The Butterfly Effect: Chaos in Disguise and Financial Markets

Dulani Jayasuriya Daluwathumullagamage

Abstract

This study analyses chaotic behavior in US and international financial markets. As chaos has properties such as nonlinearity, sensitivity to initial conditions, and fractality, the study performed several techniques including the Brock-Dechert-Scheinkman test to identify the existence of chaos in several asset classes in the US markets. Filtering is conducted using several GARCH models and kernel based subsampling techniques. Moreover, this study adjusts for long memory issues in stock returns by filtering the linear structure of the returns using additional ARFIMA, FIGARCH, FIEGARCH models. Finally, we provide evidence for the presence of low complexity chaotic behavior in US and international markets in daily and high frequency data and in various financial times series.

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Contact author email: d.jayasuriya@auckland.ac.nz

1. Introduction

Past couple of decades saw at least eight major crises hitting the financial markets. However, crisis times result in market models, assumptions and theories being revaluated with new and old paradigms emerging to re-identify and re-analyze financial data that can explain such events. Following the global financial crisis in 2008, focus shifted especially on deterministic chaos and non-linear characteristics of financial markets[1]. Prior literature, most financial modelling techniques and theory were primarily able to address problems ideal or normal market conditions. For example, the frequency of wide swings in asset prices was not what one would expect from returns normally distributed[2]. Hence, given the inability of present financial models to explain extreme market events (crashes, bubbles), alternative theories are required to provide explanations and accurate tools to measure market disorders. Such an alternative view is that financial markets are governed by chaotic dynamics. Chaos appears to be random and is a nonlinear deterministic process. Chaos is an interesting concept to model financial markets due to several reasons. Chaos theory characterize financial markets possessing nonlinear characteristics resulting in persistent and self-generating processes. For example, external shocks such as unanticipated news causing investor panic and price depression is not the only cause of economic fluctuations. Another cause maybe that financial or stock markets are indeed a chaotic process which is characterized by large price movements occasionally.

The objective of this study is to obtain an understanding of market characteristics by identifying the existence of chaotic dynamics. Is there chaos in financial markets? Is it a matter of winning between Chaos and the Efficient Market Hypothesis? Or a peaceful co-existence of both? The objective is to identify the presence of chaos and not to choose one theory above the other. Basically, the presence of chaos in financial markets suggest that asset price movements are not random and are similar to other dynamic systems[3]. This study identifies the presence of chaos in US and international financial time series. Namely, exchange rate data, index data and stock price data.

Chaos Theory is a significant departure from traditional Newtonian mechanics. Chaos theory analyses deterministic processes that seemingly appear to possess random behavior. The random behavior stems from sensitivities to initial conditions. In a chaos dynamics non-linear behavior is constrained from a higher deterministic structure resulting in an underlying order to the apparent random dynamics. Financial markets have been observed to be dynamic (or non-linear) systems contrary to popular asset pricing models such as the CAPM (Capital Asset Pricing Model) and portfolio theory such as the MPT (market portfolio theory). Chaos theory is the study of these types of dynamic systems.

In addition, a key principle of chaos theory is the sensitivity to initial conditions also commonly known as the Butterfly Effect. According to the “Butterfly Effect”, even a small change in the initial conditions of a dynamic system can cause large differences in its future states and behavior. Therefore, the smallest error in calculation of what affects stock prices initially would cause large inaccuracies in any long-term predictions resulting the movement of asset prices seem totally random. Given this setting, chaos theory posits that an underlying order can be observed in dynamic systems that are seemingly random and difficult to predict. Hence, identification of chaotic behavior in financial market data is

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[1]A key realization from the 2008 global financial crises was the fact that most economic and financial model assumptions were either unrealistic or failed during extreme market conditions.

[2]The assumption that financial time series are random and are drawn from a normal distribution fails to account for large price movement scenarios and extensive market crashes or highs. These extreme probability events result in “fat tails,” due to the larger area under the tails relative to the normal distribution. As market volatility increases, the normal distribution expands allowing larger variances in asset prices and shrinks to accommodate decreases in volatility. However, the inherent problem is the inadequate assumptions of financial or stock markets being a linear system forming a basis for these distributions.

[3]Financial markets’ price fluctuations observed even for a single day, or month, can be a scaled down version of how the overall market move over an extended period of time. Cycles of chaotic systems are also full of fluctuations and bifurcations.

crucial. Having identified chaotic behavior, chaos theory may be applied in the analysis of the return/performance of asset prices in financial markets as a result of meditative process.

To the authors' knowledge this is the first study to analyses chaotic behavior in US financial market data, especially using a sample including the 2008 global financial crisis. Furthermore, since Chaos theory is still a developing area, concepts are redefined or complemented continuously. However, the identification of Chaos in financial time series has several important implications. This gives better understanding of market conditions and possible predictability which leads to immense improvement in forecasting time series. Moreover, the identification of Chaos dynamics leads to better understanding and predictability of stock market bubbles and clashes which has important research, industrial and social implications. In addition, given the identification of Chaos dynamics in financial times series, new theories and tests could developed similar to other fields of science such as Physics and Engineering.

Moreover, to obtain the residuals for the chaos analysis, this study implements the realized volatility models reviewed in Jayasuriya (2016)[4]. Main findings show the chaotic behavior of certain residuals of financial time series and the identification of a general model. The study is organized as follows. Section 2 reviews prior literature. Section 3 discusses the data. Section 4 explains the theoretical and empirical methodology. Section 5 provides the empirical results. Section 6 concludes the study. All additional tables and figures can be included in an internet appendix.

2. Review of Literature

Chaos theory is the analysis of deterministic chaos, which is seemingly unpredictable behavior that is however governed by certain rules. This section amalgamates several strands of literature linked to this study including chaos theory and its primary competitors the efficient market hypothesis, fractal market hypothesis and realized volatility models etc.

2.1 Chaos theory and Efficient Market Hypothesis

The efficient market hypothesis (EMH) since its introduction in 1960s (Fama, 1965, 1970; Samuelson, 1965) has been widely accepted as a keystone of finance. However, EMH[5] has had its share of criticism over the past decades, primarily theoretical (LeRoy, 1976) and empirical (Malkiel, 2003). The most problematic issue for EMH is its inability to account for extreme events and large price swings in financial markets (Stanley, 2003). EMH assumes that all investors have homogenous expectations, returns are normally, identically and independently distributed. In addition, the standard deviation is used as a measure of total risk and future expected returns are unpredictable. According to Chaos theory returns are not normally distributed and far more complex models should be utilized to analyze financial time series and market phenomenon. Even though nonlinear models in general are able to predict certain financial times series, not all models are easily applicable to real time financial market data (Schreimber (1998)). However, deterministic chaos is a useful method to implement a set of fixed rules required to predict future financial time series using past observations of financial time series without imposing overly restrictive assumptions (Peitgen et al (2004)). Hsieh (1991) characterizes chaos as a nonlinear deterministic process, seemingly random due to an irregular oscillatory process. Chaos processes are periodically irregular and influenced by initial conditions (Brown (1995)). Devaney (1990) states that chaos theory enables the explanation of complex dynamics as a combination of simpler trends. Arnold (1992) finds that the analysis

[4]This study employs a variety of ARMA and GARCH models for the mean and residual series to avoid the residuals depicting chaotic behavior due to the lack of a good fit in the initial model.

[5]Fama (1970), in his seminal paper, states that all information available is reflected in asset prices in efficient markets. In addition, (Fama, 1991) defined the efficient market as the one where prices reflect available information to the point where marginal gain from using the information would equal the marginal cost of obtaining it.

of several significant points representing deflectors, attractors or periodicity enable the analysis of chaos. The primary purpose of this study is to test for chaotic dynamics in the US and several international financial markets. A chaotic system shows leptokurtotic distributions.

2.2 BDS test for Chaos in financial markets

Deterministic nonlinear equations resulted in a major breakthrough in many science fields including physics by generating seemingly random data. One of the primary objectives of Chaos theory is to analyze the non-linear aperiodic behavior of processes highly dependent on initial conditions. Encouraged by the wide applications of Chaos theory in science fields, econometricians have attempted to identify the presence of Chaos and its characteristics in financial time series data. Brock, Dechert, and Scheinkman (1987) and Dechert (1996) developed the key well established test for nonlinearity and detection of Chaos which is the BDS test. In addition, Savit and Green (1991) and Wu, Savit, and Brock (1993) further generalized the BDS test. Moreover, De Lima (1998) implemented an iterative version of the BDS test.

Presence of Chaos has been tested utilizing the BDS test in several prior literatures. Wiley (1992) analyses daily S&P 100 and NASDAQ index returns from 1982-1988 using the BDS statistics and is unable to identify any underlying dependence, chaotic or otherwise. Sewell et al (1996) rejects the independent and identically distributed (IID) assumption for US, Japanese, Hong Kong, Singaporean stock markets using weekly data from 1980 to 1994. Abhyankar et al (1997) rejects IID characteristics in returns data for S&P500, DAX, NIKKEI and FTSE from September to November 1991. Refer to Mayfield & LeBaron (1989); Hsieh (1991); Ahmed et al (1996); Barkoulas & Travlos (1998) for additional literature on the use of the BDS test to identify chaotic behavior in stock markets. Hsieh (1989); Vassilicos et al (1992); Chiarella et al (1994); Cecen & Erkal (1996); Serletis & Gogas (1997) attempts to identify chaotic behavior in exchange rate data. Kodres & Papell, (1991); Vaidyanathan & Krehbiel, (1992); Eldridge & Coleman, (1993); Chwee, (1998) does the same for futures data. In addition, Kohzadi & Boyd, (1995) and Frank & Stengos, (1998) conducts the same analysis for commodity prices. In general, the literature on Chaos and the BDS test discussed above provide substantial empirical evidence of nonlinear structures in several financial asset classes. However, the majority of these studies were conducted in the 90's. There is a significant dearth in research on Chaos theory following that particular decade, especially for any sample including the 2008 financial crisis as done by this study.

3. Data

This study applies the proposed methodology to three financial data sets in US financial markets and international financial time series. The financial time series utilized are exchange rate, indices and stock price data. DS1 for stock price data, DS2 for stock index data and DS3 for exchange rate data. Stock price data consists of the lead tech stocks that moves the US markets: IBM, MICROSOFT, and GOOGLE & DELL. The frequency is daily data from August 2004 to January 2010. High frequency data was also considered for the same stocks for October 2006. This year was chosen as a stable year with no crashes and therefore least likely to observe chaotic behavior. Stock index data is for S&P 500, TPOIX and Nikkei. The daily index data spans from January 1980 to January 2010. Exchange rate data is for the Euro and British pound against the US dollar. Daily exchange rate data spans from January 1999 to January 2010. The high frequency data were obtained from the Wharton Research Database. As it is widely believed that the 2008 financial crisis began in the USA and spilled over to other parts of the world, this study chooses a majority of US data. Table 1 and 2 provides the summary statistics.

4. Empirical Methodology

4.1 BDS test, univariate and multivariate GARCH models

Uncertainly of asset returns are measured by volatility. Volatility is used in many applications in finance including risk management, option pricing, quantitative trading and portfolio management. Define Volatility as the standard deviation of the return on an asset, denoted by.

- Discrete time:
- Price: P_t
- Return over $[t-1,t]$: $r_t = \log(P_t) - \log(P_{t-1})$
- Volatility: $\sigma_t^2 = Var[r_t]$
- Continuous time:
- Price: $d\log(P_t) = \mu_t dt + \sigma_t dW_t,\quad W_t$ follow a brownian motion.
- Integrated volatility: $Var[r_t] = Var[\log(P_t) - \log(P_{t-1})] = \int_{t-1}^{t} \sigma_s^2 ds$

Several studies testing for chaotic dynamics utilize the BDS test on the residuals of GARCH type models. Hence, if GARCH models perfectly fit the data, then the standardized residuals of the fitted GARCH model would be independent. Hence, to firstly remove any non-linearity before testing for chaotic dynamics in the existing series, this study uses GARCH models to model volatility. Engle (1982)'s keystone paper introduces the Autoregressive Conditional Heteroskedasticity (ARCH) model. Subsequently, Bollerslev, Engle, and Nelson (1994), Palm (1996), Shephard (1996) and a plethora of literature burgeoned on various forms of ARCH models. Although most of prior literature focuses on modeling volatility of financial time series, once cannot ignore the co-movements of financial returns. Hence this study also considers multivariate GARCH (MGARCH) models (refer Bollerslev, Engle, and Wooldridge (1988), Ng (1991)). Multivariate GARCH models are mainly employed to analyse spillover effects and correlation transmission in contagion studies (refer Tse and Tsui (2002) and Bae, Karolyi, and Stulz (2003)).

4.2. Estimation procedures for chaotic dynamics

Financial market movements often depict irregular dynamics in volume and returns that standard linear forecasting models may have difficulty predicting accurately. Most dynamics identified by traditional standard linear models are do not persist over time and considerable volatility swings render previously estimated models irrelevant. Chaotic systems are seemingly unpredictable over most time scales with the exception until the butterfly effect dominates the system. The non-linearity is expressed with the continuous change of initial conditions (butterfly effect). This would result in even two initially similar variables of a dynamic system, may deviate particularly evolve over time. Thus, in dynamic phases of a system, there can coexist simultaneously, regions of chaos, with more organized regions.

Link to Chaos: Four-step Procedure

(1) Remove autocorrelation, if present by AR, ARMA, ARIMA.

(2) Phase-space reconstruction $x_t^n = (x_{t-n-1}, \ldots., x_t)$ where 'n' is the embedded dimension.

(3) Calculate the correlation integral

$$C_n, T(\varepsilon) = \lim_{T\to\infty} \frac{1}{T^2} \#\{(t,s), 0 < t, s < T: ||x_t^n - x_s^n < \varepsilon\}$$

(4) Calculate for minute ε, $\log C_{n,T(\varepsilon)}$ ∀ $\log T(\varepsilon)$ which happens to be the slope of the graph.

$$W_n\mathrm{T}(\varepsilon) = \lim_{\varepsilon \to 0} \frac{\log C_n T(\varepsilon)}{\log T(\varepsilon)}$$

The financial times series would be consistent with chaos dynamics if V_n does not increase with n. $W_n T(\varepsilon) = \sqrt{T}[C_{n,T(\varepsilon)} - C_{1,T(\varepsilon)}n]/\sigma_{n,T(\varepsilon)}$ statistic is the ratio between the error term spread with regard to the asymptotic standard error $\sigma_{n,T(\varepsilon)}$ and the normality assumption ($C_{n,T(\varepsilon)}$ - $C_{1,T(\varepsilon)}n$). When the time series error is IID (independent and identically distributed), the statistics would have a zero value. In all other instances it is may but not necessarily identify the presence of chaos. These methods are implemented on time series of errors obtained from various forms of filtering criteria. In this study many forms of auto regressive, moving average, GARCH and kernel based filtering models are utilized as mentioned in the next section.

4.2. The BDS Test

This study analyses several financial time series' noise filtered by many techniques globally to identify the presence of chaotic noise or traditional white noise. The BDS test is implemented for the identification of the error series' random or hidden structure. Once any linear dependence has been removed (Using traditional ARIMA, GARCH, kernel based filtering or even basic first differencing) from financial time series, BDS tests for nonlinearity. In the field of finance and economics riddled with noisy data sets, BDS presents a most straightforward way to identify chaotic processes. Firstly, time series data is fitted with a mean model to obtain the residuals and then the BDS test is implemented on those residuals. Secondly, obtain the residuals from a mean model and use a GARCH model to model the residuals. Then test these residuals using the BDS test. Thirdly, kernel estimators are used for filtering the financial time series and the BDS test is applied.

