benefits.

Keywords: equity markets, real estate sector, wavelet analysis.
JEL Classifications: C49, E44, G11.

1. Introduction

The U.S. stock and real estate markets contribute to investors' portfolio diversification and denote the largest wealth components of households (Sousa, 2010). Thus, changes in their market values can greatly impact growth.

It is also apparent that booms and busts in real estate and stock markets have been prominent fingerprints of the history of U.S. business cycles, e.g. the 1987 stock market crash, the Iraq-Kuwait war of 1990-1991, the Dot-com bubble of 1997-2000 and subsequent burst in 2000-2001,the housing boom of 2005-2007 and the subprime mortgage crisis of 2007-2008. In this context, busts have always hampered U.S. economic growth or worsened the economic slowdown and increased unemployment, while booms typically boosted production.

The seminal works assessing the relationship between real estate and stock prices have generally focused on the U.S. and relied on the analysis of the correlation between the two asset prices.[1]Other and more recent studies have applied cointegration[2] and causality[3] tests. And the empirical evidence has, so far, produced mixed results, with some some authors detecting co-movement among the two asset markets (Ambrose et al., 1992; Simon and Ng, 2009), and others uncovering market segmentation(Myer and Webb, 1993; Wilson and Okunev, 1999). Additionally, while somestudies finda causal relationship running from the real estate market to the stock market (Lu et al., 2007), other pieces of research show that the stock market leads the real estate market (Gyourko and Keim, 1992; Okunev et al., 2000; Goodness et al., 2013).

[1]SeeIbbotson and Siegel (1984), Liu and Mei (1992), Quan and Titman (1999) and Liow (2012).
[2]See, for instance: Okunev and Wilson (1997) and Tsai et al. (2012), on the U.S.; Tse (2001), for Hong Kong; Liow and Yang (2005), on Singapore and Hong Kong; Goodness et al. (2013), for South Africa).
[3]See, for example: Gyourko and Keim (1992), Okunev et al. (2000), Tsai et al. (2012) and Tsai (2015), on the U.S.; Okunev and Wilson (1997), on Australia; Liow (2006), on Singapore; Sim and Chang (2006), on Korea; Goodness et al. (2011), on South Africa.

Against this background, the existing research has almost exclusively employedconventional time-domain methods. In contrast, less attention has been paid to the frequency-domain analysis. Thus, our study intends to contribute to fill this gap in the literature.

We capture the performance of the real estate market via the listed real estate investment trusts (REITs),which were established by the U.S. government in the sixties and own many types of real estate, including apartments, offices, hotels and warehouses. And while most REITs are equity REITs, the long-term path of equity REIT returns has been different from that of equity returns, which gives rise to the role played by these funds as an important source of portfolio diversification (Wilshire Associates, 2012). Additionally, we consider the long-term relationship between these two markets, because the real estate generally is a long-term investment due to its high transaction costs and relatively low liquidity (Oikarinen et al., 2011). Moreover, REITs are an established asset class that is easily accessible to investors, as they can be converted into tradable instruments such as mortgage-backed securities (MBSs).

From an econometric perspective, we rely onwavelet analysis and frequency causality, which allows the investigation of lead-lag effects and offers new insights about the co-movement between real estate and equity markets with respect to both the time- and the frequency-domain.This analytical framework is particularly suitable for our research question, as both asset markets include heterogeneous traders and investors making decisions over different time horizons. Time-scale co-movement and causality are also essential for the purpose of portfolio diversification since investment risks might be different across frequencies.

Using U.S. data over the period 1972:M1-2014:M4, our results reveal that the co-movement between U.S. real estate and equity returns indeed varies across frequencies and over time. Thus, the empirical evidence provides relevant information about the time- and frequency-domains where diversification benefits exist. It also provides important findings about the direction of causality between the two asset returns. This sheds some light about the hedging opportunities between them.

Our study is highly indebted to the works by Zhou (2010) and Chang et al. (2015). Zhou (2010) investigates the co-movement between stock and securitised real estate markets for six developed countries (i.e. Australia, Hong Kong, Japan, Singapore, United States and United Kingdom) over the period 1990-2012. The results show a strong cross-market co-movement at the long range of frequencies in Hong Kong, Japan and Singapore, but only for a selective band of frequencies in the case of Australia, the United Kingdom and the United States. Chang et al. (2015) assess the linkages between real house prices and stock prices in the United States over the period 1890-2012. Despite the generally positive co-movement, a strongly negative co-movement prevailed in the period 1998-2002. Over the same time frame, as well as in the second half of the nineties, the frequency-domain results show that both markets correlated at high frequencies.While we also rely on a wavelet analysis framework, we look, in addition, at the lead-lag effects between the two asset markets under consideration and evaluate potential non-linear causality relationships.

The remainder of the paper is structured as follows. Section 2 presents the econometric methodology. Section 3 describes the data and discusses the empirical results. Finally, Section 4 concludes.

2. Econometric Methodology

2.1 Lead-lag effects

The innovative features for the wavelet transform analysisin the U.S. stock and real estate markets include the maximal overlap discrete wavelet transform (MODWT) and multi-resolution analysis (MRA) (Percival and Walden, 2000; Gençay et al., 2002). According to the wavelet analysis, any function $f(t)$ in $L^2(R)$ can be decomposed into components associated with different scales of resolution. The wavelet representation of the function *f(t)*is defined by the following form

$$f(t)=\sum_k s_{J,k}\phi_{J,k}(t)+\sum_k d_{J,k}\psi_{J,k}(t)+\sum_k d_{J-1,k}\psi_{J-1,k}(t)+.......+\sum_k d_{1,k}\psi_{1,k}(t) \quad (1)$$

where ϕ is the scaling function (also known as the father wavelet), ψ is the wavelet function (also known as the mother wavelet), $\phi_{J,k}$ and $\psi_{J,k}$ are a scaling and a translation of ϕ and ψ, respectively, and $s_{J,k}$ and $d_{J,k}$ are the smooth and the detailed coefficients,respectively. The wavelet transform coefficients and $\phi_{J,k}(t)$ and $\psi_{j,k}(t)$ represent the approximating wavelet functions. The wavelet transformations can be expressed as

$$s_{J,k}=\int\phi_{J,k}(t)f(t)dt \quad (2)$$

$$d_{j,k}=\int\psi_{j,k}(t)f(t)dt \quad (3)$$

where J(j=1.2,..J) is the maximum integer such that 2^J is smaller than the number of observations.

The MODWT filter is directly explained from the discrete wavelet transform (DWT) filter. Let $\bar{\bar{\phi}}_{j,k}$ and $\bar{\bar{\psi}}_{j,k}$ denote the DWT scaling and wavelet filters, with *k = 1, ..., K* being the length of the filter and j^{th} the level of decomposition for a time series *X* with arbitrary sample size (*N*). Then, the MODWT scaling $\tilde{V}_{j,t}$ and wavelet $\tilde{w}_{j,t}$ coefficients are expressed as

$$\tilde{v}_{j,t}=\sum_{k=0}^{Kj-1}\bar{\bar{\phi}}_{j,k}X_{t-k \bmod N} \quad and \quad \tilde{W}_{j,t}=\sum_{k=0}^{Kj-1}\bar{\bar{\psi}}_{j,k}X_{t-k \bmod N} \quad (4)$$

where $\bar{\bar{\phi}}_{j,k}=\frac{\phi_{J,k}}{2^{j/2}}$ and $\bar{\bar{\psi}}_{j,k}=\frac{\psi_{J,k}}{2^{j/2}}$.

It should be noted that the MODWT - which is a non-decimated form of the discrete wavelet transform (DWT) - applies high-pass and low-pass filters to the input signal at each level. The wavelet variance of stochastic process X is estimated using the MODWT coefficients for scale $\tau_j = 2^{j-1}$ through:

$$\hat{\sigma}_x^2(\tau_j) = \frac{1}{\hat{N}_j} \sum_{k=L_j-1}^{N-1} (\hat{W}_{j,k})^2 \tag{5}$$

where $\hat{W}_{j,k}$ the MODWT wavelet coefficient of variable *X*.

The wavelet correlation is analogous to its Fourier equivalent, i.e. the “complex coherency” (Gençay et al., 2002). Likewise, the wavelet cross-correlation decomposes the cross-correlation between U.S. stock and real estate markets on ascale-by-scale basis. Thus, it ispossible to see how the association between the U.S. stock and real estate markets changes with time horizons. Genaçay et al. (2002) define the wavelet cross-correlation as:

$$\hat{\rho}_{x,k}(\tau_j) = \frac{\gamma_{x,k}(\tau_j)}{\hat{\sigma}_1(\tau_j)\hat{\sigma}_2^2(\tau_j)} \tag{6}$$

where $\hat{\sigma}_1(\tau_j)$ and $\hat{\sigma}_2^2(\tau_j)$ are the wavelet variances for $x_{1,t}$ and $x_{2,t}$ associated with scale τ_j, respectively,and $\gamma_{x,k}(\tau_j)$ is the wavelet covariance between $x_{1,t}$ and $x_{2,t-k}$ associated with scale τ_j . Thus, the wavelet cross-correlation gives a lead-lag relationship between the markets of interest on a scale by scale basis.

2.2 Continous-time waveletcoherence

The continuous-time wavelet transform $W_t^{\Phi}(\Theta)$ of a discrete sequence $x_m\left(m = 1, \ldots, M-1, M\right)$ with uniform time steps δ_t is defined as the convolution of x_m with the scaled and normalized wavelet. The equation can be written as

$$W_t^{\Phi}(\Theta) = \sqrt{\frac{\delta_t}{r}} \sum_{t=1}^{M} x_m, \psi_{\sigma}\left[\frac{(m'-m)\delta_t}{r}\right] \tag{7}$$

where δ is the time step. The wavelet power is defined as $\left|W_t^{\Phi}(\Theta)\right|^2$.