The study firstly conducted the usual stationarity & normality tests including ADF tests to identify unit roots. The following mean models were fitted to all data sets: MA (1), MA (2), MA (3), AR(1), AR(2), AR(3), ARMA(1,1), ARMA(1,2), ARMA(2,1), ARMA(2,2). Refer to Table 1A, 4A, 7A, 10A, 13A, 15A, 16A, 19A, and Table 22A. PACF and ACF patterns are analyzed to identify lags (refer to Figure 4A, 5A, 6A and 7A). AIC, BIC and Likelihood ratio tests are conducted to identify the best mean model (refer to Tables 2A, 5A, 8A, 11A, 14A, 18A, 20A and Table 23A). For each of the above models' residuals GARCH (1, 1), EGARCH (1, 0), IGARCH, FGARCH, TGARCH, AVGARCH, gjrGARCH models (refer Tables 3A, 6A, 9A, 12A, 15A, 17A, 21A and Table 24A) and multivariate GARCH models (BEKK (1, 1) and MGARCH) were implemented. Refer to Tables 25A and 26A. Kernal based sub sampling techniques of flat-top Bartlett, Tukey-Hanning, Epanechnikov and Zhang et al. (2005) are also implemented. Finally, the BDS test was implemented on all the residuals from all filtering models for both daily and high frequency data.

The fundamental idea behind the analysis is that the mean model residuals should ideally be linear and independent. Hence, ignored non-linearity would be portrayed in any dependencies found in the residuals. However, if the nonlinear dynamics stem from a known non-deterministic system, this would imply the absence of chaos (Brock et al, 1993). Hence, if the null hypothesis of the BDS test is accepted, this implies that the GARCH-type process or the filtering model used is able to explain any non-linear structure in the data and the opposite gives evidence of Chaos.

5. Empirical Findings

According to PACF, ACF and AIC, BIC and log likelihood criterion, AR (1) model emerges as the best mean model that fits all data sets. Refer to Figures 4A, 5A, 6A and 7A.

5.1 High Frequency Data

A 5-minute interval is selected for the sampling method for the high frequency data. According to the PACF, ACF and the AIC and BIC and log likelihood criterions we found that the AR (1) model is the best mean model that fits high frequency Stock data. While for the residuals, EGARCH (1, 0) model is observed to have a better fit.

5.2 Realized Volatility Results

The GARCH (1, 1) model is observed to have a better fit for all three datasets DS1, DS2 and DS3. Both flat-top Bartlett kernel and Zhang et al. (2005) subs sample estimators show the same distribution. Bartlett methods, Tukey-Hanning and Epanechnikov show less variance and are similar as observed by all graphs. Refer to Figures 8A, 9A, 10A, 11A, 12A, 13A, 14A, 15A, 16A, 17A, 18A, 19A, 20A, 21A, 22A and 23A.

5.3 Empirical Results on Chaos Dynamics

Chaos theory assumes that the underlying process is non-linear and deterministic. Several prior literatures especially during the 1990's show evidence of chaotic behavior in financial time series and some are inconclusive (Hsieh, 1993).

We accept IID assumption for some models for DS3 data (USD vs. EUR). We accept IID for most models for DELL & IBM high frequency data. Implication from these results are that stock returns are non-stationary and that returns are generated by stochastic nonlinear processes. This study gives evidence of low complexity chaotic behavior in various financial times series across the world. Reasons for rejection of IID may be due to structural change, i.e., financial and technological innovations, regulation changes etc. Refer to Table 3 for BDS test results for IBM. All remaining series of MICROSOFT, GOOGLE daily returns series, stock index returns of the S&P500, Nikkei and Topix, EURO vs GBP and USD vs GBP rejects the IID assumption and shows low chaotic behavior. Within key tech stock returns, EMH (for DELL and IBM) and Chaos (for MICROSOFT and GOOGLE) is observed to co-exist. However, overall market indices for US (S&P 500) and Japan (Nikkei and Topix) exhibit chaotic behavior according to this study's findings. Finally, this study finds that exchange rate data for US dollar against the Euro and GBP exhibit chaotic behavior. Additional robustness tables on the BDS test for different sub samples are available upon request.

6. Policy Implications

Several policy implications and generalizable lessons can be derived from this study on the US financial and international financial time series and high frequency data. Moreover, understanding of chaotic behavior in financial markets have become vital especially following the 2008 global financial crisis and its worldwide socio and economic consequence. Chaos theory states that financial markets are predictable to some extent and only the scale of movements changes and that the nature of risk is universal. Consequently, the same leptokurtic distributions are observed for any time frequency. Identification of chaotic behavior in financial time series allows risk managers and other market participants to gain insights into the true nature of risk. Thoroughly analyzing patterns at certain intervals would help in identifying causes behind these particular distributions such as market sentiment, psychology, or changes in external and internal conditions.

One of the primary contributions of chaos theory especially to modern financial markets is that it recognizes the recurrence of extreme volatility and turbulence, bubbles and crises. Chaos theory approach to financial markets is that it accepts the presence of many more bubbles, crashes and extreme outcomes yet to emerge and raise awareness of the same. This is that traditional neoclassical theories omit as a fact. Hence, the identification of Chaos in financial markets and the results from this study may encourage market participants and managers to better accept and understand the changing dynamics, risks and crashes and better plan for adverse outcomes such as the 2008 global financial crisis. Enable market participants to weather market bubbles and extreme volatility events.

Chaotic systems are highly sensitive to initial conditions and display non-linear behavior. Financial market participants and especially risk managers should be aware of the presence of chaos and its characteristics in financial markets. Especially, since chaos highlights the fact that risk is endogenous in financial markets resulting in it being amplified within system. Chaotic behavior is observed to be more pronounced in crisis times especially with the presence of spillover effects. Given the results in this study of presence of chaos in various US and international financial time series it is important to: 1) Utilize non-linear models for modelling financial times series; and 2) complement any quantitative model used with stress tests to analyze extreme events. Chaotic systems allow rapid adaptability since they continuously evolve over different critical stages and transition phases. These techniques would allow rapid adaptability in case of sudden unexpected asset price movements in financial markets. Finally, ever evolving chaos and fractal theories provide market participants and risk managers' better quantitative models to identify and mitigate risk. This would enable firms and market participants largely dominated by thinking derived from traditional paradigms to renew their risk management practices and trading models. Subsequently better equipping them for present days' volatile financial markets.

7. Conclusion

There are interesting applications of Chaos dynamics and this study identifies the presence of such in several financial times series. Claims of Chaos dynamics in financial markets would attract many opportunists pursing high gains. However, the identification of Chaos in financial time series has several important implications. This gives better understanding of market conditions and possible predictability which leads to immense improvement in forecasting time series. Moreover, the identification of Chaos dynamics leads to better understanding and predictability of stock market bubbles and clashes which has important research, industrial and social implications. In addition, given the identification of Chaos dynamics in financial times series, new theories and tests could developed similar to other fields of science such as Physics and Engineering. Linear models has been a significant part of scientific discussion for over a century. The first key deviations from the linear notion was made in thermodynamics, and subsequently in quantum mechanics. The final conclusion was the establishment of the Newtonian theories and chaos theory proposing new mechanisms to model complex realities and processes. This study addresses an important question regarding financial time series that has again become the focus following the global financial crisis. Is the apparent randomness of financial time series returns explicable through a deterministic process? In other words, is there a nonlinear structure characterized by low dimensional chaos? This study applies the BDS test to several financial time series to identify presence of chaotic dynamics. In addition, this study conducts an empirical analysis of univariate and multivariate GARCH models for several financial time series. Moreover, the most important methods for consistent estimation of mean and residual models are presented. Finally, the main empirical findings using univariate and multivariate methods are summarized.

In terms of the mean and residual models for financial time series, the study finds that stock returns are not independent and identically distributed (IID). In addition, the study finds that general ARCH-type models are unable to fully capture the nonlinearity in financial time's series highlighting the need of conditional heteroskedastic models. Finally, this study gives evidence of low complexity chaotic dynamics in financial time series in terms, of stock returns, exchange rates, index returns both on a daily frequency and high frequency for stock returns. The findings of this study has the following implications: Firstly, unconditional density functions should not be fitted generally on financial time's series especially stock returns with IID assumptions. The nonlinear dependence must first be removed. Secondly, for financial time's series when modelling nonlinearity, efforts should be directed towards modeling conditional heteroskedasticity and not solely conditional mean changes. Thirdly, conditional heteroskedasticity models along with nonparametric estimates of the standardized residual density may provide useful conditional probability distributions.

References

Aït-Sahalia, Y., Hansen, L.P., Scheinkman, J., (2010). Operator Methods for Continuous time Markov Models. In: Aït-Sahalia, Y., Hansen, L.P. (Eds.), Handbook of Financial Econometrics, North-Holland, and pp. 1–66.

Aït-Sahalia, Mancini, L., (2008). Out of sample forecasts of quadratic variation. Journal of Econometrics 147, 17–33.

Aït-Sahalia, Mykland, P.A., Zhang, L., (2005). How often to sample a continuous-time process in the presence of market microstructure noise. Review of Financial Studies 18, 351–416.

Albin, P. S, (1982), The metalogic of economic predictions, calculations and propositions. Center for study of system structure and Industrial complexity, City University of New York, New York, NY 10019 U.S.A. p 329-358.

Alexander, S, (1961), Price movements in speculative markets: trends or random walks. Reprinted with permission of Industrial Management Review, Vol. 2, No 2 (May) p7-26.

Allman, W. F, (1994) The Simple Laws of a Complex World. U.S. News & World Report;4/25/94, Vol. 116 Issue 16. Pages: 84.

Ambrose B, Weinstock E and Griffiths, M, (1993) Fractal Structure in the Capital Markets Revisited. Financial Analysts Journal, Vol. 49, No. 3 (May - June) Pages: 73-77.

Andersen, T.G., Bollerslev, T. and Diebold, F.X. (2005), "Roughing It Up: Including Jump Components in the Measurement, Modeling and Forecasting of Return Volatility," Manuscript, Northwestern University, Duke University and University of Pennsylvania.

Andersen, T., Bollerslev, T., Diebold, F.X. and Vega, C. (2003), "Micro Effects of Macro Announcements: Real-Time Price Discovery in Foreign Exchange," American Economic Review, 93, 38-62.

Andersen, T.G., Bollerslev, T., Diebold, F.X. and Wu, J. (2006), "Realized Beta: Persistence and Predictability," in T. Fomby (ed.) Advances in Econometrics: Econometric Analysis of Economic and Financial Time Series in Honor of R.F. Engle and C.W.J. Granger, in press.

Andersen, T.G., Bollerslev, T., Diebold, F.X. and Wu, J. (2005), "A Framework for Exploring the Macroeconomic Determinants of Systematic Risk," American Economic Review, 95, 398-404.

Areal, N.M.P.C., Taylor, S.J., (2002). The realized volatility of FTSE-100 futures prices. Journal of Futures Markets 22, 627–648.

Arthur, B. W., Durlauf, S,N, Lane, D. A, (1997), The Economy as an Evolving Complex System II, Proceedings Volume XVII. Reading, M.A: Addison-Wesley. p 1-14.

Arthur, B.W., Holland, J. H., LeBaron, B, Palmer, R, Tayler, P, (1997), Asset Pricing Under Endogenous Expectations in Artificial Stock Market, The Economy as an Evolving Complex System II, Proceedings Volume XVII. Reading, M.A: Addison-Wesley, p15-39.

Assaf, A, (2009), Extreme observations and risk assessment in the equity markets of MENA region. Tail measures and Value-at-Risk International Review of Financial Analysis, p109–116.

Bandi, F., Russell, J., (2006). Separating microstructure noise from volatility. Journal of Financial Economics 79, 655–692.

Bandi, F., Russell, J., (2008). Microstructure noise, realized volatility, and optimal sampling. Review of Economic Studies 75, 339–369.

Bandi, F., Russell, J., Zhu, Y., (2008). Using high-frequency data in dynamic portfolio choice. Econometric Reviews 27, 163–198.

Barndorff-Nielsen, O.E. and Shephard, N. (2004), "Power and Bipower Variation with Stochastic Volatility and Jumps" (with discussion), Journal of Financial Econometrics, 2, 1-48.

Barndorff-Nielsen, O. E. And Shephard, N. (2005a), “Econometrics of Testing for Jumps in Financial Economics Using Bipower Variation,” Journal of Financial Econometrics.
Barndorff-Nielsen, O.E. and Shephard, N. (2005b), “Impact of Jumps on Returns and Realised Variances: Econometric Analysis of Time-deformed Levy Processes,” Journal of Econometrics.
Bianchi, S, Pianese, A, (2007) Modeling stock price movements:Multifractality or Multifractionality?. Quantitative Finance, Vol. 7, No. 3 (June) p301-319.
Blackledge, J. M, (2008) Application of the Fractal Market Hypothesis for Macroeconomics Time Series Analysis. ISAST Transactions on Electronic and Signal Processing. No. 1 Vol 2. p 89.
Borland, L, Bouchaud, J, Muzy, J, Zumbach, G, (2005), The Dynamics of financial markets- Mandelbrot’s Multifractal cascades and beyond.
Bouchard, J, Potters, M, (2000), Theory of Financial Risks: From Statistical Physics to Risk Management. Cambridge University Press. United Kingdom.p1-218.
Bradley, B.O, Taqqu, M.S, (2003) Financial Risk and Heavy Tails, Handbook of Heavy Tailed Distributions in Finance. Elsevier Science B.V. Amsterdam, Netherlands. p35-104.
Brock, W, (1991) Understanding Macroeconomic Time Series Using Complex Systems Theory. Structural Change and Economic Dynamics 2. p119-41.
Brownlee, S, (1996), Complexity meets the business World. U.S. News & World Report, Vol. 121, Issue 13.
Bunt, T, Murphy, R, (2008), Are Residual Economic Relationships normally distributed? Testing an assumption of neoclassical economics. Rev Austrian Econ 21. p329-340.
Burlando, T, (1994), Chaos and Risk Management. Risk Management Volume 41 No 4 (April), p54-56.
Butler, C, (1999), Mastering Value at Risk: A set-by-step- guide to understanding and applying VaR. Financial Times Pitman Publishing, p1-241.
Byrne, D.S, (1998), Complexity Theory and the Social Sciences: An Introduction. Routledge.
Calvet, L, Fisher, A, (2001), Forecasting multifractal volatility. Journal of Econometrics, p27–58.
Calvet, L, Fisher, A, (2002), Multifractality in asset returns: theory and evidence, The Review of Economics and Statistics 94, Vol. 84, No. 3 (August), p381-406.
Carlsons, M, (1997), Mirowski's Thesis and the "Integrability Problem" in Neoclassical Economics. Journal of Economic Issues, Vol. 31, No. 3 (September), p741-760.
Carr, P.H, (2004), Does God play dice? Insights from the fractal Geometry of Nature. Zygon, vol. 39, no. 4.
Carrier, D, (1993), Will Chaos kill the auctioneer?, Review of Political Economy, p299 - 320.
Caulkin, S, (1995), Chaos Inc. Across the Board, Vol. 32, Issue 7.
Chen, X., Hansen, L.P., Scheinkman, J., (2009). Nonlinear principal components and long run implications of multivariate diffusions. The Annals of Statistics 37, 4279–4312. 234 T.G. Andersen et al. Journal of Econometrics 160 (2011) 220–234.
Chorafas, D, (1994), Chaos Theory in the Financial Markets. Applying fractals, fuzzy logic, genetic algorithms, swarm simulations and Monte Carlo method to manage market chaos and volatility. Probus Publishing Company. England.
Cootner, P, (1964) The random character of stock market prices. The M.I.T Press. Cambridge.
Cootner, P. H., (1964b), Refinement and Empirical Testing in the random character of stock market prices. The M.I.T Press. Cambridge, 1964. p79-83.
Cootner, P, (1964c), Introduction. The Random Walk Hypothesis Re-examined in The random character of stock market prices. The M.I.T Press. Cambridge, 1964. p189-197.