In our study, the wavelet coherence is a suitable tool for measuring the extent of synchronization between U.S. equity and real estate returns over time and across scales. It is defined as the ratio of the cross-power spectrum of two series over the product of each series' power spectrum and can be interpreted as the "local" correlation between the series under consideration. Let X_t and Y_t be the equity and real estate return series with wavelet power spectra, $W_t^{\Phi}(\Theta)$ and $W_t^{\Gamma}(\Theta)$, respectively, and the cross-wavelet power spectrum, $W_t^{\Phi\Gamma}(r) = W_t^{\Phi}(\Theta) * W_t^{\Gamma}(\Theta)$. Following Torrence and Webster (1999), their wavelet coherence measure is computed as

$$R_t^2(\Theta) = \frac{\left|Q\left(\Theta^{-1} W_t^{\Phi\Gamma}(\Theta)\right)\right|^2}{Q\left|\left(\Theta^{-1}\left|W_t^{\Phi}(\Theta)\right|^2\right)\right| \cdot Q\left|\left(\Theta^{-1}\left|W_t^{\Gamma}(\Theta)\right|^2\right)\right|} \tag{8}$$

where Q refers to a smoothing operator. The numerator in Eq. (8) is the absolute squared value of the smoothed cross-wavelet spectrum, while the denominator represents the smoothed wavelet power spectra (Torrence and Webster, 1999). The value of the wavelet squared coherence $R_t^2(\Theta)$ is bounded between 0 and 1, with a high value showing strong co-movement between equity and real estate returns. However, unlike the standard correlation coefficient, the wavelet coherence measure only takes positive values. As in Torrence and Compo (1998), Monte Carlo simulation methods are used to generate the statistical significance of coherence.

2.3 Non-parametric Granger causality

Finally, we investigate the causal relationship between U.S. real estate and equity markets with both linear and non-linear causality tests. It is well known that the linear Granger causality test has a low power against non-linear relationships and might, thereby, over-reject the causal relationship between the variables of interest. Thus, a non-parametric statistical method for detecting non-linear Granger causality was proposed by Hiemstra and Jones (1994).

Diks and Panchenko (2005, 2006) show that this method may sometimes over-reject the null hypothesis. The authors propose an alternative approach, whereby the null hypothesis for the non-parametric Granger causality test is specified as

$$H_0: Y_{t+1}|(X_t^{\ell_X}; Y_t^{\ell_Y}) \sim Y_{t+1}|Y_t^{\ell_Y} \tag{9}$$

where ℓ_X and ℓ_Y represents the lag length of each variable, respectively. For a strictly stationary bivariate time series, this implies a statement about the invariant distribution of the $(\ell_X + \ell_Y + 1)$-dimensional vector $W_t = (X_t^{\ell_X}, Y_t^{\ell_Y}, Y_{t+1})$.

In order to simplify and compress the expression, we redefine Y_{t+1} as Z_t and, by further assuming that $\ell_X = \ell_Y = 1$, the time index notation may be dropped. Thus, Eq. (9) can be restated as follows:

$$H_0: Z|(X = x; Y = y) \sim Z|(Y = y). \tag{10}$$

Eq.(10) can be reformulated in terms of ratios of joint distributions. The joint probability density function, $f_{X,Y,Z}(x, y, z)$, and the marginal probability density function, $f_Y(y)$, should satisfy the relationship:

$$\frac{f_{X,Y,Z}(x, y, z)}{f_Y(y)} = \frac{f_{X,Y}(x, y)}{f_Y(y)} \frac{f_{Y,Z}(y, z)}{f_Y(y)}, \tag{11}$$

which means that X and Z are independent, conditional on $Y = y$ for each fixed value of y. Thus, we arrive at the final form of the null hypothesis:

$$H_0: q \equiv E\left[f_{X,Y,Z}(X, Y, Z) f_Y(Y) - f_{X,Y}(X, Y) f_{Y,Z}(Y, Z)\right] = 0. \tag{12}$$

Let $\hat{f}_W(W_i)$ be a local density estimator of a d_w-variate random vector W at w_i, defined by $\hat{f}_W(w_i) = (2\varepsilon_n)^{-d} w(n-1)^{-1} \sum_{j,j\neq i} I_{ij}^W$, where $I_{ij}^W = I(\|W_i - W_j\| < \varepsilon_n)$ with $I(\cdot)$, which is the indicator function and ε_n, the bandwidth. Then, the test statistic is a scaled sample version of q in Eq.(12):

$$T_n(\varepsilon_n) = \frac{n-1}{n(n-2)} \sum_i \left(\hat{f}_{X,Z,Y}(X_i, Z_i, Y_i) \hat{f}_Y(Y_i) - \hat{f}_{X,Y}(X_i, Y_i) \hat{f}_{Y,Z}(Y_i, Z_i)\right) \tag{13}$$

For the lag length of $\ell_X = \ell_Y = 1$ and if $\varepsilon_n = Cn^{-\beta}\left(C > 0, \frac{1}{4} < \beta < \frac{1}{3}\right)$, Diks and Panchenko (2006) prove that the test statistic $T_n(\cdot)$ satisfies

$$\sqrt{n}\frac{(T_n(\varepsilon_n) - q)}{S_n} \xrightarrow{D} N(0,1), \tag{14}$$

where $\xrightarrow{D}$ denotes convergence in distribution and S_n is an estimator of the asymptotic variance of $T_n(\cdot)$. We follow the Diks and Panchenko (2006)'s suggestion to implement a one-tailed version of the test, rejecting the null hypothesis if the left-hand side of Eq (14) is too large. The bandwidth variable, ε_n, is set to 1, as if the bandwidth parameter is set to larger (smaller) values than 1, this will generally result in smaller (larger) *p*-values of the causality test (Bekiros and Diks, 2008).

3. Empirical Evidence

3.1. Data

We select a sample period from January 1972 until April 2014 to give a special focus on the link between equity and real estate markets over the recent episodes of the global financial crisis and economic recessions around the world. Real estate returns are computed using data for the FTSE NAREIT US Real Estate Index, while equity returns are calculated using information for the S&P 500 index. Both indicesare expressed in U.S. dollars and obtained from Datastream International.Returns are computed by taking the difference in the logarithm of two consecutive prices.

Figure 1 plots the two asset price indices and Figure 2 displays the return series. It can be seen that they experienced an upward trend over the sample period with some very large swings. However, the price patterns between these markets are not alike.In the early seventies, the breakdown of the Bretton Woods system of fixed exchange rates, together with the spike in international oil prices and the crash in property prices, catalyzed a prolonged global recession, leading to corrections in the equitymarket in 1972-1975. And the real estate sectorcrisis in the nineties has its roots in the difficulties of the savings and loan industry of the mid-eighties (Reinhart and Rogoff, 2011). The instability in real estate and equity prices can also be linked withvarious economic and financial crisis episodes, such as the Asian financial crisis of 1997-1998, the dot-com bubble burst in 2000-2001, the terrorist attack of September 2001,the Gulf war of 2003, the commodity price boom-bust cycle of 2006-2009, and the Global Financial Crisis of 2007-2008.

[INSERT FIGURE 1 HERE]

[INSERT FIGURE 2 HERE]

Table 1 reports the descriptive statistics for U.S. real estate and equity returns.Average equity returns are larger than real estate returns; they are also less volatile in terms of their unconditional standard deviation. Moreover, both return series are negatively skewed have kurtosis coefficientslarger than three. The Jarque-Bera test (JB) reveals that both real estate and equity returns are not normally distributed. In addition, we compute the Ljung-Box statistic to test the null hypothesis of absence of autocorrelation in squared returns and the Engle (1982)'s test for conditional heteroscedasticity. The results of the Ljung-Box test show significant autocorrelation for both equity and real estate returns at order10. The ARCH test also provides strong evidence of conditional heteroscedasticity.In addition, the unconditional correlation between the U.S. equity and real estate returns is positive, albeit small (0.52). This suggests that there aresome diversification benefits from investing in both assets. Finally, the Augmented Dickey and Fuller (1981) testsshow that while asset prices are non-stationary, returns are stationary at the conventional levels.

[INSERT TABLE 1 HERE]

3.2. Lag-lead relationship

Following Percival and Walden (2000), weconsider the Daubechies least asymmetric wavelet filter of level 8 (i.e. *LA=8*) with periodic boundary conditions in the MODWT multi-resolution decomposition. Equity and real estate returns are decomposed into six series ranging from high to low frequencies, that is, *D1, D2, D3, D4, D5* and *D6* correspond to 2-4, 4-8, 8-16,16-32, 32-64 and 64-128 months, respectively, and *S6* captures the time trend of the original series.

Figure 3 summarizes the MODWT based wavelet Variance analysis in equity and real estate returns. It can be seen that the estimated wavelet variances decrease as the scale increases. Interestingly, across different time scales, both equity and real estate returns exhibit a significant change in volatility. And the results fail to corroborate the existence of a linear relationship between wavelet variances.

[INSERT FIGURE 3 HERE]

In Figure 4, we display the lead-lag relationships between the two return series using the wavelet cross-correlations obtained from the different wavelet scales. At the highest frequencies (i.e., *D1-D2*), such relationships are not significantly different from zero at all leads and lags. This indicates that asset returns are independent and past real estate returns do not have predictive power for future equity returns.For medium frequencies (i.e., *D3-D4*), there are several cases of significantly positive or negative correlation between equity and real estate returns. Finally, at low frequencies (i.e., *D5-D6*), the empirical findings show that both returns exhibit a positive and significant correlation for leads and lags up to five at the frequency 32-64 months; and share a positive correlation after five lags, which is not attractive in terms of diversification benefits.

[INSERT FIGURE 4 HERE]

3.3.Market co-movement

The wavelet application to U.S. equity and real estate returns seems to have both time- and frequency-domain representations, as a reflex of the heterogeneity of market participants at various time horizons. Indeed, short-term and long-term investors will be more interested in short-term and long-term price movements, respectively.

Figure 5 displays the cross-wavelet coherence of the co-movement between the two asset returns. The frequency (in years)is plotted in the vertical axis, while the time (in years) is depicted in the horizontal axis. The color code for power ranges from blue (low coherence or low correlation in volatility) to red (high coherence or high correlation in volatility).The thick blue contour denotes the 95% confidence level, which is estimated from Monte Carlo simulations using phase randomized surrogate series. The (downward pointing) cone of influence indicates the region affected by the so-called "edge effects".