Cootner, P, (1964d), Comments on the Variation of Certain Speculative Prices in The random character of stock market prices. The M.I.T Press. Cambridge, 1964. p333-337.
Corradi, V., Distaso, W., Swanson, N.R., (2009a). Predictive density estimators for daily volatility based on the use of realized measures. Journal of Econometrics 150, 119–138.
Corradi, V., Distaso, W., Swanson, N.R., (2009b). Predictive inference for integrated volatility. Unpublished Manuscript. University of London.
Corsi, F., (2009). A simple approximate long memory model of realized volatility. Journal of Financial Econometrics 7, 174–196.
Crouchy, M, Galal, D, Mark, R, (2001), Risk Management. Mc-Graw Hill, p1-717.
Culp, C. L, (2002), The Art of Risk Management. John Wiley and Sons, Inc. p1-572.
Dangl, T, Lehar, A, (2001), Value-at-Risk vs. Building Block Regulation in Banking (August). EFA 2001 Barcelona Meetings; AFA 2001 New Orleans Meetings.
David, P.A, (2005), Path Dependence in economic processes: implications for policy analysis in dynamical contexts in The Evolutionary Foundations of economics. Cambridge University Press. United Kingdom. p151-194.
Deo, R., Hurvich, C., Lu, Y., (2006). Forecasting realized volatility using a long-memory stochastic volatility model: estimation, prediction and seasonal adjustment. Journal of Econometrics 131, 29–58.
De Bondt, W, Thaler, R, (1985), Does the Stock Market Overreact? Journal of Finance, XL (3), (July) Pages: 793-805.
De Bondt, W, (2003), Bubble Psychology, Asset Price Bubbles. The Implications for Monetary, Regulatory, and International Policies, Cambridge, MIT Press 2003, p205-16.
Dell'Anno,R., Shneider,F., (2009), A Complex Approach to estimate Shadow Economy: The Structural Equation Modeling, Coping with the complexity of economics Milan, Springer. Italy
Dewatripont, M., Tirole, J., (1994), The prudential Regulation of Banks (Chapter 2). MIT Press. p13-45.
Diebold, F.X. and Strasser, G. (2005), "On the Correlation Structure of Microstructure Noise in Theory and Practice," Manuscript in progress, University of Pennsylvania.
Diebold, F.X. and Vega, C. (2002), "Filtering Microstructure Noise," Manuscript, University of Pennsylvania.
Di Matteo, T.,(2007), Multi-scaling in Finance. Quantitative Finance, Vol. 7, No. 1 (February) 2007, p21-36.
Dopfer, K.,(2005), Evolutionary economics: a theoretical framework in The Evolutionary Foundations of economics. Cambridge University Press. United Kingdom.
Dosi, G.,Marengo, L.,Fagiolo, G.,(2005), Learning in evolutionary environments in The Evolutionary Foundations of economics. Cambridge University Press. United Kingdom.
Doyne, F.J., Fabrizio, L.,(2004), The Long Memory of the Efficient Market, Studies in Nonlinear Dynamics & Econometrics 8, no. 3.p1-33.
Duffie,D.,Pan,J.P.,(1997), An overview of Value at Risk. The Journal of Derivatives (Spring). p7-49.
Durlauf, S.,(2007), What should policymakers know about economic complexity? Department of Economics. University of Wisconsin and Santafe Institute.
Easley, D. and O'Hara, M. (1987), "Price, Trade Size, and Information in Securities Markets." Journal of Financial Economics, 19, 69-90.
Easley, D. and O'Hara, M. (1992), "Time and the Process of Security Price Adjustment," Journal of Finance, 47, 576-605.
Eisele,W.,Knobloch,A.P.,(2000), Value at Risk: Tool for Managing Trading Risks, Risk Management Challenge and Opportunity. Springer-Verlag Berlin Heidelberg. p1-415.
Elliott, G. (1998), "On the Robustness of Cointegration Methods when Regressors Almost Have Unit Roots," Econometrica, 66, 149-158.

Embrechets,P.,Resnick,S., Samorodsky,G., (1998), Living ate the Edge of Chaos. Risk Magazine, 11 (11). p1-10.
Fama,E.,(1963),Mandelbrot and the Stable Paretian Hypothesis, The random character of stock market prices. The M.I.T Press. Cambridge, Massachusetts, 1964.p297 – 306.
Fama,E.,(1965), The behaviour of Stock-Market Prices. Journal of Business, XXXVIII,(1), January, p34-105.
Fama, E., (1970). Efficient Capital Markets: A Review of Theory and Empirical Work, Journal of Finance, XXV (2) May, p382-417.
Fama,E., (1971) Risk, Return, and Equilibrium. The Journal of Political Economy, Vol.79, No. 1 (Jan. - Feb),p30-55.
Fama,E.,French,K.R..(2003) The Capital Asset Pricing Model: Theory and Evidence. CRSP Working Paper No. 550; Tuck Business School Working Paper No. 03-26. (August).
Fabozzi,F.J.,Ortobelli,S.,Rachev,S.,Rachev,S.T.,Stoyanov,S.,(2005) The Poper Use of Risk Measures in Portfolio Theory. International Journal of Theoretical and Applied Finance. Vol 8., No. 8. p1107- 1133.
Fabozzi,F.J.,Rachev,S.,Svetlozar, Sun,W.,(2007), Fractals or I.I.D: Evidence of Long-Range Dependence and Heavy Tailedness from Modeling German Equity Market Returns. Journal of Economics of Business 59. p575 – 595.
Fusaro,P.,(2008) The professional risk management guide to the energy market. McGraw Hill Finance and Investing, United States.
Gaigalas,R.,(2006) A poisson bridge between fractional Brownian motion and stable Lévy motion. Stochastic Processes and their Applications. Volume 116, Issue 3 (March).p447-462.
Garcia, R., Lewis, M.A., Pastorello, S., Renault, E., 2011. Estimation of objective and risk-neutral distributions based on moments of integrated volatility. Journal of Econometrics 160 (1), 22–32.
Garcia, R., Meddahi, N., (2006). Comment on realized variance and market microstructure noise. Journal of Business and Economic Statistics 24, 184–191.
Ghysels, E., Santa-Clara, P., Valkanov, R., (2006). Predicting volatility: getting the most out of return data sampled at different frequencies. Journal of Econometrics 131, 59–96.
Ghysels, E., Sinko, A., (2006). Volatility forecasting and microstructure noise. Unpublished Manuscript. University of North Carolina.
Gleason,J.T.,(2000), Risk: The New Management Imperative in Finance. Bloomberg Press, P.O Box 888, Princeton, NJ 08542.
Glen,P.,(2005),The Efficient Capital Market Hypothesis, Chaos Theory and the insider requirements of the Securities Exchange Act of 1934: The Predictive Power of Form 4 Filings. Fordham Journal of Corporate and Financial law. Vol. XI. 2005. p85-160.
Grandmont,J.M.,(1998), Expectations Formation and stability of large socioeconomic systems. Econometrica, Vol. 66, No. 4 (July)
Grassberger & I. Procaccia, (1982), “Characterization of strange attractors”, Physical Review Letters, 1983, 50(5), 346-349
Glosten, L.R., and Milgrom, P.R. (1985), “Bid, Ask, and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders,” Journal of Financial Economics, 14, 71-100.
Hansen, P.R., Lunde, A., (2006). Realized variance and market microstructure noise. Journal of Business and Economic Statistics 24, 127–161.
Hansen, L.P., Scheinkman, J., (1995). Back to the future: generating moment implications for continuous time Markov processes. Econometrica 63,
767–804.
Hull, J., White, A., (1987). The pricing of options on assets with stochastic volatilities. Journal of Finance XLII, 281–300.

Haken,H.,(2005), Synergetics: from physics to economics in The Evolutionary Foundations of economics. Cambridge University Press. United Kingdom. p70-87.
Hall,J.,Brorsen,W.,Irwin,S.,(1989),The Distribution of Futures Prices: A Test of the Stable Paretian and Mixture of Normals Hypotheses. The Journal of Financial and Quantitative Analysis, Vol. 24, No. 1 (March). p105 -116.
Haridas,A.,(2003),Order and Disorder: The Chaotic Nature of Financial Markets. IIBM Management Review. June 2003. p19-25.
Harmantzis,F., Miao,L.,Chien,Y.,(2005), Empirical Study of Value-at-Risk and Expected Shortfall Models with Heavy Tails (August 23). p1-19.
Harper,I.,KellerJ.G.,Pfeil,C.M.,(2000) Does Risk Management Make Financial Markets Riskier?,Risk Management Challenge and Opportunity. Springer-Verlag Berlin Heidelberg.p1-415.
Hasbrouck, J. (1999a), "Security Bid/Ask Dynamics with Discreteness and Clustering: Simple Strategies for Modeling and Estimation," Journal of Financial Markets, 2, 1-28.
Hasbrouck, J. (1999b), "The Dynamics of Discrete Bid and Ask Quotes," Journal of Finance, 6, 2109-2142.
Hasbrouck, J. (2004c), "Empirical Market Microstructure: Economic and Statistical Perspectives on the Dynamics of Trade in Securities Markets," Manuscript, Stern School of Business, New York University.
Hayek,F.A.,(1967), The Theory of Complex Phenomenon in Studies in philosophy, politics and economics. p22-42.
Herbet,S.,(1962),The Architecture of Complexity Proceedings of the American Philosophical society. December.p467-82.
Herbert,S.,(1987), Decision Making and Problem Solving. Interfaces (Sep/Oct) Vol. 17 Issue 5. p11-31.
Hirschey,M.,(2001), Investments: Theory and applications. First Edition. United States.
Hodgson,G.,(2007), Evolutionary and Institutional Economics as the New Mainstream? Evolutionary Institutional Economics. Econ. Rev. 4 (1).p7 -25.
Holtofr,C., Rudolf,M.,(2000), Market Risk: Benchmark and Standard Model, Risk Management Challenge and Opportunity. Springer-Verlag Berlin Heidelberg. p1-415.
Hsieh,C.Y.,Ye,M.H., (1991), Economics, Philosophy and physics. M.E Sharpe Inc. New York, United States of America. p1-196.
Hudson,R.L.,(2004),"Introducing a Maverick in science". Prelude of the book the (Mis)behaviour of Markets by Benoit Mandelbrot.
Ikenberry,D., Lakonishok,J.,(1989), Seasonal Anomalies in Financial Markets: A survey in A Reappraisal of the Efficiency of Financial Markets, NATO. Advanced Science Institute Series: Springer-Verlag.
Jacod, J., Protter, P., (1998). Asymptotic error distributions for the Euler method for stochastic differential equations. The Annals of Probability 26, 267–307.
Jaditz T. & Sayers C., (1993), "Using out of sample forecasting performance to evaluate model specification.", Bureau of Labor Statistics, 1993, 1(1), 1-12
James F., (2003). "Comment on 'Exact solutions to chaotic and stochastic systems'", Chaos: An Interdisciplinary Journal of Nonlinear Science, Mar, 13(1), 123
Jayasuriya D,D. (2016), Realized Volatility and Market Microstructure, working paper.
Koopman, S.J., Jungbacker, B., Hol, E., (2005). Forecasting daily variability of the S&P 100 stock index using historical, realized and implied volatility measures. Journal of Empirical Finance 12, 445–475.
Kyle, A. (1985), "Continuous Auctions and Insider Trading," Econometrica, 53, 1315-1335.
LeBaron B., (1994), "Chaos and Nonlinear Forecastability in Economics and Finance", University of Wisconsin, 1994, 2, 1-10 Lorenz E., 1963, "Deterministic non-periodic flow", Journal of Atmospheric Sciences, 1963, 20, 130-141

Lo, A.W. and MacKinlay, A.C. (2001), A Non-Random Walk Down Wall Street. Princeton: Princeton University Press.
Mandelbrot B., 1999, “A Multifractal Walk down Wall Street”, Scientific American, 1999, 1, 70-74P.
Martens, M., (2002). Measuring and forecasting S&P 500 index futures volatility using high-frequency data. Journal of Futures Markets 22, 497–518.
Meddahi, N., (2001a). An eigenfunction approach for volatility modeling. CIRANO Working Paper. 2001s-70.
Meddahi, N., (2002b). A theoretical comparison between integrated and realized volatility. Journal of Applied Econometrics 17, 479–508.
Meddahi, N., (2002c). Moments of continuous time stochastic volatility models. Working Paper. Université de Montréal.
Merton, R.C., (1980). On estimating the expected return on the market: an exploratory investigation. Journal of Financial Economics 8, 323–361.
Owens, J.P. and Steigerwald, D.G. (2005), “Noise Reduced Realized Volatility: A Kalman Filter Approach,” in T. Fomby and D. Terrell (eds.), Advances in Econometrics: Econometric Analysis of Economic and Financial Time Series, Volume 20, Part A, forthcoming.
Patton, A.J., (2011). Volatility forecast comparison using imperfect volatility proxies. Journal of Econometrics 160 (1), 246–256.
Pong, S., Shackleton, M.B., Taylor, S.J., Xu, X., (2004). Forecasting currency volatility: a comparison of implied volatilities and AR(FI)MA models. Journal of Banking and Finance 28, 2541–2563.
Roll, R. (1984), “A Simple Implicit Measure of the Effective Bid-ask Spread in an Efficient Market,” Journal of Finance, 39, 1127-1139.
Sun, X., (2006). Best quadratic unbiased estimators of integrated variance in the presence of market microstructure noise. Working Paper. University of California at San Diego.
Thomakos, D.D., Wang, T., (2003). Realized volatility in the futures market. Journal of Empirical Finance 10, 321–353.
Zhang, L., (2006). Efficient estimation of stochastic volatility using noisy observations: a multi-scale approach. Bernoulli 12, 1019–1043.
Zhang, L., Mykland, P.A., Aït-Sahalia, Y., (2005). A tale of two time scales: determining integrated volatility with noisy high-frequency data. Journal of the American Statistical Association 100, 1394–1411.
Zhou, B., (1996). High-frequency data and volatility in foreign exchange rates. Journal of Business and Economic Statistics 14, 45–52.
Zumbach, G., Corsi, F., Trapletti, A., (2002). Efficient estimation of volatility using high-frequency data. Unpublished Manuscript. Olsen & Associates, Zürich, Switzerland.

Tables and Figures

Appendix

Table 1: Summary Statistics for DS1,2 and 3

Variable	Mean	Std Dev	Min	Max	Std Skewness	Std Kurtosis	Q1	Q2	No of Obs
Stock Data (DS1)									
GOOGLE	0.0013	0.02	-0.12	0.18	0.50	9.73	0.01	0.01	1402
MSFT	0.0001	0.02	-0.12	0.17	0.15	14.63	-0.01	0.01	1402
IBM	0.0003	0.01	-0.09	0.11	0.05	9.21	-0.01	0.01	1402
DELL	-0.0006	0.02	-0.15	0.13	-0.43	9.41	-0.01	0.01	1402
Indices (DS2)									
NIKKEI	0.0001	0.01	-0.16	0. 13	-0.27	13.10	-0.01	0.01	7829
S&P	0.0003	0.01	-0.23	0.11	-1.25	32.64	0.00	0.01	7829
TOPIX	0.0001	0.01	-0.16	0.13	-0.33	14.27	0.00	0.01	7829
Currencies (DS3)									
USD-EUR	0.0001	0.01	-0.03	0.03	0.14	4.49	0.00	0.00	2870
USD-UBP	0.0000	0.01	-0.04	0.03	-0.37	5.95	0.00	0.00	2870

Notes: Q1 is the test statistic for the Box-Pierce portmanteau test of the differenced logarithmic series based on the autocorrelation coefficients up to order 20. Q2 is the portmanteau test statistic of the squared differenced logarithmic series.