[INSERT FIGURE 5 HERE]

In-phase denotes that the variables havea cyclical effect on each other and out-of-phase (or anti-phase) shows that the variables have an anti-cyclical effect on each other. The three-dimensional setting enables us to detect areas of co-movement among returnsthat vary over time and across frequencies. Areas of strong co-movement in the time-frequency space imply lower benefits from portfolio diversification. Our results show that the relationship between U.S. equity and real estate returns was not homogeneous across scales, since the arrows point right and left, and down and up, constantly.

Moreover, Figure 5 reveals a relatively high degree of co-movement between asset returns at specific frequencies.In particular, at the 4-14 months and 6-12 monthsof scale, the arrow series are in-phase, corresponding to the periods of 1973-1977 and 1986-1993. Over the periods of 1981-1990 and 2006-2012, we find strong synchronization between U.S. equity and real estate returns at the 0-6 months, 8-18 months and 24-36 months of scale. However, in the periods 1979-1983and 2007-2011, the arrows point right and down at the 8-12 months and 16-20 months of scale,indicating that the real estate returns lead equity returns.

3.4. Non-linear causality

As a final assessment, we investigate the relationship between equity and real estate returns at various time-scales using the linear causality test of Granger (1969)and the non-parametric causality test of Diks and Panchenko (2006).

The results are summarized in Panels A and B of Table 2. They show that there is a bi-directional linear causality between asset returns at different short-term scales (i.e. *D1-D3*),which is consistent with the non-linear case, except for the *D2* frequency scale (in the linear case) and the *D6* frequency scale (in the non-linear case).

Unidirectional causality is found from real estate returns to equity returns at the *D4-D5* frequency scales. In the case of the non-linear causality test, there is bi-directional causality between the two asset returns at the time-domain. Finally, the causality from equity to real estate returns is stronger in the non-linear framework than in linear one.

[INSERT TABLE 2 HERE]

4. Conclusion

Understanding the co-movement between equity and real estate returns are crucial for portfolio allocation, risk management, and policy-decision making. A strong co-movement wouldimply that both markets are driven by similar economic factors, thus indicatingpoor diversification opportunities. Incontrast, a low co-movement makes them desirable from a diversification perspective.

Our work shows that theco-movement between U.S. real estate and equity returns varies both across frequencies and over time. The results from the nonlinear causality investigation also provide important insights about the direction of causality between the two asset returns, which sheds some light about the scale-dependent hedging opportunities between them. What stocks investors could take away from our findings is that the benefits from diversifying into real estate assets is the least at the medium and low frequencies, particularly over the 6 to 48 month horizon. In the time domain, the real estate market offers the least diversifying potential for stock investment in the 1970s, 1980s and from the year 2005 to the end of our sample (2014) given their strong synchronization. Higher allocation rates to real estate assets by investors following various stock market downturns and crises could be an explanation of this strong synchronization.

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List of Tables

Table 1: Descriptive statistics for U.S. equity and real estate returns.

Variables	Mean (%)	Std. Dev.	Skew.	Kurt.	J-B	$Q^2(10)$	ARCH(10)	ADF	Corr. with DLSP500
DLSP500	0.57	0.04	-0.86	5.98	250.77^{+}	20.31^{++}	1.90^{++}	-21.02[0]$^{+}$	1
DLNARALL	0.10	0.05	-1.60	14.24	2887.30^{+}	135.87^{+}	11.29^{+}	-9.17[3]$^{+}$	0.52

Notes: J-B, $Q^2(10)$ and ARCH(10) refer respectively to the empirical statistics of Jarque-Bera (J-B) test for normality, Ljung-Box test for 10-order serial autocorrelation in squared returns, and Engle (1982)'s test for conditional heteroscedasticity. Figures in squared bracketsindicate the lag length. +,++,+++ indicates the rejection of the null hypotheses at the 1%,5% and 10% level.

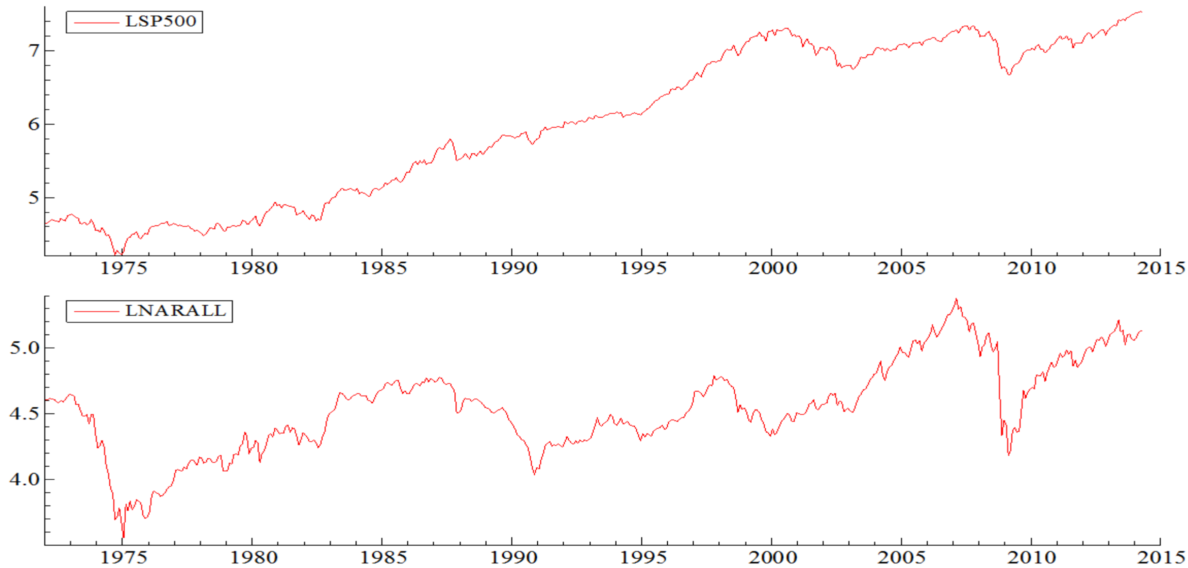
Table 2: Causality tests.

Null Hypothesis	Time domain		Frequency Bands (months)													
			D1: 2-4 months		D2:4-8 months		D3:8-16 months		D4:16-32 months		D5:32-64 months		D6:64-128 months		S6: Trend original	
	F-stat.	Prob.	F-Stat.	Prob.	F-Stat.	Prob.	F-Stat	Prob.	F-Stat.	Prob.	F-Stat.	Prob.	F-Stat.	Prob.	F-Stat.	Prob.
Panel A: Linear Causality																
$S\&P500 > NARALL$	1.95(1)[a]	0.37	10.87(4)[ab]	0.00	12.19(4)[ab]	0.00	5.29(4)[ab]	0.07	2.11(4)[ab]	0.34	0.37(4)[ab]	0.83	19.40(4)[ab]	0.00	4.59(3)[ab]	0.00
NARALL $> S\&P500$	47.06(1)[a]	0.00	46.92(4)[ab]	0.00	22.57(4)[ab]	0.00	27.44(4)[ab]	0.00	15.71(4)[ab]	0.00	10.16(4)[ab]	0.00	24.99(4)[ab]	0.00	5.51(3)[ab]	0.06
Panel B: Non-linear Causality																
$S\&P500 > NARALL$	1.71(2)	0.04	1.84(2)	0.03	3.25 (2)	0.00	3.75(2)	0.00	4.08(2)	0.00	2.55(2)	0.00	1.39(2)	0.08	0.41(2)	0.65
NARALL $> S\&P500$	0.11(2)	0.54	1.26(2)	0.10	0.35(2)	0.36	2.44(2)	0.00	1.32(2)	0.09	0.13(2)	0.44	1.78(2)	0.03	2.18(2)	0.01

Notes: The number of lags in the linear causality test is identified based on the AIC([a])or SIC([b])criteria of the VAR model. Figures in brackets () indicate the lag length. The cointegrating vectors are identified using the trace statistic of the Johansen (1996)test. Causality is investigated with aVAR specification (the null of no cointegration was not rejected).The number of lags in the nonlinear causality test is $\ell x=\ell y=1$.