Table 2: Summary Statistics of for High Frequency Data

Variable	Mean	Std Dev	Min	Max	Std Skewness	Std Kurtosis	Q1	Q2	No of Obs
GOOGLE-HF	0.0001	0.02	-0.70	0.69	-0.12	841.55	1691.50	0.00	1716
MSFT-HF	0.0004	0.14	-2.90	2.90	0.04	215.60	0.00	0.00	1716
IBM-HF	0.0001	0.45	-7.09	7.09	0.01	214.64	0.00	0.00	1716
DELL-HF	0.0000	0.15	-3.85	3.85	0.02	526.61	0.00	0.00	1716

Table 3: BDS test results for High frequency IBM data

Embedding Dimensions =	2346			
Epsilon for close points =	0.1602	0.3203	0.4805	0.6406
Standard Normal =				
dim				
[2]	2.6339	2.6339	3.1788	3.1394
[3]	2.2218	2.2218	2.7314	2.6983
[4]	1.8696	1.8696	2.3476	2.3125
[5]	1.5904	1.5904	2.0457	2.0077
p-value=				
dim				
[2]	0.0084	0.0084	0.0015	0.0017
[3]	0.0263	0.0263	0.0063	0.007
[4]	0.0615	0.0615	0.0189	0.0208
[5]	0.1117	0.1117	0.0408	0.0447

Appendix

Table 1A: Coefficients of the mean models for IBM

Model	Coefficient	Coefficient	Coefficient		Intercept
AR(1)	-0.02				0.00
	0.03				0.00
AR(2)	-0.02	-0.01			0.00
	0.03	0.03			0.00
AR(3)	-0.02	0.00	0.08		0.00
	0.03	0.03	0.03		0.00
MA(1)	-0.02				0.00
	0.03				0.00
MA(2)	-0.02	0.00			0.00
	0.03	0.03			0.00
MA(3)	-0.02	0.01	0.09		0.00
	0.03	0.03	0.03		0.00
ARMA(1,1)	-0.01	-0.01			0.00
	0.41	0.41			0.00
ARMA(1,2)	-0.01	-0.01	0.00		0.00
	0.41	0.41	0.41		0.00
ARMA(2,1)	-0.70	-0.05	0.67		0.00
	0.20	0.03	0.20		0.00
ARMA(2,2)	-0.38	-0.81	0.34	0.77	0.00
	0.23	0.11	0.23	0.13	0.00

Table 2A: Test results for IBM

Model/Test	**BIC**	**JB-Test**	**Lung-Box**	**ADF**
AR(1)	3925	2251	0.76	-10.82
AR(2)	3933	2251	0.76	-10.81
AR(3)	3945	2251	0.76	-11.13
MA(1)	3925	2251	0.76	-10.81
MA(2)	3933	2251	0.76	-10.81
MA(3)	3945	2251	0.76	-11.14
ARMA(1,1)	3933	2251	0.76	-10.82
ARMA(1,2)	3311	2251	0.76	-10.80
ARMA(2,1)	3941	2251	0.76	-9.52
ARMA(2,2)	3952	2251	0.76	-10.76

Table 3A: Coefficients of the residual models for IBM

Model	omega	alpha	beta	mu	gamma
After fitting data into AR(1)					
GARCH(1,1)	0.00	0.13	0.84		
EGARCH(1,0)	-0.14	-0.08	0.98	0.00	0.13
After fitting data into AR(2)					
GARCH(1,1)	0.00	0.13	0.84		
EGARCH(1,0)	-0.14	-0.08	0.98	0.00	0.13
After fitting data into AR(3)					
GARCH(1,1)	0.00	0.12	0.85		
EGARCH(1,0)	-0.11	-0.08	0.99	0.00	0.10
After fitting data into MA(1)					
GARCH(1,1)	0.00	0.13	0.84		
EGARCH(1,0)	-0.14	-0.08	0.98	0.00	0.13
After fitting data into MA(2)					
GARCH(1,1)	0.00	0.13	0.84		
EGARCH(1,0)	-0.14	-0.08	0.98	0.00	0.13
After fitting data into MA(3)					
GARCH(1,1)	0.00	0.12	0.85		
EGARCH(1,0)	-0.11	-0.08	0.99	0.00	0.10
After fitting data into ARMA(1,1)					
GARCH(1,1)	0.00	0.13	0.84		
EGARCH(1,0)	-0.14	-0.08	0.98	0.00	0.13
After fitting data into ARMA(1,2)					
GARCH(1,1)	0.00	0.13	0.84		
EGARCH(1,0)	-0.14	-0.08	0.98	0.00	0.13
After fitting data into ARMA(2,1)					
GARCH(1,1)	0.00	0.13	0.84		
EGARCH(1,0)	-0.13	-0.08	0.98	0.00	0.12
After fitting data into ARMA(2,2)					
GARCH(1,1)	0.00	0.14	0.84		
EGARCH(1,0)	-0.15	-0.08	0.98	0.00	0.13

Table 4A: Coefficients of the mean models for Dell

Model	Coefficients	Coefficients	Coefficients	Coefficients	Intercept
AR(1)	-0.05				0.00
	0.03				0.00
AR(2)	-0.05	-0.04			0.00
	0.03	0.03			0.00
AR(3)	-0.05	-0.04	-0.03		0.00
	0.03	0.03	0.03		0.00
MA(1)	-0.06				0.00
	0.03				0.00
MA(2)	-0.06	-0.05			0.00
	0.03	0.03			0.00
MA(3)	-0.06	-0.04	-0.02		0.00
	0.03	0.03	0.03		0.00
ARMA(1,1)	0.51	-0.57			0.00
	0.17	0.17			0.00
ARMA(1,2)	0.36	-0.42	-0.03		0.00
	0.30	0.30	0.04		0.00
ARMA(2,1)	0.42	-0.02	-0.48		0.00
	0.24	0.03	0.24		0.00
ARMA(2,2)	0.06	0.16	-0.11	-0.20	0.00
	1.84	1.05	1.83	1.13	0.00

Table 5A: Test results for Dell

Model/Test	**BIC**	**JB-Test**	**Lung-Box**	**ADF**
AR(1)	3295	2443	3.66	-9.71
AR(2)	3303	2443	3.66	-9.65
AR(3)	3311	2443	3.66	-9.59
MA(1)	3295	2443	3.66	-9.70
MA(2)	3303	2443	3.66	-9.62
MA(3)	3311	2443	3.66	-9.56
ARMA(1,1)	3304	2443	3.66	-9.51
ARMA(1,2)	3311	2443	3.66	-9.52
ARMA(2,1)	3311	2443	3.66	-9.52
ARMA(2,2)	3318	2443	3.66	-9.53

Table 6A: Coefficients of the residual models for Dell

Model	omega	alpha	beta	mu	gamma
After fitting data into AR(1)					
GARCH(1,1)	0.00	0.03	0.96		
EGARCH(1,0)	-0.06	-0.04	0.99	0.00	0.08
After fitting data into AR(2)					
GARCH(1,1)	0.00	0.03	0.96		
EGARCH(1,0)	-0.06	-0.04	0.99	0.00	0.08
After fitting data into AR(3)					
GARCH(1,1)	0.00	0.03	0.96		
EGARCH(1,0)	-0.06	-0.04	0.99	0.00	0.08
After fitting data into MA(1)					
GARCH(1,1)	0.00	0.03	0.96		
EGARCH(1,0)	-0.06	-0.04	0.99	0.00	0.08
After fitting data into MA(2)					
GARCH(1,1)	0.00	0.03	0.96		
EGARCH(1,0)	-0.06	-0.04	0.99	0.00	0.08
After fitting data into MA(3)					
GARCH(1,1)	0.00	0.03	0.96		
EGARCH(1,0)	-0.06	-0.04	0.99	0.00	0.08
After fitting data into ARMA(1,1)					
GARCH(1,1)	0.00	0.03	0.96		
EGARCH(1,0)	-0.06	-0.03	0.99	0.00	0.08
After fitting data into ARMA(1,2)					
GARCH(1,1)	0.00	0.03	0.96		
EGARCH(1,0)	-0.06	-0.04	0.99	0.00	0.08
After fitting data into ARMA(2,1)					
GARCH(1,1)	0.00	0.03	0.96		
EGARCH(1,0)	-0.06	-0.04	0.99	0.00	0.08
After fitting data into ARMA(2,2)					
GARCH(1,1)	0.00	0.03	0.96		
EGARCH(1,0)	-0.06	-0.04	0.99	0.00	0.08

Table 7A: Coefficients of the mean models for Google

Model	Coefficients	Coefficients	Coefficients	Coefficients	Intercept
AR(1)	0.00				0.00
	0.03				0.00
AR(2)	0.00	-0.01			0.00
	0.03	0.03			0.00
AR(3)	0.00	-0.01	0.00		0.00
	0.03	0.03	0.03		0.00
MA(1)	0.00				0.00
	0.03				0.00
MA(2)	0.00	-0.01			0.00
	0.03	0.03			0.00
MA(3)	0.00	-0.01	0.00		0.00
	0.03	0.03	0.03		0.00
ARMA(1,1)	0.00	0.00			0.00
	1.89				0.00
ARMA(1,2)	0.00	0.00	-0.01		0.00
	2.03	2.03	0.03		0.00
ARMA(2,1)	0.04	-0.01	-0.04		0.00
	1.89	0.03	1.89		0.00
ARMA(2,2)	0.04	-0.98	-0.04	0.96	0.00
	0.01	0.02	0.02	0.03	0.00

Table 8A: Tests for Google

Model/Test	BIC	JB-Test	Lung-Box	ADF
AR(1)	3280	2706	0.00	-10.44
AR(2)	3287	2706	0.00	-10.43
AR(3)	3294	2443	3.66	-10.44
MA(1)	3280	2443	3.66	-10.44
MA(2)	3287	2443	3.66	-10.43
MA(3)	3294	2443	3.66	-10.44
ARMA(1,1)	3287	2443	3.66	-10.45
ARMA(1,2)	3294	2443	3.66	-10.43
ARMA(2,1)	3294	2443	3.66	-10.43
ARMA(2,2)	3308	2443	3.66	-10.52

Table 9A: Coefficients of the residual models for Google

Model	omega	alpha	beta	mu	gamma
After fitting data into AR(1)					
GARCH(1,1)	0.00	0.06	0.93		
EGARCH(1,0)	-0.12	-0.04	0.98	0.00	0.16
After fitting data into AR(2)					
GARCH(1,1)	0.00	0.03	0.93		
EGARCH(1,0)	-0.12	-0.04	0.98	0.00	0.16
After fitting data into AR(3)					
GARCH(1,1)	0.00	0.06	0.93		
EGARCH(1,0)	-0.12	-0.04	0.98	0.00	0.16
After fitting data into MA(1)					
GARCH(1,1)	0.00	0.06	0.93		
EGARCH(1,0)	-0.12	-0.04	0.98	0.00	0.16
After fitting data into MA(2)					
GARCH(1,1)	0.00	0.06	0.93		
EGARCH(1,0)	-0.12	-0.04	0.98	0.00	0.16
After fitting data into MA(3)					
GARCH(1,1)	0.00	0.06	0.93		
EGARCH(1,0)	-0.12	-0.04	0.98	0.00	0.16
After fitting data into ARMA(1,1)					
GARCH(1,1)	0.00	0.06	0.93		
EGARCH(1,0)	-0.12	-0.04	0.16	0.98	0.16
After fitting data into ARMA(1,2)					
GARCH(1,1)	0.00	0.06	0.93		
EGARCH(1,0)	-0.12	-0.04	0.98	0.00	0.16
After fitting data into ARMA(2,1)					
GARCH(1,1)	0.00	0.06	0.93		
EGARCH(1,0)	-0.12	-0.04	0.98	0.00	0.16
After fitting data into ARMA(2,2)					
GARCH(1,1)	0.00	0.07	0.93		
EGARCH(1,0)	-0.14	-0.04	0.98	0.00	0.17

Table 10A: Coefficients of the mean models for Microsoft

Model	Coefficients	Coefficients	Coefficients	Coefficients	Intercept
AR(1)	-0.10				0.00
	0.03				0.00
AR(2)	-0.11	-0.09			0.00
	0.03	0.03			0.00
AR(3)	-0.10	-0.08	0.07		0.00
	0.03	0.03	0.03		0.00
MA(1)	-0.12				0.00
	0.03				0.00
MA(2)	-0.10	-0.08			0.00
	0.03	0.03			0.00
MA(3)	-0.09	-0.08	0.07		0.00
	0.03	0.03	0.03		0.00
ARMA(1,1)	0.45	-0.55			0.00
	0.28	0.26			0.00
ARMA(1,2)	-0.48	0.39	-0.14		0.00
	0.14	0.14	0.03		0.00
ARMA(2,1)	-0.58	-0.15	0.47		0.00
	0.13	0.03	0.13		0.00
ARMA(2,2)	-0.84	-0.50	0.75	0.36	0.00
	0.20	0.18	0.21	0.19	0.00

Table 11A: Tests for Microsoft

Model/Test	BIC	JB-Test	Lung-Box	ADF
AR(1)	3589	7907	13.67	-10.52
AR(2)	3602	7907	13.67	-10.35
AR(3)	3612	7907	13.67	-10.54
MA(1)	3590	7907	13.67	-10.48
MA(2)	3601	7907	13.67	-10.27
MA(3)	3611	7907	13.67	-10.55
ARMA(1,1)	3599	7907	13.67	-10.15
ARMA(1,2)	3611	7907	13.67	-10.35
ARMA(2,1)	3612	7907	13.67	-10.41
ARMA(2,2)	3621	7907	13.67	-10.52

Table 12A: Coefficients of the residual models for Microsoft

Model	omega	alpha	beta	mu	gamma
After fitting data into AR(1)					
GARCH(1,1)	0.00	0.09	0.88		
After fitting data into AR(2)					
GARCH(1,1)	0.00	0.09	0.88		
EGARCH(1,0)	-0.09	-0.01	0.99	0.00	0.10
After fitting data into AR(3)					
GARCH(1,1)	0.00	0.09	0.88		
EGARCH(1,0)	-0.08	-0.01	0.99	0.00	0.09
After fitting data into MA(1)					
GARCH(1,1)	0.00	0.09	0.88		
EGARCH(1,0)	-0.09	-0.01	0.99	0.00	0.09
After fitting data into MA(2)					
GARCH(1,1)	0.00	0.10	0.87		
EGARCH(1,0)	-0.09	-0.01	0.99	0.00	0.10
After fitting data into MA(3)					
GARCH(1,1)	0.00	0.09	0.88		
EGARCH(1,0)	-0.08	-0.01	0.99	0.00	0.09
After fitting data into ARMA(1,1)					
GARCH(1,1)	0.00	0.10	0.87		
EGARCH(1,0)	-0.09	-0.01	0.99	0.00	0.10
After fitting data into ARMA(1,2)					
GARCH(1,1)	0.00	0.09	0.88		
EGARCH(1,0)	-0.08	-0.01	0.99	0.00	0.09
After fitting data into ARMA(2,1)					
GARCH(1,1)	0.00	0.09	0.88		
EGARCH(1,0)	-0.08	-0.01	0.99	0.00	0.09
After fitting data into ARMA(2,2)					
GARCH(1,1)	0.00	0.09	0.88		
EGARCH(1,0)	-0.08	-0.01	0.99	0.00	0.09