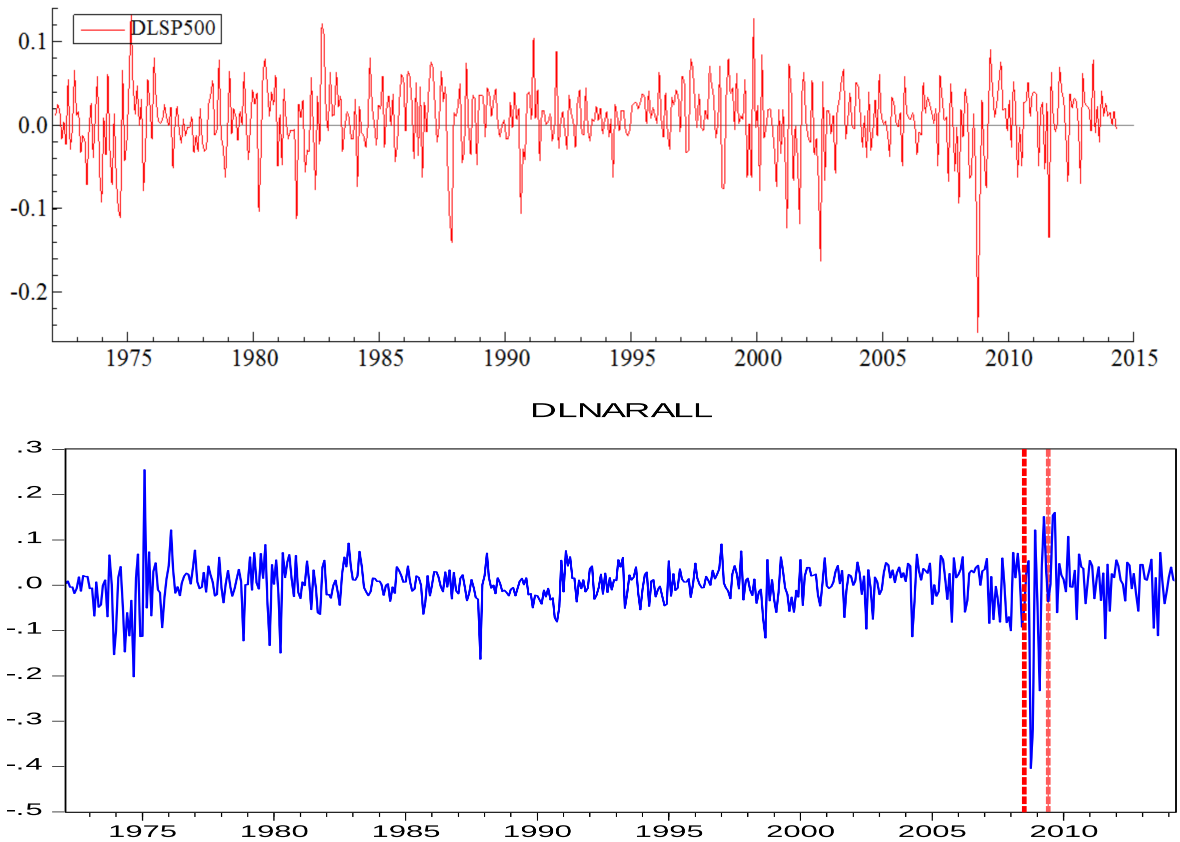
List of Figures

Figure 1: Dynamics of U.S. equity and real estate price indices.



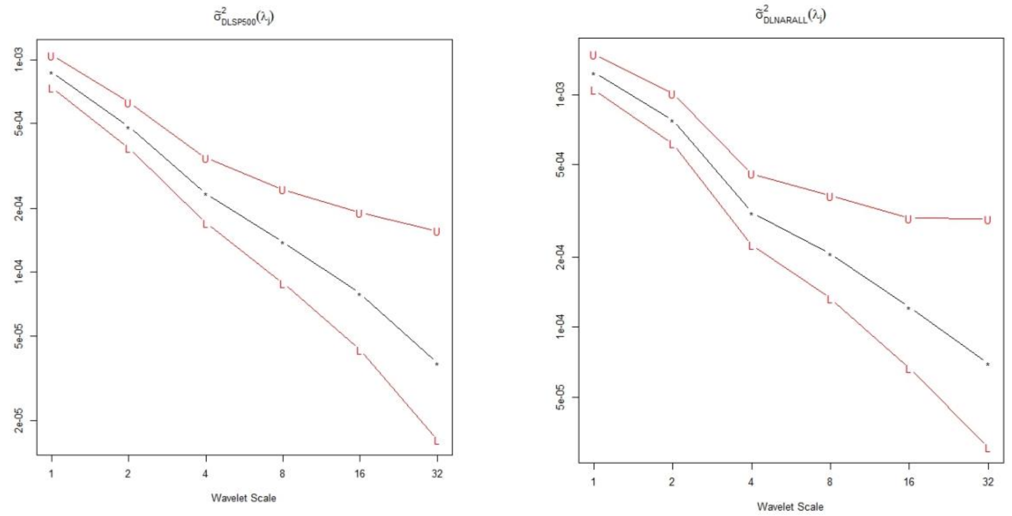
Notes: The logarithm for FTSE NAREIT US Real Estate Market Index is presented in LNARALL and the logarithm for S&P 500 is presented in LS&P500. Sample period: 1972:M1-2014:M4. Source: Datastream International.

Figure 2: U.S. equity and real estate returns.



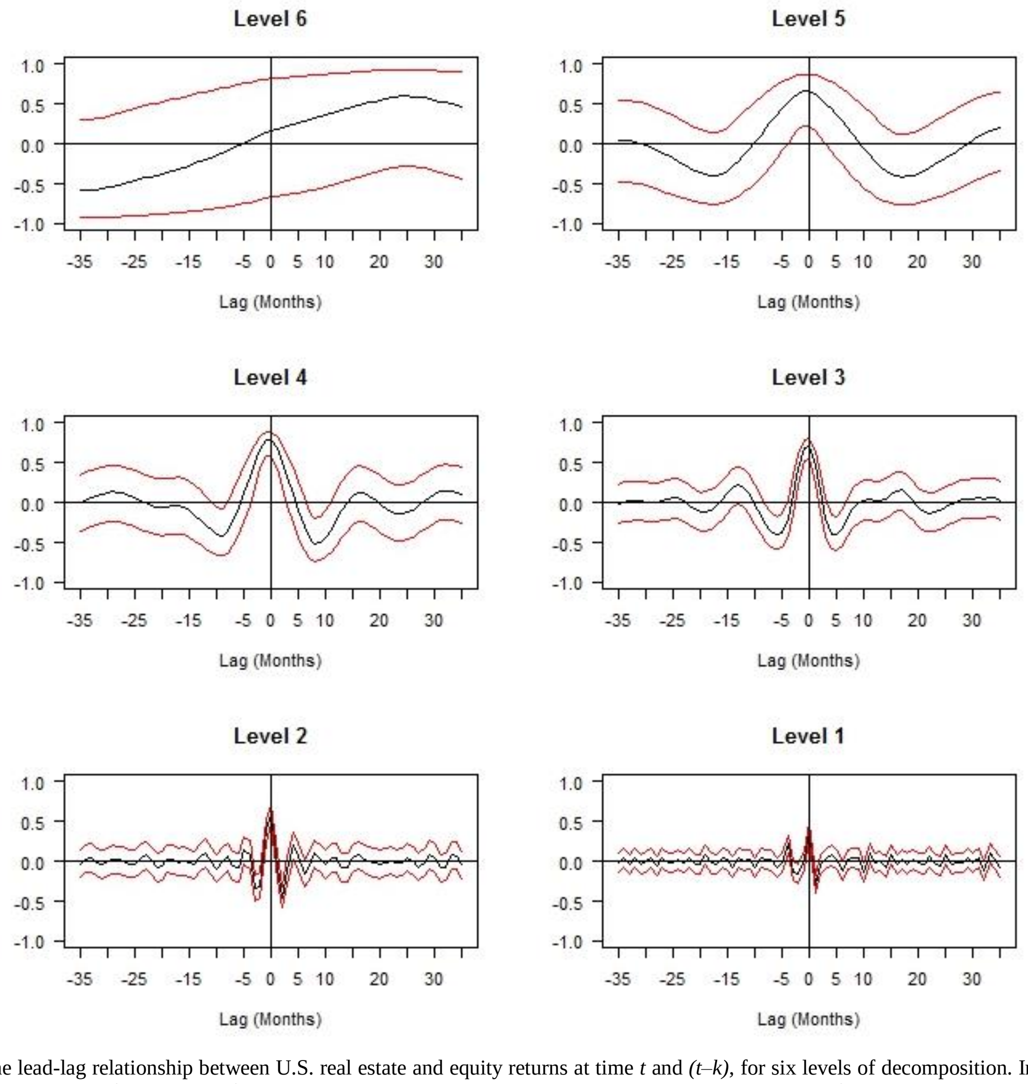
Notes: See notes of Figure 1.