Table 13A: High Frequency data for DELL

Model	Coefficient	Coefficient	Intercept
AR(1)	-0.50		0.00
	0.02		0.00
AR(2)	-0.67	-0.33	0.00
	0.02	0.02	0.00
MA(1)	-0.98		0.00
	0.00		0.00
ARMA(1,1)	0.00	-0.98	0.00
	0.02	0.00	0.00

Table 14A: High Frequency data for DELL diagnostic tests

Model	Log likelihood	AIC	BIC	J-B Test	Box-Test	ADF
AR(1)	1081	-2155	1088	19600000.00	429.94	-19.14
AR(2)	1181	-2354	1196	19600000.00	429.94	-19.09
MA(1)	1413	-2820	1420	19600000.00	429.94	-13.33
ARMA(1,1)	1413	-2818	1428	19600000.00	429.94	-13.33

Table 15A: HF Data, Coefficients of the residual models for Dell

After fitting data into AR(1)					
Model	**omega**	**mu**	**alpha1**	**beta1**	**gamma11**
GARCH(1,1)	0.00		0.40	0.60	
FGARCH(1,1)	0.00	0.00	0.11	0.87	
IGARCH(1,1)	5.881781e-	0.00	0.41	0.59	
EGARCH(1,1)	-0.35		0.08	0.92	0.26
TGARCH(1,1)	0.00	0.00	0.10	0.92	0.03

Table 16A: High Frequency data for GOOGLE

Model	Coefficient	Coefficient	Intercept
AR(1)	-0.49		0.00
	0.02		0.00
AR(2)	-0.66	-0.33	0.00
	0.02	0.02	0.00
MA(1)	-0.87		0.00
	0.01		0.00
ARMA(1,1)	0.00	-0.87	0.00
	0.03	0.01	0.00

Table 17A: HF Data, Coefficients of the residual models for GOOGLE

Model	omega	mu	alpha1	beta1	gamma11
GARCH(1,1)	0.00		0.70	0.31	
FGARCH(1,1)	0.00	0.00	0.10	0.90	
IGARCH(1,1)	0.57	0.00	0.43	0.57	
EGARCH(1,1)	-2.14		0.38	0.82	0.40
TGARCH(1,1)	0.00	0.00	0.11	0.91	0.05

Table 18A: HF Data, Diagnostic tests for GOOGLE

Model	Log likelihood	AIC	BIC	J-B Test	Box-Test	ADF
AR(1)	4219	-8432	4227	50300000	421	-17.81
AR(2)	4316	-8624	4331	50300000	421	-17.24
MA(1)	4462	-8919	4470	50300000	421	-11.93
ARMA(1,1)	4462	-8917	4477	50300000	421	-11.92

Table 19A: HF Data, Coefficients of the mean models for IBM

Model	Coefficient	Coefficient	Intercept
AR(1)	-0.50		0.00
	0.02		0.01
AR(2)	-0.66	-0.33	0.00
	0.02	0.02	0.00
MA(1)	-1.00		0.00
	0.00		0.00
ARMA(1,1)	0.00	-1.00	0.00
	0.02	0.00	0.00

Table 20A: HF Data, Diagnostic tests for IBM

Model	Log likelihood	AIC	BIC	J-B Test	Box-Test	ADF
AR(1)	-831	1667	-823	3200000	427	-19.92
AR(2)	-730	1467	-715	3200000	427	-19.45
MA(1)	-485	975	-477	3200000	427	-12.52
ARMA(1,1)	-485	977	-470	3200000	427	-12.53

Table 21A: HF Data, Coefficients of the residual models for IBM

Model	omega	mu	alpha1	beta1	gamma11
GARCH(1,1)	0.00		1.00	0.00	
FGARCH(1,1)	0.00	0.00	0.06	0.91	
IGARCH(1,1)	0.56	0.00	0.45	0.56	
EGARCH(1,1)	-3.52		-1.15	0.33	1.34
TGARCH(1,1)	0.00	0.00	0.13	0.90	0.22

Table 22A: HF Data, Coefficients of the mean models for Microsoft

Model	Coefficient	Coefficient	Intercept
AR(1)	-0.48		0.00
	0.02		0.00
AR(2)	-0.64	-0.33	0.00
	0.02	0.02	0.00
MA(1)	-0.86		0.00
	0.01		0.00
ARMA(1,1)	0.02	-0.86	0.00
	0.03	0.01	0.00

Table 23A: HF Data, Diagnostic tests for Microsoft

Model	Log likelihood	AIC	BIC	J-B Test	Box-Test	ADF
AR(1)	1142	-2278	1149	3230000	396	-18.22
AR(2)	1244	-2480	1259	3230000	396	-17.46
MA(1)	1377	-2749	1385	3230000	396	-11.83
ARMA(1,1)	1378	-2747	1392	3230000	396	-11.73

Table 24A: HF Data, Coefficients of the residual models for Microsoft

Model	omega	mu	alpha1	beta1	gamma11
GARCH(1,1)	0.00		0.05	0.95	
FGARCH(1,1)	0.00	0.00	0.08	0.90	
IGARCH(1,1)	0.59	0.00	0.41	0.59	
EGARCH(1,1)	-0.18		-0.09	0.94	0.18
TGARCH(1,1)	0.00	0.00	0.07	0.94	0.27

Table 25A: Daily Data, Coefficients of the BEKK(1,1) MGARCH model

Data	Variable	Mean Model	Coefficients	k=4			
DS1	DELL	AR(1)	0.01	0.00	0.00	0.00	0.01
			0.00	0.00	0.00	-0.01	0.01
	MSFT		0.46	-0.14	0.03	-0.06	0.03
			0.25	0.47	-0.21	0.00	0.25
	IBM		0.42	0.01	-0.09	0.17	0.11
			0.06	0.25	-0.85	-0.13	0.03
	GOOGLE		0.06	-0.92	0.81	0.19	-0.05
			-0.39	-0.20	-0.01	0.07	-0.83
			0.09	-0.18			
DS1	DELL	MA(1)	0.02	0.01	-0.01	0.00	0.00
			0.00	0.01	-0.01	0.00	0.02
	MSFT		-0.40	0.62	0.34	0.17	0.02
			0.30	0.14	0.17	0.08	0.37
	IBM		0.23	0.04	-0.15	0.48	-0.12
			-0.10	0.01	0.11	-0.47	0.54
	GOOGLE		-0.09	-0.43	0.68	0.23	-0.17
			-0.50	0.24	0.41	-0.24	-0.54
			0.16	0.33			
DS1	DELL	ARMA(1,1)	0.01	0.00	0.00	0.00	0.01
			0.01	0.00	0.00	0.00	0.01
	MSFT		-0.40	0.30	0.42	0.18	0.04
			0.18	-0.05	0.10	0.06	0.22
	IBM		0.21	0.06	0.04	0.27	-0.16
			-0.13	-0.43	-0.60	0.68	-0.02
	GOOGLE		-0.06	-0.87	-0.28	0.20	-0.14
			-0.34	0.17	0.01	0.01	-0.86
			1.14	-0.60			

Table 26A: High Frequency Data, Coefficients of the BEKK(1,1) MGARCH model

Data	Variable	Mean Model	Coefficients	k=4			
DS1	DELL	AR(1)	0.00	0.07	-0.07	-0.17	-0.18
			0.25	0.01	-0.01	0.00	0.05
	MSFT		0.38	0.00	0.00	0.00	-0.01
			0.58	0.00	0.07	0.05	0.02
	IBM		0.36	-0.05	-0.01	0.06	0.48
			-0.31	-0.23	0.00	0.04	-0.03
	GOOGLE		-0.09	-0.22	-0.06	0.05	-0.89
			0.01	0.11	-0.46	0.03	-0.06
			0.00	0.20			
DS1	DELL	MA(1)	0.01	0.01	-0.06	-0.12	-0.02
			-0.27	0.00	0.00	0.00	0.00
	MSFT		-0.61	-0.03	0.00	0.01	0.73
			-0.96	0.01	2.45	1.55	-0.26
	IBM		0.02	1.26	0.03	-0.03	-0.36
			3.21	-0.41	0.01	0.03	-0.01
	GOOGLE		-0.83	-0.36	-0.05	-0.46	-5.16
			0.05	0.21	0.40	0.07	-0.01
			0.00	0.28			
DS1	DELL	ARMA(1,1)	0.01	0.01	0.01	0.21	-0.03
			0.19	0.00	-0.01	0.00	0.00
	MSFT		-0.25	-0.02	-0.02	-0.03	-0.19
			-0.12	0.01	0.46	-3.23	-0.21
	IBM		0.44	0.31	-4.33	1.08	1.64
			2.50	-0.19	-0.18	0.01	-0.02
	GOOGLE		-1.72	0.35	0.20	-0.07	1.04
			1.69	-0.18	-0.15	0.00	-0.02
			0.01	-0.01			

Figure 1A: Prices for DS1- Google, Microsoft, IBM & DELL

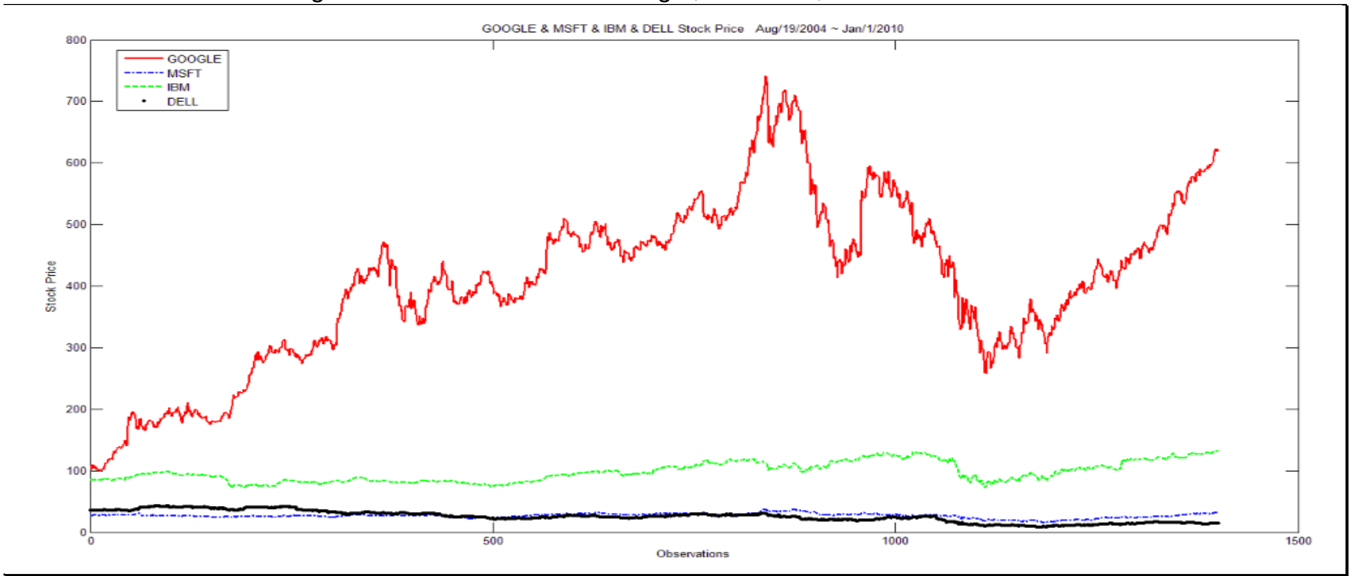

Figure 2A: Index Values for DS2-Nikkei, TOPIX, S&P500

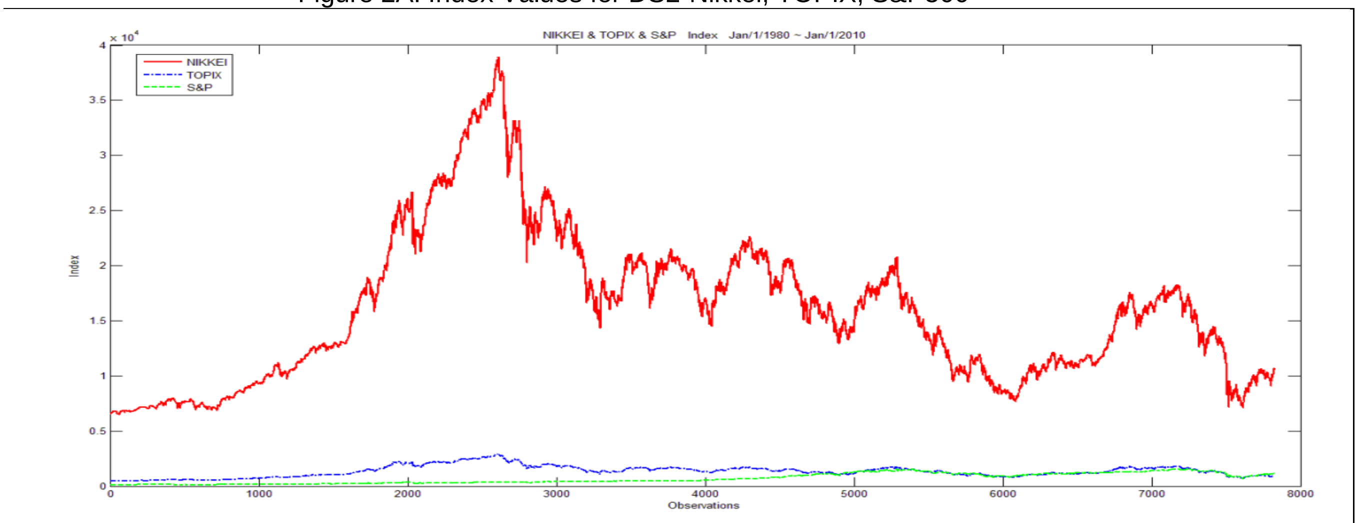

Figure 3A: Exchange rates for DS3-USD/EUR and USD/GBP

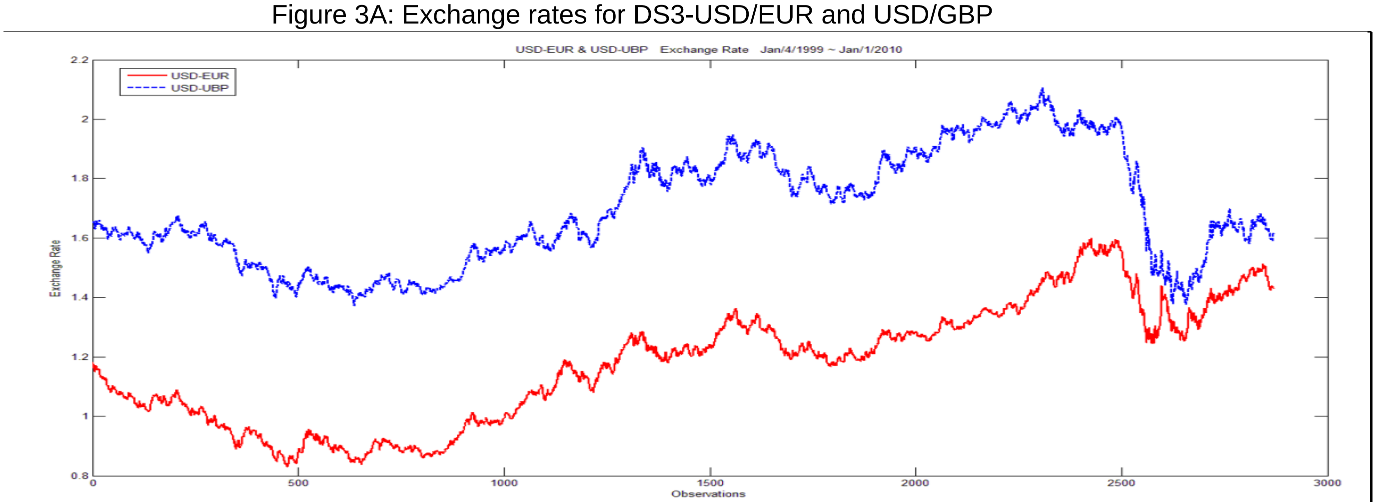

Figure 4A: PACF and ACF plots for daily data: DELL

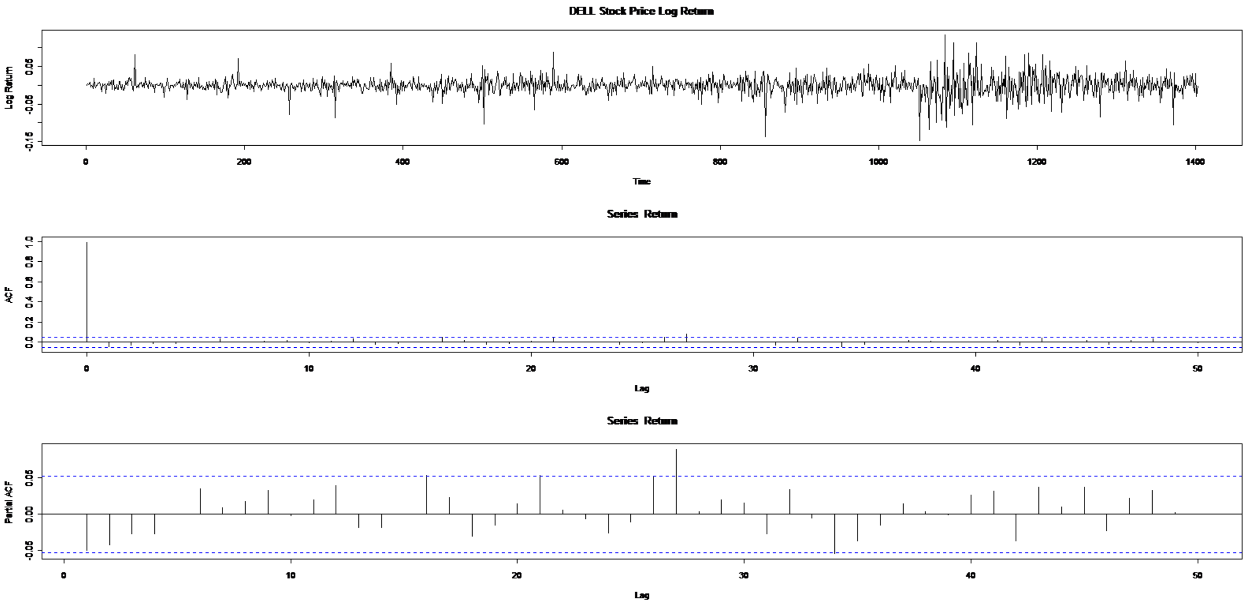

Figure 5A: PACF and ACF plots for daily data: S&P 500 index returns

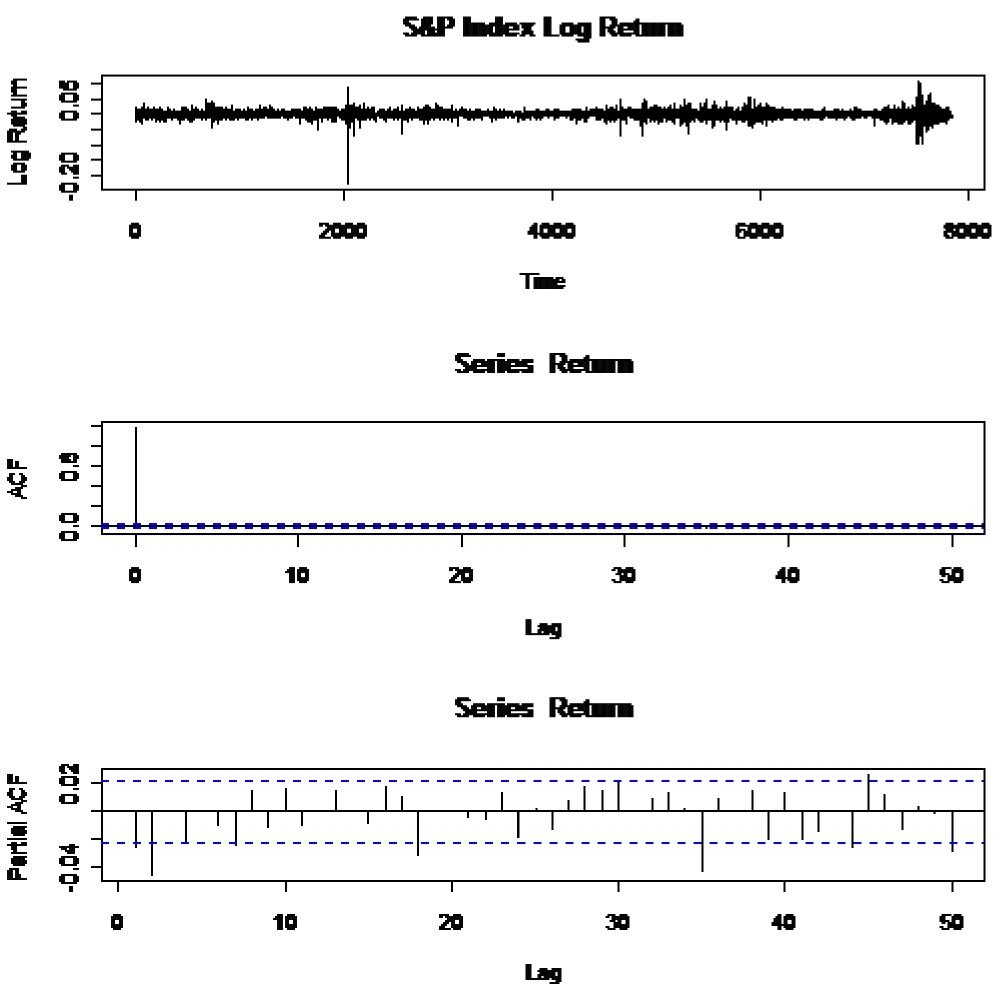

Figure 6A: PACF and ACF plots for daily data: TOPIX Index

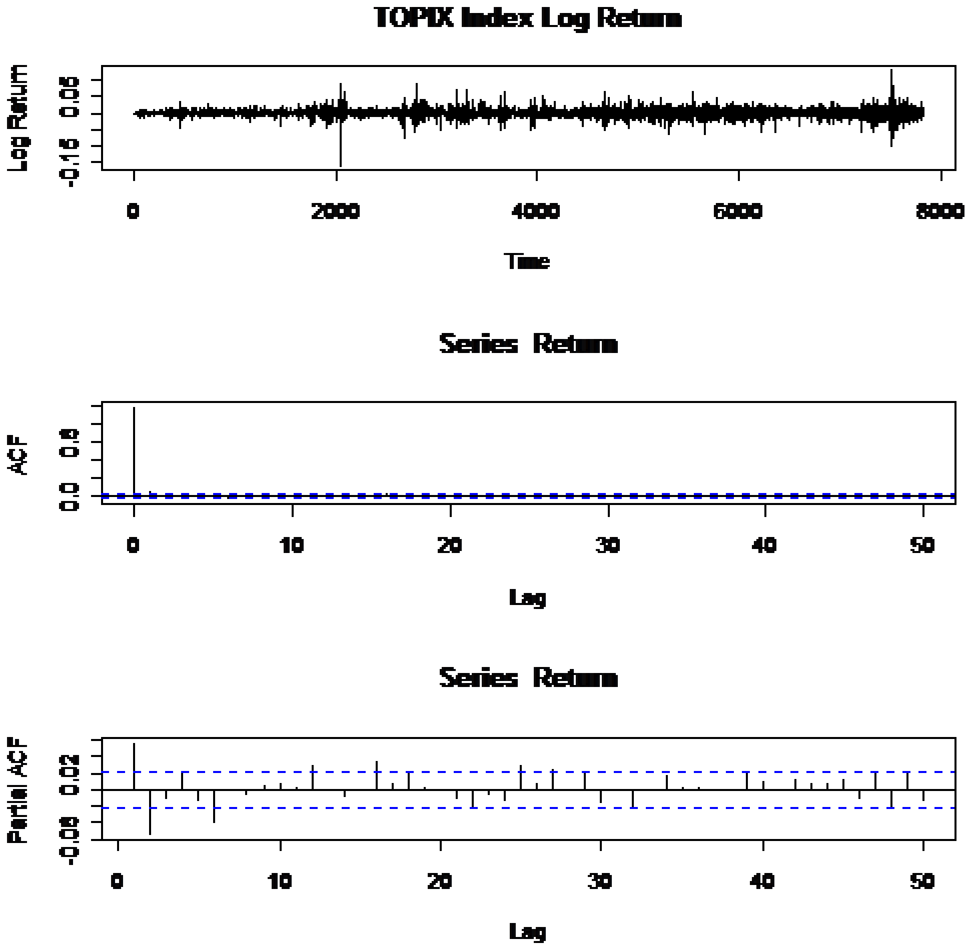

Figure 7A: PACF and ACF plots for daily data: USD vs EUR

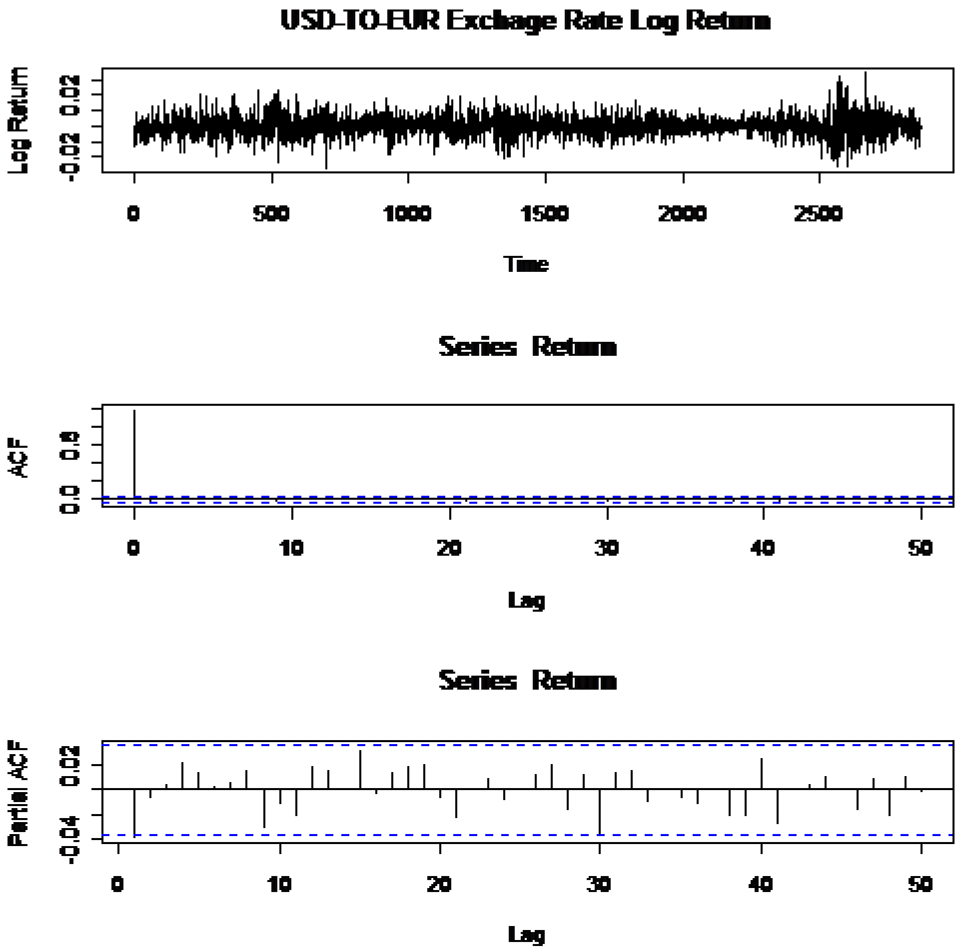

Figure 8A: Kernel estimates of Dell daily returns.

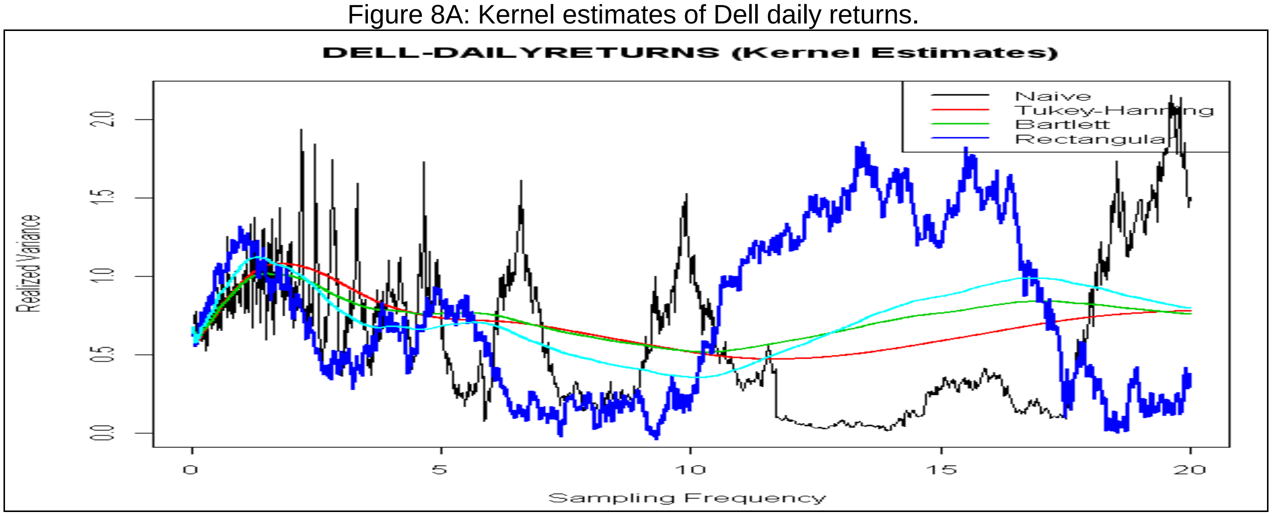

Figure 9A: Sampling estimates for Dell daily returns.

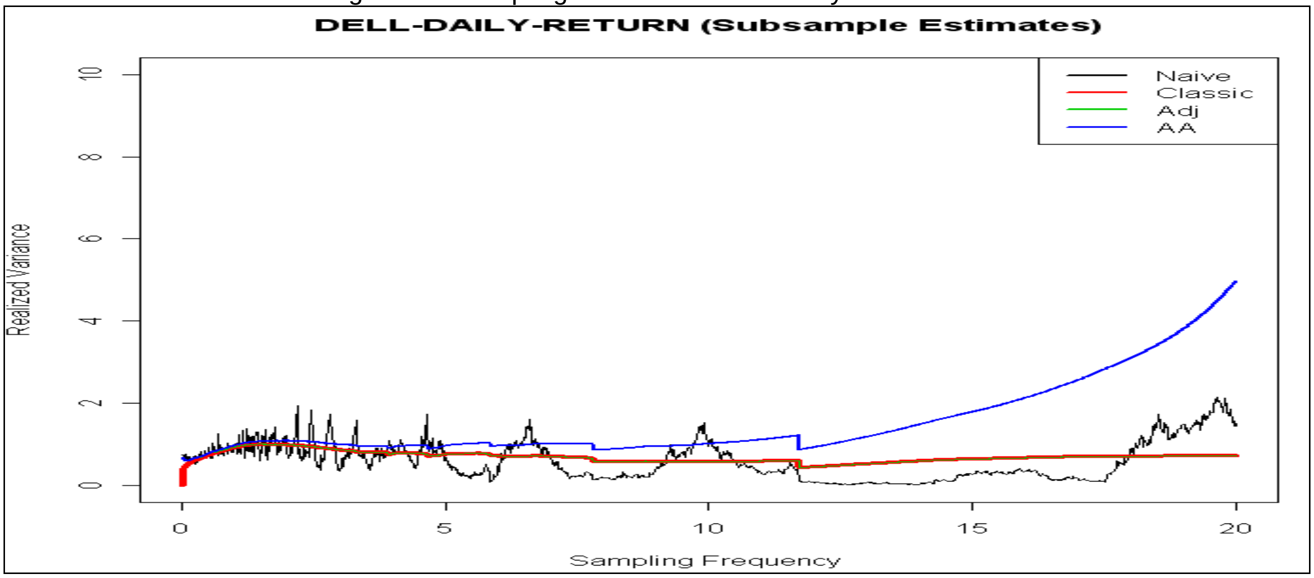

Figure 10A: Kernel estimates for Google daily returns.