Figure 3: Variance analysis for U.S. equity and real estate returns.



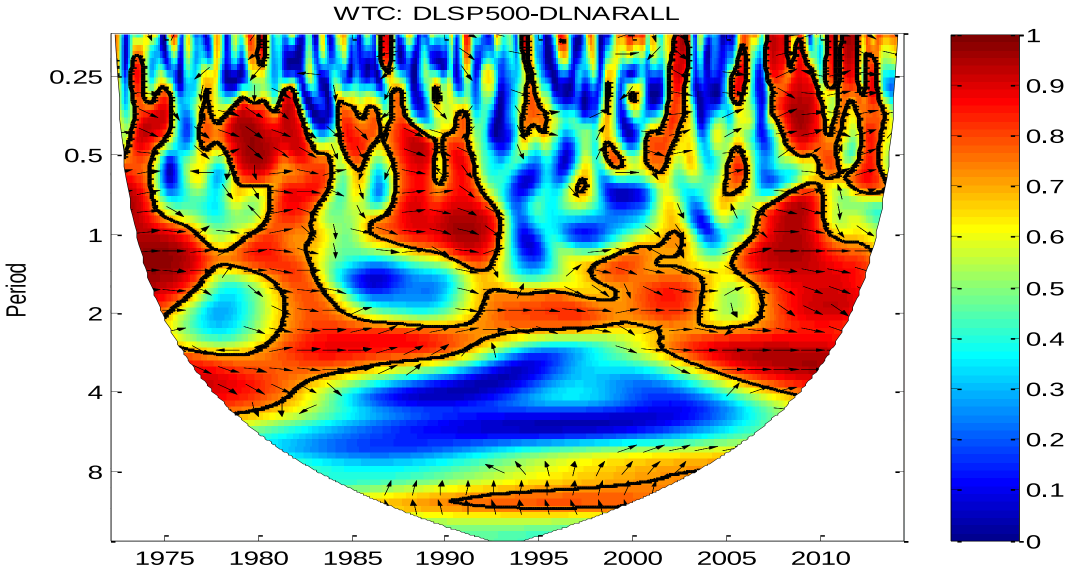
Notes: The variance estimates based on the 5%level of significance or 95% confidence internal in association with upper (U) and lower (L) bounds.

Figure 4: Wavelet cross-correlation in U.S. equity and real estate returns.



Notes: The lead-lag relationship between U.S. real estate and equity returns at time *t* and *(t–k)*, for six levels of decomposition. In the application, the short-term fluctuations of the series show smaller correlation compared to the long-term. In fact, the magnitude of the cross-correlation increases as the frequency band is increased.

Figure 5: Wavelet coherence.



Notes: Phase arrows indicate the direction of co-movement among the returns series of both U.S. equity - real estate pairwise. Arrows pointing to the right denote perfectly phased variables. The direction "right-up" indicates lagging real estate, whilst the "right-down" direction indicates leading real estate versus the U.S. stock index. Arrows pointing to the left correspond to out-of-phase variables. The direction "left-up" indicates leading real estate, whilst the "left-down" direction indicates lagging real estate. In-phase variables represent a cyclical relationship and out-of-phase (or anti-phase) variables show anti-cyclical behavior. The thick black contour lines indicate the 5% significance intervals estimated from Monte Carlo simulations with phase-randomized surrogate series. The cone of influence, which marks the region affected by edge effects, is shown with a lighter shade black line. The color legend for spectrum power ranges from Blue (low power) to Red (high power). Y-axis measures frequency (scale in years) and X-axis represents the time period studied (in years).