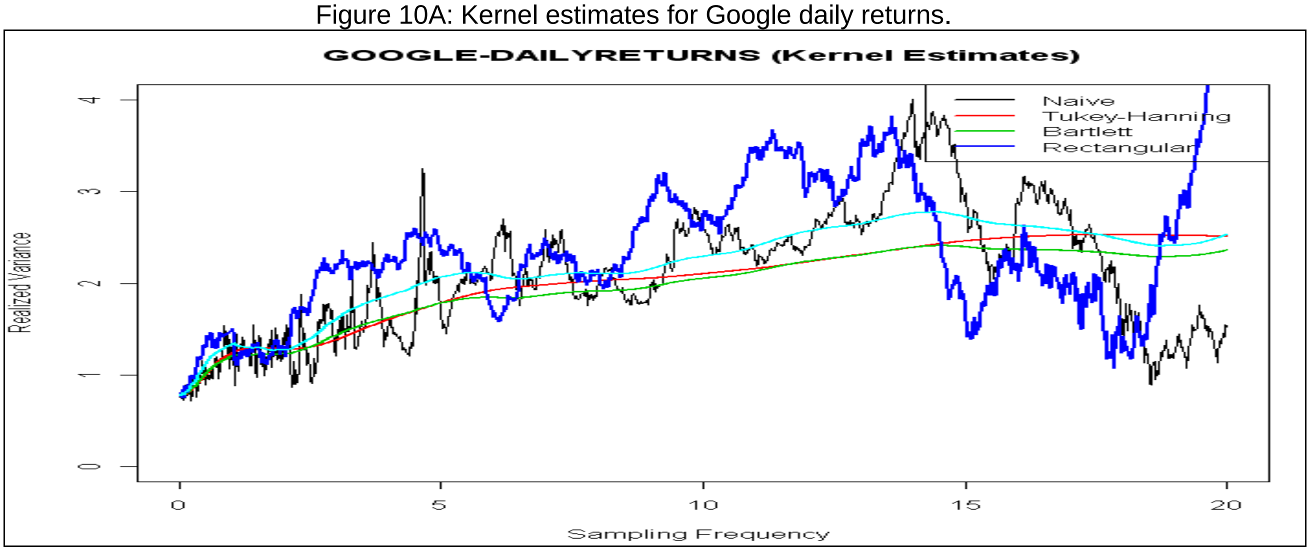

Figure 11A: sub sampling estimates for Google daily returns.

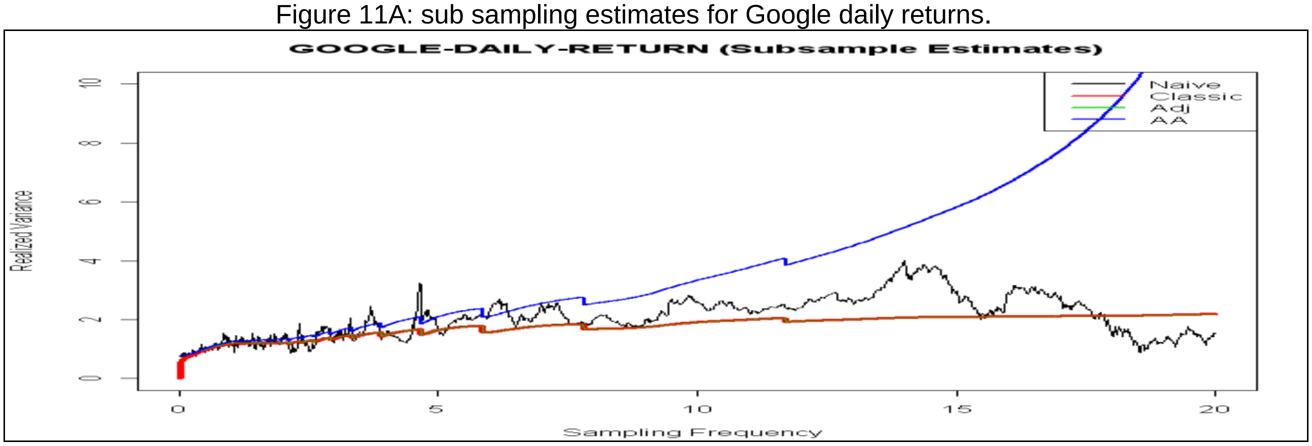

Figure 12A: Kernel estimates for IBM daily returns.

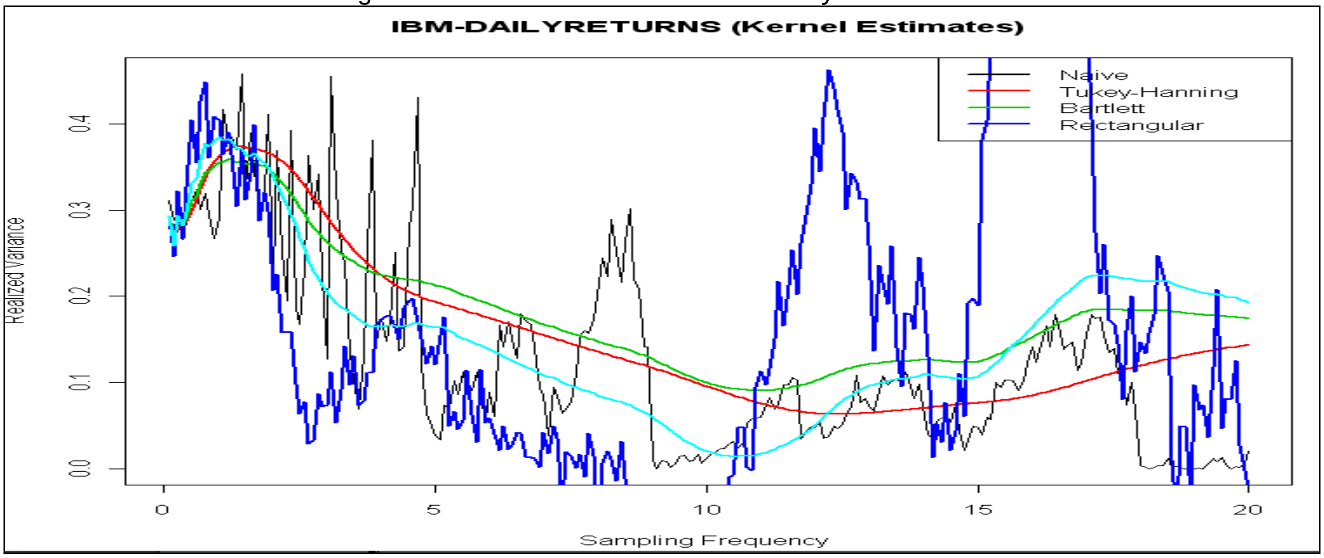

Figure 13A: Sub sampling estimates for Google daily returns.

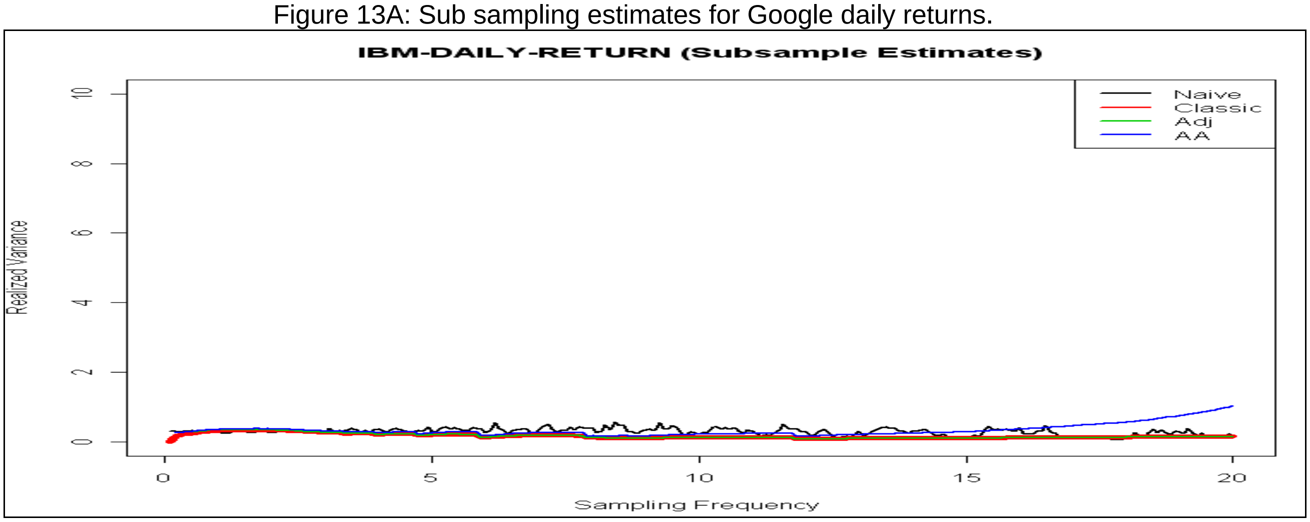

Figure 14A: Kernel estimates for Microsoft Daily returns.

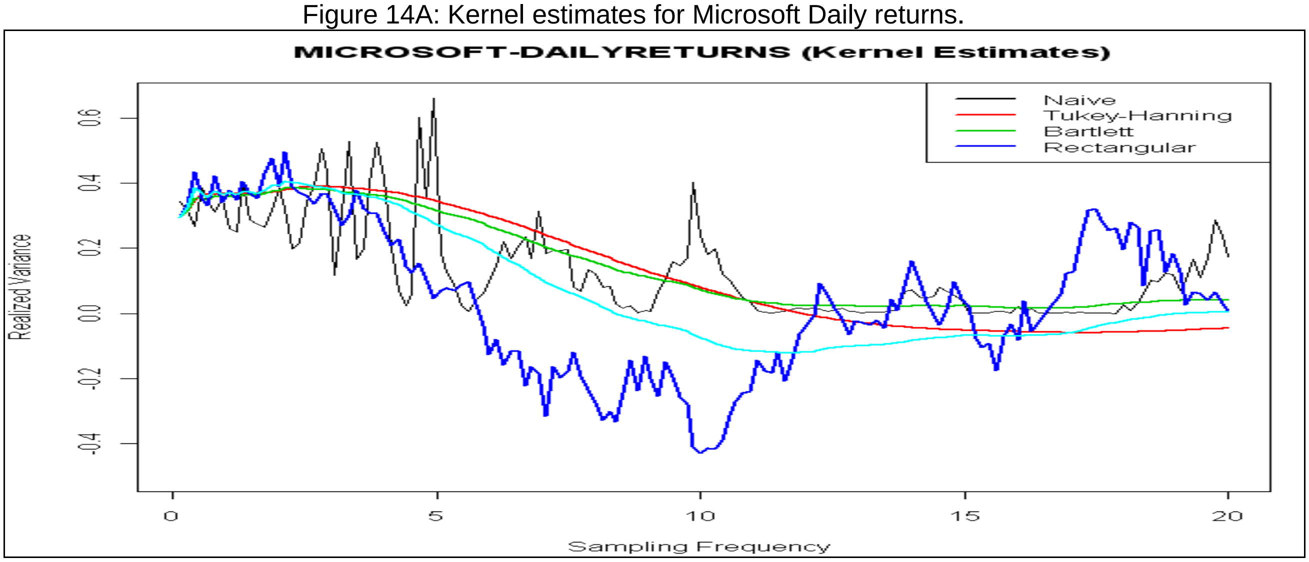

Figure 15A: Sub sampling estimates for Microsoft daily returns.

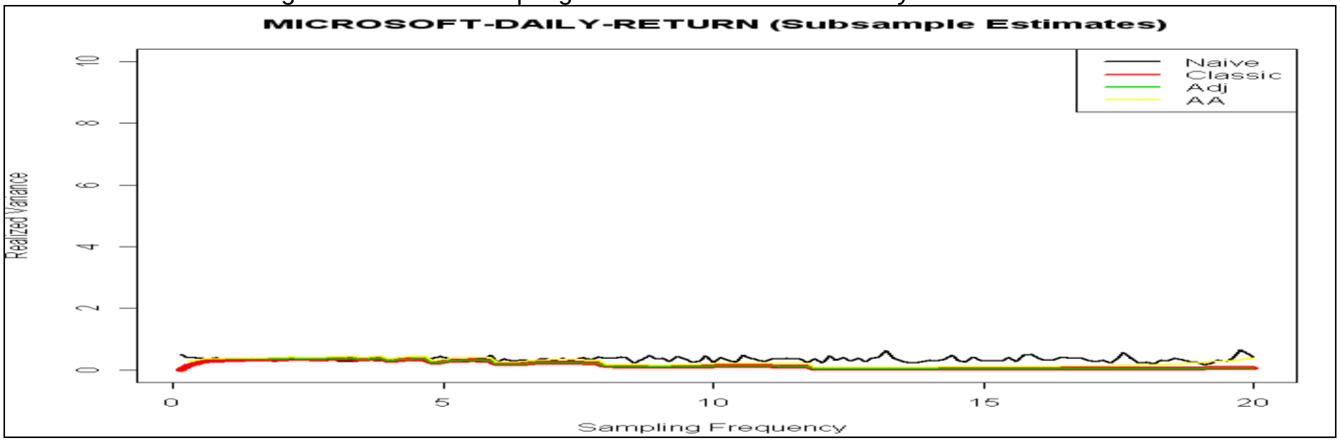

Figure 16A: Kernel estimates for Dell High Frequency data.

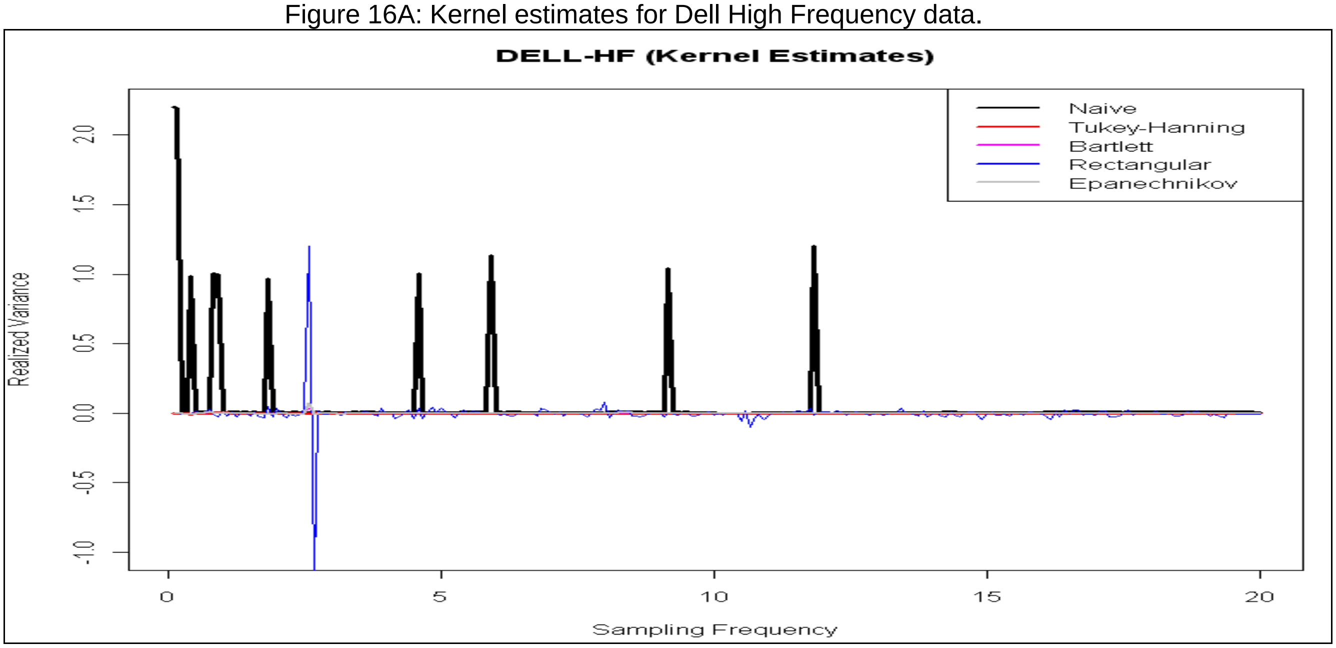

Figure 17A: Sub sampling estimates for Dell High Frequency data

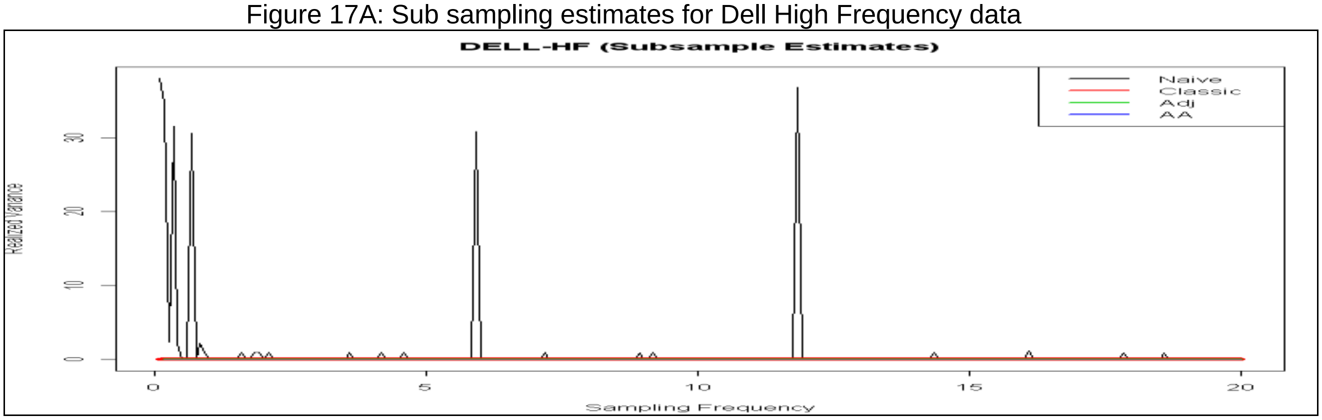

Figure 18A: Kernel estimates for Google High frequency data.

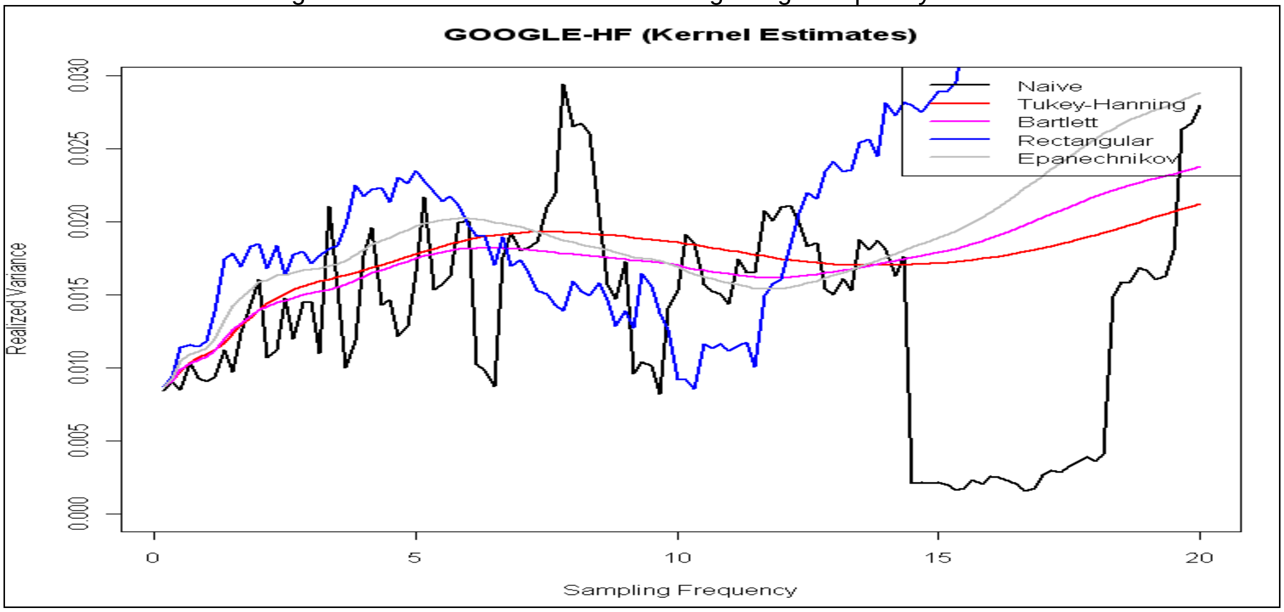

Figure 19A: Sub sampling estimates for Google High frequency data.

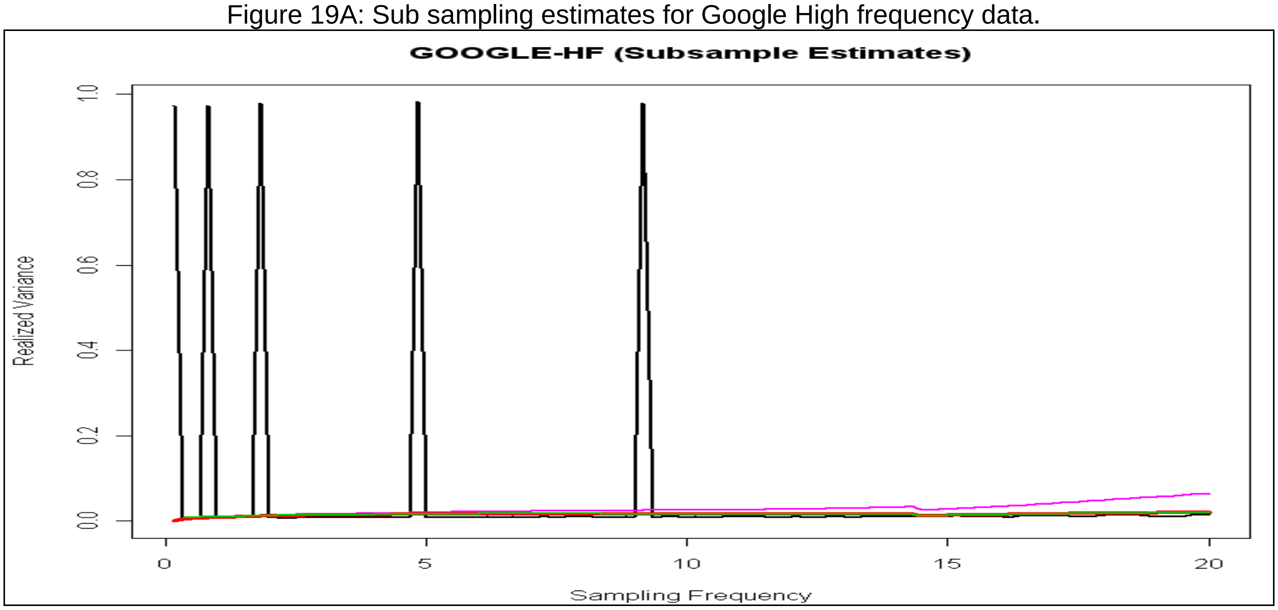

Figure 20A: Kernel estimates for IBM High frequency data.

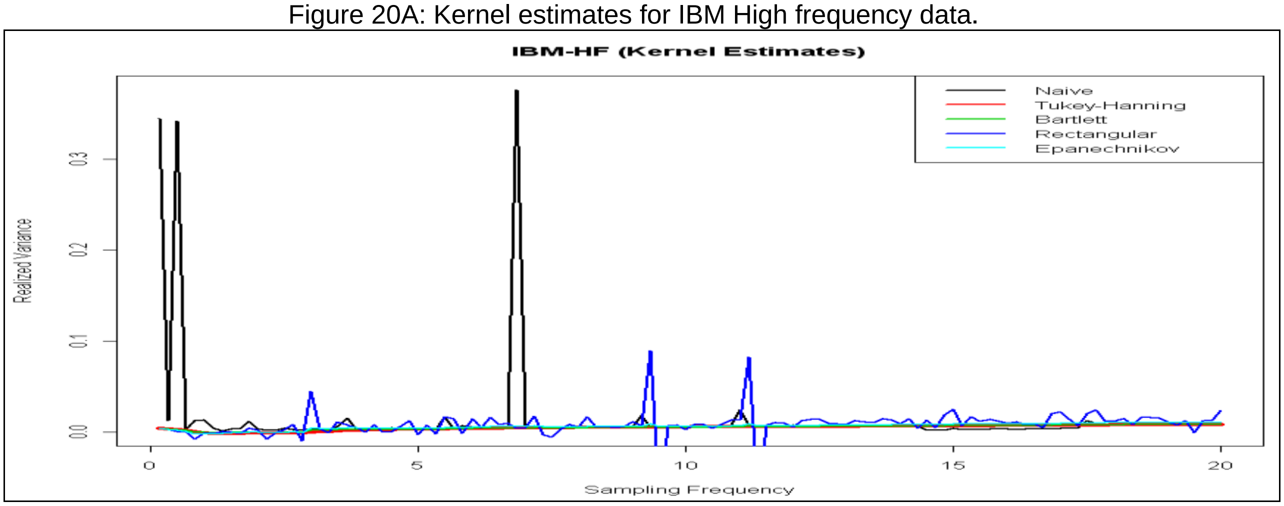

Figure 21A: Sub sampling estimates for IBM high frequency data.

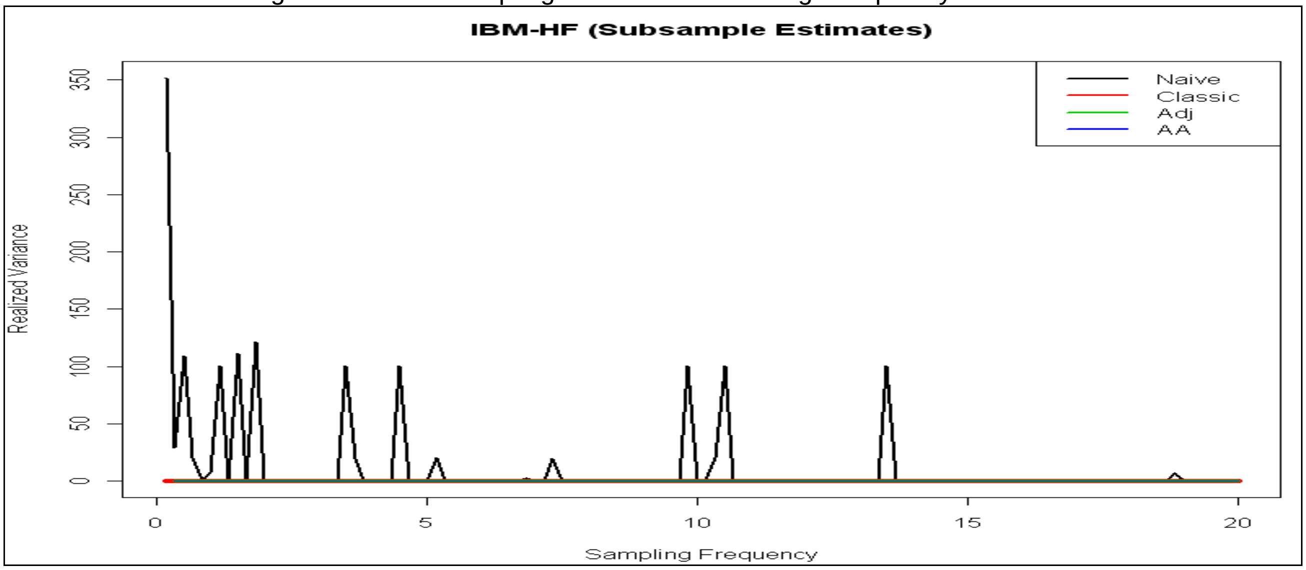

Figure 22A: Kernel estimates for Microsoft high frequency data.

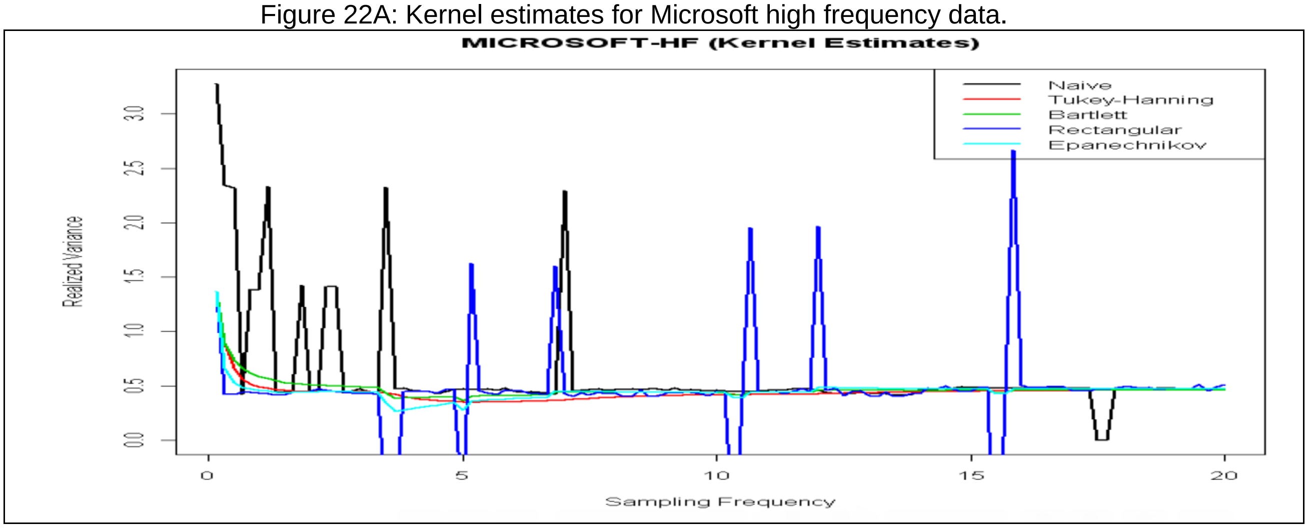

Figure 23A: sub sampling estimates for Microsoft High frequency data.